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BROWNIAN MOTION AS A MECHANISM FOR PARTICLE PRODUCTION*

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Abstract: An application of the theory of Brownian motion to particle-production problems in hadron physics is attempted. Simple examples of one-dimensional random walk in rapidity space and the Langevin-type equation for a pion density field are analyzed.

1. Introduction

Studies of Brownian motion[1] are principally concerned with random motion exhibited by a physical quantity $\psi(z)$. $\psi(z)$ is a process in which the variable ψ (displacement velocity, fluctuating voltage, pion field, etc.) does not depend in a completely definite way on the independent variable z, as in a causal process. In the case of a random process, functions $\psi(z)$ are different in different observations therefore only certain probability distributions are directly observable. In fact, a random process $\psi(z)$ is completely described (or defined) by the following set of joint probability distributions

$$W_1(\psi z),\ W_2(\psi_1 z_1; \psi_2 z_2), \dots . \tag{1}$$

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2. *Classical example*

Consider the motion of a classical charged particle which is unobserved. The motion of such a particle is recorded only by means of radiation the charged particle emits during its motion.

According to the classical theory, the charged particle will emit radiation only if it is accelerated. If this happens by some unknown causes, probably through random collisions with other particles of the medium, the charged particle will perform random motion (Brownian motion) inside the medium.

Can we say something about the angular distribution of the emitted radiation starting from a standard theory of Brownian motion?

3. *Emission of particles in hadron physics*

Similar reasoning can be extended to production processes in hadron physics, where the motion of an unobserved underlying field (a quark field, for example) is considered inside a hadron medium. A consequence of the random walk of the underlying field is the emission of real particles such as pions, kaons, etc. The underlying field is put into random motion by means of energy gained during high-energy collisions of two hadrons.

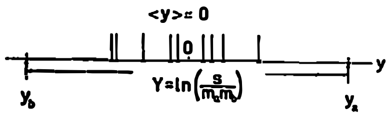

Fig. 1. A typical event population of the rapidity space.

4. *One-dimensional random walk in rapidity space*

High-energy collisions are usually analyzed[2)] by resolving a particle's momentum $\vec{q}$ into components transverse to the incident beam, $\vec{q}_L$, and a component along the incident beam, q_L. The rapidity is then defined by

$$Y = \frac{1}{2} \ln \frac{E_q + q_L}{E_q - q_L}. \tag{2}$$

A possible c. m. rapidity configuration in an n-particle inclusive reaction

$$a + b \rightarrow 1, 2, \ldots, n + \text{anything}$$

is shown in Fig. 1. Suppose now that this configuration is achieved by random walk along the rapidity line y (Fig. 1).

A particle is emitted after each displacement along the straight line y. The length of the average displacement is $\overline{\Delta y}$. Each step is taken either in the forward or in the backward direction with probabilities ω_f and ω_b such that

$$\omega_f + \omega_b = 1. \tag{3}$$

$\omega_f \neq \omega_b$ is expected if colliding particles are of different type.

After taking n such steps, the particle can be emitted from any point between $-n\,(\overline{\Delta y})$ and $n\,(\overline{\Delta y})$.

We may now require to find the probability $W(n,m)$ that the particle be emitted at the point $y = m\,(\overline{\Delta y})$ after taking n steps. The required probability is easy to find and is in fact the Bernoullian distribution

$$W(n,m) = \frac{n!}{\left(\frac{1}{2}(n+m)\right)!\left(\frac{1}{2}(n-m)\right)!}\,\omega_f^{\frac{1}{2}(n+m)}\,\omega_b^{\frac{1}{2}(n-m)}. \tag{4}$$

Consider a particular case of pp scattering. Here $\omega_f = \omega_b = \frac{1}{2}$ and (4) reduces to

$$W(n,m) = \frac{n!}{\left(\frac{1}{2}(n+m)\right)!\left(\frac{1}{2}(n-m)\right)!}\left(\frac{1}{2}\right)^n. \tag{5}$$

From (5) it follows that

$$\langle y\rangle = 0 \text{ and } \langle y^2\rangle = n\,(\overline{\Delta y})^2. \tag{6}$$

It is of interest to take n very large and $m \ll n$. In this case the probability can be expressed by the asymptotic formula

$$W(n,m) = \left(\frac{2}{n\pi}\right)^{1/2} \exp\left(-m^2/2n\right). \tag{7}$$

When n is large, it is convenient to introduce the net displacement y from the origin and write

$$W(n,m) = \frac{1}{2\,(\pi D)^{1/2}} \exp\left(-y^2/4D\right), \tag{8}$$

where

$$D = \frac{1}{2}\,n\,(\overline{\Delta y})^2.$$

We recognize the Gaussian distribution in (8). Note that in (8) the variable y is restricted to the region $y \varepsilon (-\infty, \infty)$.

