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THE TIME DEPENDENCE OF THE VELOCITY OF ROTATING ARC MOTION

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Abstract: Experimental investigations have shown that the velocity of rotating arc changes with time, and that this variation may affect considerably the properties of the rotationg arc as a spectrochemical source. In the present work theoretical relations are derived describing qualitatively the time dependence of the arc motion velocity, and a comparison is made with experimental data. It was found that the rise time to a steady arc motion depends essentially on the anode temperature and that it is of the order of magnitude of the rise time to reach stationary temperature field in the anode. The effect of individual parameters on the duration of the transient time has been investigated and a possibility of reducing the rise time to the steady state is pointed out. The results obtained suggest an explanation of the nature of friction force between the anode spot and the anode in the case where the arc is driven by electrodynamic forces. All the measurements were performed with a free burning electric arc between graphite electrodes at atmospheric pressure.

1. Introduction

The possibility of using the rotating electric arc in spectrochemistry increased the interest in determination of its dynamic characteristics. The velocity of plasma column and optimum conditions for spectrochemical analyses have been investigated by various experimental methods[1,2]. Most of the relevant measurements were carried out in a stable condition of electric arc motion. It was found that[3,4] there is a transient period during which the plasma velocity varies, and a way was proposed to determine the rise time to the stable condition of the arc motion. To our knowledge, no detailed theoretical and experimental studies of this phenomenon have been carried out by other authors.

2. *Theory of arc motion transient state*

In analyzing the time dependence of rotating arc motion we use a differential equation derived on the basis of MHD considerations. This equation may be written in the form[5)]

$$\frac{df}{dt} + \frac{4B\varrho_n}{3d\varrho}\Delta f^2 + \frac{2c}{\varrho d^2 \pi R} f = \frac{IB}{\varrho d^2 \pi^2 R}, \tag{1}$$

where f is the rotation frequency of the plasma column $f = \frac{\omega}{2\pi}$, R is the anode radius, d is the cylindric arc radius, ϱ is the arc plasma density, ϱ_n is the neutral gas density, c is the coefficient of friction near electrode, I is the arc current intensity, and B is the magnetic induction. The function Δf is of the form $\Delta f = f(1 - e^{-1/f\tau_g})$, where τ_g denotes the time of relaxation for the neutral gas macroscopic velocity. The solution of differential equation (1) should describe the time dependence of rotation frequency, and also the velocity of motion of the rotating arc plasma column. Assuming that only f varies with time, the above equation could be solved by numerical methods. But values of c, I and R also vary with time, so that the above equation can not be solved without knowing the functions $c(t)$, $I(t)$ and $R(t)$. For instance, to determine the radius of a cylindric arc column, $d(t)$, it is necessary to solve the equation of energy balance assuming that the arc radius varies due to the arc cooling as it moves through the surrounding neutral gas. In analyzing the equation (1) we shall assume that d is time independent because of the low relative velocity of the plasma column and of the surrounding neutral gas motion in the case of a rotating arc. We shall also neglect the time variation of the inner radius of cylindric anode caused by the anode material dissipation, because this phenomenon affects considerably the rotation frequency only long time after the arc ignition. This phenomenon will be discussed in the conclusion. We also found that the time dependence of rotating arc frequency is defined by a set of equations and not merely by equation (1).

In further analysis of the time dependence of rotation frequency we suppose, by analogy to relaxation processes, that all processes leading to steady state of individual quantities follow an exponential law. To describe such a variation we introduce the notation of time constants to reach steady-state values of the arc current intensity τ_I, of the coefficient of friction near electrode τ_c and of the plasma column rotation frequency τ_f. The transition to stationary arc current intensity is affected not only by the arc but also by the entire circuit including connections of the instruments and the source, i. e. by the entire circuit topology. The circuit time constant used in our experimental device is much smaller than the rise time to the steady-state rotation frequency, i. e. $\tau_I \ll \tau_f$. In the course of the experiment it was observed that the arc current varies during the rise time to the stable state of the arc motion if the circuit was left to itself. This variation will also be neglected, because the current changes can be compensated.