If the net displacement y is restricted to the region $y \varepsilon (-Y/2, Y/2)$, $Y = |y_a - y_b|$, then the normalized distribution becomes

$$W(n, m) = \frac{1}{\text{Norm}} \exp(-y^2/4D), \tag{9}$$

where

$$\text{Norm} = 2(\pi D)^{1/2} \Phi(Y/4\sqrt{D}) \tag{10}$$

and

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt; \quad \Phi(x) \to 1 \text{ as } x \to \infty.$$

As a consequence of this restriction, the average number of emitted particles is

$$\langle n \rangle \simeq \frac{1}{2|\overline{\Delta y}|} y. \tag{11}$$

5. Random processes in hadron physics

Let us consider a process $a + b \to 1, 2, 3, \ldots, n$. The n-particle contribution to the unitarity equation[3] is

$$A_n(Y, \vec{p}) = \frac{1}{4s} \int d^2 B \, e^{i\vec{p}\vec{B}} \prod_{i=1}^{n} dz_i \, | T_n(Y, \vec{B}; \{z_i\} |^2, \tag{12}$$

where

$$z_i \equiv (\vec{q}_{iT}, y_i); \quad dz_i = \frac{d^2 q_{iT} dy_i}{(2\pi)^2 4\pi}$$

and $\vec{B}$ is the relative impact parameter between particles a and b.

Normalization is such that

$$A_n(Y, 0) = s\sigma_n(Y)$$

and (13)

$$\sum_{n=1}^{\infty} \sigma_n(Y) = \sigma_{\text{incl}}(Y).$$

Let us write

$$| T_n(Y, \vec{B}; \{z_i\}) |^2 = \Pi_n(z_1 z_2 \ldots z_n). \tag{14}$$

For simplicity, the dependence on the variables $(Y, \vec{B})$ is suppressed.

Consider now a set of real-valued functions $\psi(z)$ integrable in a certain domain Ω. Given a functional $F\{\psi\}$, an average of $F\{\psi\}$ over all ψ will be denoted by

$$\langle F\{\psi\}\rangle. \tag{15}$$

The characteristic functional

$$G\{\xi\}_\psi = \langle \exp i \int \xi(z)\psi(z)\,\mathrm{d}z \rangle \tag{16}$$

is defined in the space of all square integrable functions $\xi(z)$. The n-point correlation function

$$\langle \psi(z_1)\ldots\psi(z_n)\rangle, \tag{17}$$

which is obtained from

$$\left.\frac{\delta^n G\{\xi\}}{\delta\xi(z_1)\ldots\delta\xi(z_n)}\right|_{\xi=0} = i^n \langle \psi(z_1)\ldots\psi(z_n)\rangle, \tag{18}$$

is identified with

$$\Pi_n(z_l\ldots z_n) = \langle \psi(z_1)\ldots\psi(z_n)\rangle. \tag{19}$$

The field $\psi(z)$ thus plays the role of a fundamental underlying density field which is treated as a random variable. If

$$\langle \psi(z)\rangle = \Pi(z) \neq 0, \tag{20}$$

we can introduce a new field

$$\Phi(z) = \psi(z) - \langle \psi(z)\rangle, \tag{21}$$

so that

$$\langle \Phi(z)\rangle = 0. \tag{22}$$

Notice that

$$G\{\xi\}_\psi = \exp\left(i \int \xi(z)\langle \psi(z)\rangle\right)\, G\{\xi\}_\Phi. \tag{23}$$