To determine the time dependence of the coefficient of friction near electrode it is necessary to set a model of the anode spot motion along the edge of a cylindrical anode. Our experiments, as well as those of other autors[6,7], show that at low velocities the anode spot motion is discontinuous and that the spot jumps to certain points. For the sake of an easier theoretical description we shall assume that the anode spot motion is continuous, which may be justified only if the entire anode area over which the spot moves was heated to a temperature which does not differ considerably from the anode material (graphite) sublimation point. The problem of the time dependence of the coefficient of friction near electrode will be considered later. We suppose that the friction coefficient is constant in the time, which is equivalent to the assumption of transition to the stable state of arc motion being a function of the plasma column inertial characteristics only, and that it is independent of friction changes near electrode. With the assumptions quoted the equation (1) can be solved analytically. Since we are interested only in the order of magnitude of the time constant to reach the stable state of arc motion due to the plasma column properties, τ_{in}, we solve the equation only for the case where $f\,\tau_g > 1$. In that case the function f can be expanded in a series, in which we retain the first two terms so that equation (1) takes the form ($\varrho_n = \varrho$)

$$\frac{\mathrm{d}f}{\mathrm{d}t} + \frac{4R}{3\,d\tau_g^2} + \frac{2c}{\varrho d^2 \pi R} f = \frac{I\,B}{\varrho d^2 \pi^2 R}. \tag{2}$$

Hence the time dependence of rotation frequency for that case is given by the relation

$$f = f_\infty \left(1 - \mathrm{e}^{-t/\tau_{in}}\right), \tag{3}$$

where

$$f_\infty = \frac{3\,I\,B\,\tau_g^2 - 4\,R^2\,\pi^2\,\varrho\,d}{6\,c\,\pi\,\tau_g^2}, \quad \tau_{in} = \frac{\varrho\,d^2\,\pi\,R}{2\,c}.$$

Using numerical values corresponding to the experimental conditions it is found that τ_{in} is of the order of 10^{-3} s. As shown in the work[8] the constant c in the case of graphite electrodes has a value of 10^{-6} kg/s. The constant indicating the time required to reach stable arc motion calculated in such a way is in disagreement with the experimental value which is of the order of 1 s. Hence it can be concluded that $\tau_f \gg \tau_{in}$, i. e. that the time dependence of the arc rotation frequency is not considerably affected by the inertial properties of the arc.

3. *Effect of the anode temperature*

Experimental measurements of the time dependence of the velocity of rotating arc motion show that the rise time to the stationary state is of the order of 10 s. The processes considered so far which may affect the rise time to the stationary state lead to the duration of the transition time of the order of 10^{-3} s. We shall therefore consider also the influence of the anode on the transition to the stable arc motion and estimate the order of magnitude of the time required for establish a stationary temperature field in the anode.

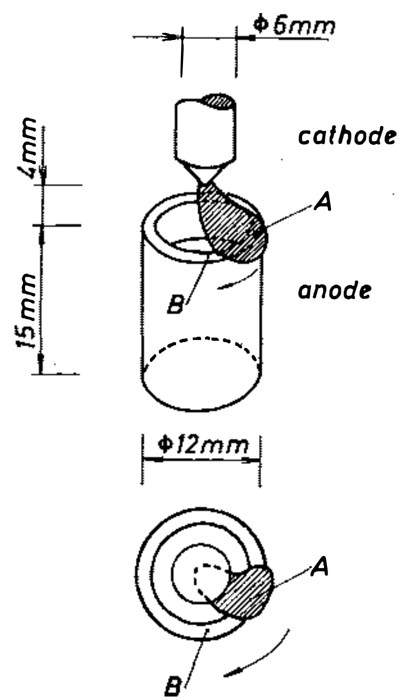

Fig. 1. The electrode system with a schematical view of the arc rotating along the anode edge from point A to point B.

Let us consider how the friction coefficient (c), which was introduced as a quantity characterizing the effect of the arc-anode interaction on rotating arc motion, may be changed. We shall make the assumption that the probability of formation of anode spot at a certain point of the anode depends on the anode temperature at this point. This assumption is justified, because the anode spot can be considered as a source of charged particles. Anode heating gives rise to thermoionic emission, described by Dushman's equation. Since the emission current intensity directly depends on anode temperature, it can be assumed that the arc would be more readily formed if the anode temperature at the point observed is higher. This statement may be illustrated by an example. If we observe an arc burning between a cathode located on the axis of the electrode system and a cylindrical anode, we find that the anode spot will statistically move in two opposite directions. This displacement can be hindered if the dissipation of heat from the anode in a direction normal to the anode wall is considerably increased, so that the anode edge temperature in the possible directions of the anode spot motion is lowered. In such a case the anode spot remains at the locus where it was formed, since it can not come out of the »temperature well«.

Fig. 1 shows the shape of the anode along the edge of which the arc moves. Let us assume that under the action of the Lorentz force the arc moves in the direction from point A to point B. The higher the temperature at point B, the more readily the arc will pass from point A to point B. If the possibility of passing from point A to point B, or the resistance to this passage, is conceived as a »friction force«, then this friction force will be inversely proportional to the anode temperature in front of the arc in the direction of its motion. In order to determine qualitatively the time variation of the friction force between the anode spot and the anode we shall examine the variation of the temperature of a cylindric anode along the edge of which the anode spot moves. If the arc is formed at point A the electrode temperature at this point will be approximately equal to the graphite sublimation temperature. If the arc immediately after its formation begins to move in the direction indicated in Fig. 1, then the temperature at point B may be considered to be approximately equal to room temperature. Of course, this also holds in the case when the velocity of the anode spot motion is higher than the velocity of propagation of the temperature field through the anode. Assuming that the time of relaxation of the electrode temperature, τ_E, is longer than the period of arc revolution $\tau_E > T$, when the arc comes again to point A the temperature at the latter will be higher than room temperature, hence the value of the coefficient of friction near the electrode will be lower during the second period. In subsequent comings of the spot to point A a further increase in temperature and decrease in the coefficient of friction near electrode will occur. This process will proceed until a stationary temperature field is established in the anode. Such a picture of the process points out the possibility of discontinuous change of the coefficient of friction near electrode during the rise time to the stable state of electric arc motion. The friction coefficient can be considered to be invariable within one period of revolution. Since in our experiment the period of revolution of the arc is of the same order of magnitude as the rise time to its stable motion due to inertial properties, we conclude that in the course of each period a new steady-

-state is established. At the end of each period the rotation frequency of the arc is equal to the steady-state frequency for the value of the friction coefficient in the last period. In the course of one period the variation of the velocity of anode spot motion is described by Equ. (3) and follows an exponential law. From such a consideration it may be concluded that the time variation of the arc rotation frequency is related to the time variation of the coefficient of friction near electrode, i.e. that the rise time to the stable state of arc motion is of the order of magnitude of the rise time to steady electrode temperature. A mathematical interpretation

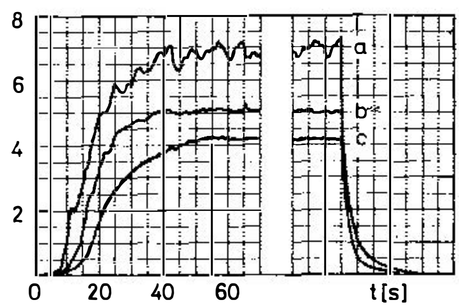

Fig. 2. The time variation of luminosity from a narrow zone of the anode at 3 mm below its edge.