The random field Φ is Gaussian if its correlation functions satisfy the decomposition property[4)]

$$\langle \Phi(z_1)\ldots\Phi(z_{2l+1})\rangle = 0, \quad l = 0, 1, 2, \ldots,$$

$$\langle \Phi(z_1)\ldots\Phi(z_{2l})\rangle = \sum_{\substack{\text{pairs}\\(ij)}} \prod_{\alpha=1}^{l} \langle \Phi(z_{i\alpha})\Phi(z_{j\alpha})\rangle, \tag{24}$$

$$l = 1, 2, \ldots .$$

In this case

$$G\{\xi\}_{\psi} = \exp\left(i \int \xi(z)\, \Pi(z)\, \mathrm{d}z\right) \exp\left(-\frac{1}{2}\int \mathrm{d}z\, \mathrm{d}z'\, \xi(z)\, C(z, z')\, \xi(z')\right), \tag{25}$$

where

$$C(z, z') = \Pi(z, z') - \Pi(z)\, \Pi(z'). \tag{26}$$

It is interesting to note that the condition $C(z, z') = 0$ leads to the AABS unitary model[5].

So far we have done nothing more than state the consequences of treating $\Phi(z)$ as a Gaussian random variable. The real physics enters the problem when we specify either the form of $\langle \Phi(z)\, \Phi(z') \rangle$ or the form of the Langevin equation for $\Phi(z)$.

6. *Langevin equation*

Let us start from the simplest Langevin equation of motion for $\Phi(z)$, suggested by the classical one-dimensional Brownian motion,

$$\partial_z \Phi = -\gamma \Phi + N(z), \quad \gamma > 0. \tag{27}$$

Here $N(z)$ plays the role of a randomly fluctuating noise »force« with the following properties:

a) $N(z)$ has a white noise spectrum, i. e., its correlation time is zero,

$$\langle N(z) \rangle = 0,$$

$$\langle N(z)\, N(z') \rangle = c\, \delta(z - z'); \tag{28}$$

b) $N(z)$ is a stationary Gaussian process. The solution of (27) can be written in the integral form as

$$\Phi(z) = \int_{-\infty}^{z} \mathrm{e}^{-\gamma(z-z')}\, N(z')\, a\, z'. \tag{29}$$

The correlation function is easy to find

$$\langle \Phi(z)\, \Phi(z') \rangle = \frac{c}{2\gamma} \exp(-\gamma\, | z - z' |). \tag{30}$$

It is interesting to note that we can also determine the differential equation satisfied by the correlation function itself, which is

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2} - \gamma^2\right) \langle \Phi(z)\, \Phi(z') \rangle = - c\, \delta(z - z'). \tag{31}$$

7. *Integral-equation method*

Let us consider the eigenvalue problem of the integral kernel $\langle \Phi(z)\, \Phi(z') \rangle$

$$\int_{\Omega} \langle \Phi(z)\, \Phi(z') \rangle\, u_l(z')\, \mathrm{d}z' = \lambda_l\, u_l(z). \tag{32}$$

Since $\langle \Phi(z)\, \Phi(z') \rangle$ is supposed to be positive definite and symmetric, its eigenvalues λ_l are all non-negative. The eigenfunctions $u_l(z)$ are then real and form an orthonormal and complete set. The spectral representation of the random function $\Phi(z)$ is defined by the expansion

$$\Phi(z) = \sum_l \Phi_l\, u_l(z), \tag{33}$$

where $\{\Phi_l\}$ now become a new set of stochastic variables describing the random field $\Phi(z)$. The continuous form of the probability density can then be obtained in the form

$$P\{\Phi\} = N^{-1} \exp\left(-\frac{1}{2} \int_{\Omega} \mathrm{d}z\, \mathrm{d}z'\, \Phi(z) \langle \Phi(z)\, \Phi(z') \rangle^{-1} \Phi(z')\right), \tag{34}$$

where N is defined in such a way that[4)]

$$\int \mathrm{d}\{\Phi\}\, P\{\Phi\} = 1. \tag{35}$$

Note that

$$\langle \Phi(z)\, \Phi(z') \rangle^{-1} = \sum \frac{1}{\lambda_l} u_l(z)\, u_l(z'). \tag{36}$$

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BROWNOVO GIBANJE KAO MEHANIZAM PRODUKCIJE ČESTICA

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Sadržaj

U radu je primijenjena teorija Brownova gibanja na probleme produkcije čestica u hadronskoj fizici. Analizirani su jednostavni slučajevi kao što su jednodimenzionalno »random« gibanje u prostoru rapiditeta čestica, te Langevinova jednadžba za pionsko polje.