of such conception consists in eliminating the term df/dt from Equ. (1) in the case where the variation of frequency is considered within a time interval much longer than the period of revolution of the electric arc. Eliminating the term df/dt we are no more able to describe the actual time dependence of the angular velocity of rotating arc motion, but only the variation of the rotation frequency averaged over a time interval of the order of magnitude of the rise time to the stable state of motion of the arc due to its inertial properties. In the case where $f \cdot \tau_g > 1$ the time dependence of the rotation frequency of the plasma column is given by the relation

$$f = \frac{3\, IB\, \tau_g^2 - 4\,\pi^2 R^2\, \varrho\, d}{6\, c\,(t)\, \pi\, \tau_g^2} \tag{4}$$

In order to determine qualitatively the form of the function $f(t)$, taking into account the decrease of the coefficient of friction near electrode as the electrode temperature rises, we shall write $c = \frac{A}{T(t)}$ (where A is the coefficient of proportionality) and give the following boundary conditions for the anode temperature: $T(0) = T_0$ and $T(\infty) = T_s$, where T_0 and T_s are the anode temperature before arc ignition and the sublimation point of the anode material respectively. For the time dependence of the anode temperature we may write the following equation which satisfies the boundary conditions

$$T(t) = T_s + (T_0 - T_s)\,\mathrm{e}^{-t/\tau_E} = T_s\left[1 - \left(1 - \frac{T_0}{T_s}\right) e^{-t/\tau_E}\right]. \tag{5}$$

If $T_0/T_s \ll 1$ we may write

$$T(t) = T_s(1 - \mathrm{e}^{-t/\tau_E}) \tag{6}$$

and for the time dependence of the coefficient of friction near electrode

$$c(t) = \frac{c_\infty}{1 - \mathrm{e}^{-t/\tau_c}}, \tag{7}$$

where $c_\infty = \frac{A}{T_s}$. Substituting Equ. (7) into expression (4) we obtain a function which describes qualitatively the transition to the stable state of electric arc motion in the form

$$f(t) = f_\infty(1 - \mathrm{e}^{-t/\tau_c}), \tag{8}$$

where it is assumed that $\tau_E = \tau_c \cong \tau_f$. Instead of $c \to \infty$ at $t = 0$, which follows from Equ. (7), it is more realistic to assume that $c \to C_0 = \frac{A}{T_0}$ at $t = 0$. The condition $f\tau_g > 1$, however, is valid for frequencies of the order of 10^3 Hz which are attained only after a definite time interval from the moment of electric arc ignition. Equ. (8) does not describe adequately the change in frequency immediately after the arc ignition, irrespective of the boundary condition quoted. In our previous paper[4] a way to determine the time dependence of rotation frequency by using Equ. (8) was proposed and its agreement with experimental values was checked. Namely, the apparatus was designed to measure the quantity $\Omega(t_2) = \int_{t_1}^{t_2} f(t)\,\mathrm{d}t$, where the time interval $t_2 - t_1$ is constant and equal to $\Delta t = 1$ s. It is evident that the boundary condition $\lim \Omega(t_2) = \overline{f}_2\,\Delta t$, $t_2 \to \infty$ is satisfied, where $\overline{f} = f_\infty$, i. e. for $\Delta t = 1$ s the quantity $\Omega(t_2)$ is numerically equal to the rotation frequency of the plasma column in steady-state. By plotting the curve $\Omega = F(t_2)$ versus t_2 it is possible to determine the value of the time constant τ_f.

4. *Experimental results*

To check experimentally the derived relations the following two methods were used: the electrode luminosity measurement, and measurement of $\Omega(t_2)$ with the device described in[3]. An accurate measurement of the anode edge temperature was not carried out, and we have only studied qualitatively the time variation of the temperature and estimated the rise time to stationary anode temperature field. By using simultaneously the two methods it would be possible to compare the time variation of temperature with that of rotation frequency.

Fig. 2 shows the time variation of electrode luminosity at 3 mm below the upper edge of the electrode. The electrode image was projected by two mirrors perpendiculary to the slit of a monochromator. The recording was carried out at a wavelength of 7650 Å with non-cooled electrodes:

in the absence of magnetic field, Fig. 2a,
— in a magnetic field, Fig. 2b, and
— with a water-cooled electrode in a magnetic field Fig. 2c.

The results show that the rise time to a stationary temperature field in non-cooled electrodes is shorter than in the case of water-cooled electrode, irrespective of magnetic field applied. Fluctuations observed in the absence of magnetic field are due to random spot motion along the anode edge, although the recording was carried out at 3 mm below the anode edge. After arc extinction the luminosity of water-cooled electrode decreases more rapidly than that of non-cooled electrodes. Obviously, the rise time to stationary luminosity refers to the electrode cross-section at 3 mm below the upper edge of the electrode, but it is nevertheless of the order of the rise time to a steady arc motion. According to the assumption made it is natural to draw the following conclusions. If all the parameters of rotating arc remain unchanged and only the regime of cooling the anode is changed than the consequences of such a change should lead to the following changes in the plasma column:

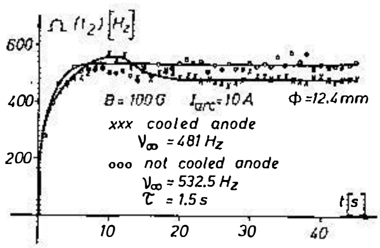

Fig. 3. The time variation of the quantity Ω (t_2) for water-cooled anode and for non--cooled anode.

— since the stationary temperature field in the anode is established more rapidly when no cooling is applied, it should be expected that the rise time to stable motion of the rotating arc will be shorter;
— when the stationary temperature field is established in the anode, the anode temperature at a point through the anode spot passes changes periodically in the time with a period equal to that of the electric arc revolution. When the anode spot passes through point A its temperature may be considered to be as high as the graphite sublimation point. As the spot goes avay from point A the temperature of it decreases due to heat dissipation by conduction and radiation. When the spot comes again to point A the temperature suddenly rises to the graphite sublimation point. We shall assume that the radiation heat loss is not changed considerably when the regime of cooling by water of the electrode is changed. Because of decrease in heat dissipation by conduction in the case where no cooling is applied, it is to be expected that the anode temperature at a certain point will decrease more slowly than in the case of water-

-cooled electrode. In other words, at the end of one period of revolution the anode temperature at a point immediately in front of the anode spot in the direction of arc motion will be higher if no cooling is applied, hence the coefficient of friction near the electrode will be lower. Consequently it is to be expected that the rotation frequency of an electric arc in steady-state will be higher when no cooling is applied.

Fig. 3 shows the time variation of $\Omega(t_2)$ in the case of water-cooled and non-cooled anodes. The experiment confirms the validity of the assumptions on the change of the arc rotation frequency due to the change of the anode temperature at a point immediately in front of the arc.

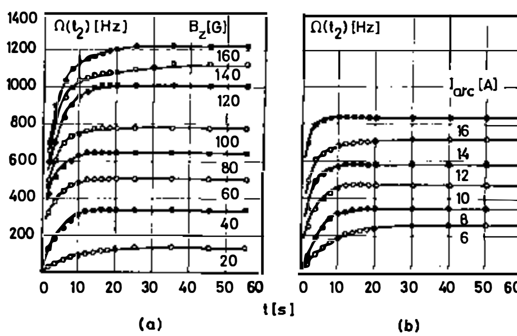

Fig. 4. The time variation of the quantity $\Omega(t_2)$ for anode diameter of 10 mm (b) at arc currents ranging from 6 to 16 A in a constant magnetic field of $B_z = 65$ G, and (a) in magnetic fields ranging from 20 to 160 G at a constant arc current of 12 A.

The appearance of the peak on the curve reffering to cooled electrodes is not fully elucidated. It has been found experimentally that the quantity R in the relation for the steady-state plasma column rotation frequency corresponds to the inner radius of the cylindric anode. The anode material dissipation leads to an increase of its inner radius, which causes a lowering of the rotation frequency. When no cooling is applied the dissipation rate is higher, i. e. the inner radius increases more rapidly and attains its maximum value. If the anode is cooled, R changes more slowly hence the rotation frequency is lowered only after the steady-state has been established, when R reaches its maximum value. However we could not find experimentally differences in the rate of change of the inner diameter of the cooled and non-cooled anodes. The peak on the curve referring to cooled electrodes might arise from the difference in the way of anode spot motion. Namely, the higher the anode edge temperature, the greater the possibility of discontinuous arc motion. In the work[6] it was shown that in the case of discon-continuous motion of anode spot the dependence of the rotation frequency on the product IB is more pronounced than in other cases.

TABLE

B_z = 65 G	I_{arc} [A]	6	8	10	12	14	16		
	τ_f [s]	6.1	4.2	3.2	2.4	2.5	1.4		
I_{arc} = 12 A	B_z [G]	20	40	60	80	100	120	140	160
	τ_f [s]	5	4.1	4.5	1.4	3.5	3	3	3.3

The time dependence of $\Omega(t_2)$ in the case of varying electric current intensity and magnetic induction strength of the external magnetic field is shown in Fig. 4, while the values of time constant for the corresponding curves are given in the Table.

5. *Conclusions*

In describing the time variation of the velocity of rotating arc motion it is necessary to take into consideration the effect of individual parameters on the rise time to the stable state of arc motion. Assuming that the time variations of all parameters follow an exponential law, one may introduce time constants defining the rate of change of individual parameters. If the quantities τ_I, τ_{in}, τ_c and τ_f are the time constants to reach steady-state values of arc current intensity, of the arc rotation frequency due to inertial properties, of the anode temperature field and of the arc rotation frequency respectively, then they may be connected by the following relation: $\tau_I \ll \tau_{in} \ll \tau_c \cong \tau_f$. Hence it can be stated that the rise time to the stationary motion of a rotating arc is determined by the variation of the anode edge temperature. This conclusion points out the way the rotating arc should be designed to be used as a spectrochemical source, with a transient as short as possible. Since it is of interest to reach optimum rotation frequency in a time interval as short as possible, a good source may be obtanied by maximum reduction of heat losses at the anode surface.

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VREMENSKA ZAVISNOST BRZINE KRETANJA ROTIRAJUĆEG ELEKTRIČNOG LUKA

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Sadržaj

Eksperimentalnim istraživanjem pokazano je da se brzina kretanja rotirajućeg električnog luka menja u toku vremena i da ova pojava može biti od velikog uticaja na osobine rotirajućeg električnog luka kao spektrohemijskog izvora.

Izvedene su relacije koje kvalitativno opisuju promenu brzine kretanja električnog luka tokom vremena i izvršeno je njihovo poređenje sa eksperimentalnim podacima. Utvrđeno je da vreme uspostavljanja stacionarnog stanja kretanja luka bitno zavisi od temperature anode i po redu veličine jednako je sa vremenom uspostavljanja stacionarnog temperaturskog polja u anodi.

Ispitan je uticaj pojedinih parametara na dužinu trajanja prelaznog režima i ukazano na mogućnost smanjenja vremena potrebnog za uspostavljanje stacionarnog stanja.

Dobiveni rezultati ukazuju i na jednu mogućnost objašnjenja prirode sile trenja između podnožja električnog luka i elektrode, pri pokretanju luka elektrodinamičkim silama. Sva merenja vršena su sa lukom koji slobodno gori među grafitnim elektrodama na atmosferskom pritisku.