FIZIKA 8 (1976) 23-28

SPATIAL EVOLUTION OF NONLINEARLY COUPLED ION-ACOUSTIC WAVES

M. ČERČEK and R. TAVZES

University of Ljubljana and Institute J. Stefan, Ljubljana

Received 13 October 1975

Abstract: Investigation of the spatial evolution of nonlinearly coupled ion-acoustic waves propagating along a weakly collisional ECR plasma column in a magnetic field is reported. An ion-acoustic wave of frequency f_1 or two waves of different frequencies f_1 and f_2 are launched by applying a sinusoidal voltage to a grid immersed in the plasma. An axially movable grid is used in determination of the damping and phase velocity of f_1, f_2 and nonlinearly generated $2f_1$ and $f_1 - f_2$ waves. The measured spatial growth and decay are qualitatively in accordance with those obtained by nonlinear fluid theory of ion wave propagation.

1. *Introduction*

In the past years the nonlinear effects in wave propagation in a plasma have been studied by many authors. Some recent works have been published on nonlinear coupling of ion-acoustic waves in a plasma. Ohnuma and Hatta[1] have studied the propagation of ion-acoustic waves with frequencies f_1 and f_2 and nonlinearly coupled waves with frequencies $f_1 - f_2$ and $2f_1$ in a collisional plasma without a magnetic field. Sato[2] has published similar results obtained in a Q machine plasma. Mix et al.[3] have measured coupling coefficients of the second harmonic wave and made quantitative comparisons with theory. Ohnuma[4] has investigated harmonic generation of ion waves propagating perpendicularly to a magnetic field. Finally, Ohnuma et al.[5] have observed an amplitude oscillation of the second harmonic ion-acoustic wave. We have measured spatial evolution of ion-acoustic waves with frequency $f_1 - f_2$[6] and $2f_1$, generated by nonlinear coupling of waves with frequencies f_1 in a weakly collisional plasma in a magnetic field.

2. *Theory*

In order to obtain the wave coupling equations we make use of fluid equations and follow the derivation of Mix et al.[3]. The desired coupled mode equations are then

$$\frac{\mathrm{d}X_1}{\mathrm{d}z} = ik_1 \frac{v_s}{v_s + U_0} X_2(z) X_3(z) - \frac{X_1(z)}{\delta_1}, \tag{1}$$

$$\frac{\mathrm{d}X_2}{\mathrm{d}z} = ik_2 \frac{v_s}{v_s + U_0} X_1(z) X_3^*(z) - \frac{X_2(z)}{\delta_2}, \tag{2}$$

$$\frac{\mathrm{d}X_3}{\mathrm{d}z} = ik_3 \frac{v_s}{v_s + U_0} X_1(z) X_2^*(z) - \frac{X_3(z)}{\delta_3}, \tag{3}$$

where the asterisk denotes a complex conjugate quantity. The normalized density perturbations are written as $X_1(z) = n_1/N_0$ and k_1, v_s and U_0 are wave numbers, ion-acoustic phase velocity and average drift velocity, respectively. We consider δ_s' as empirical, from experiment known quantities.

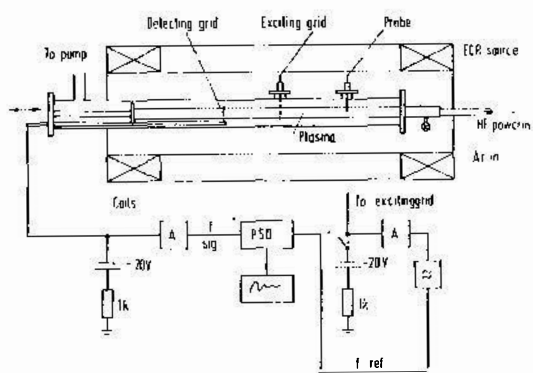

Fig. 1. Block-diagram of the experimental set up.

Assuming that f_1 and f_2 waves are only linearly damped, as confirmed by experiment, we obtain the solutions

$$X_1(z) = X_1(0)\, e^{-z/\delta_1}, \tag{4}$$

$$X_2(z) = X_2(0)\, e^{-z/\delta_2}. \tag{5}$$

The solution for spatial variation of the $f_3 = f_1 - f_2$ wave amplitude is therefore

$$X_3(z) = k_3 \frac{v_s}{v_s + U_0} X_1(0) X_2(0) \left[\frac{1}{\delta_1} + \frac{1}{\delta_2} - \frac{1}{\delta_3}\right]^{-1} \left[e^{-z/\delta_3} - e^{-z\left(\frac{1}{\delta_1} + \frac{1}{\delta_2}\right)}\right]. \tag{6}$$

We have assumed that the amplitude of the nonlinearly generated f_3 wave is zero at the grid ($z = 0$). The amplitude of the wave first increases with z, reaches a maximum and then falls off. We notice also that the amplitude is proportional to the initial amplitudes of the launched waves.

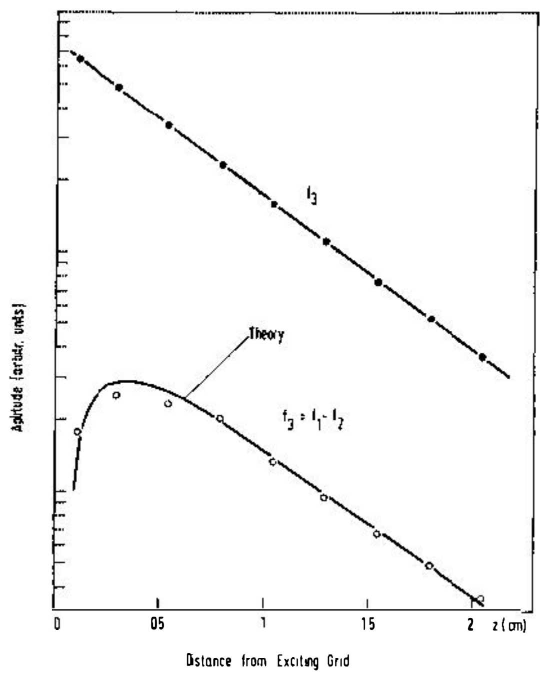

Fig. 2. The amplitude of the $f_1 - f_2$ (0) wave as a function of distance from the exciting grid. The amplitude of the launched $f_3 = f_2 - f_2$ (0) wave is also shown.

Similarly we obtain the solution of the spatial variation of the nonlinearly generated second harmonic $f_2 = 2f_1$ wave amplitude

$$X_2(z) = k_1 \frac{v_s}{v_s + U_0} X_1^2(0) \left[\frac{2}{\delta_1} - \frac{1}{\delta_2}\right]^{-1} [e^{-z/\delta_2} - e^{-2z/\delta_1}]. \tag{7}$$

The spatial evolution of the amplitude is the same as in the case of $f_3 = f_1 - f_2$ wave and it is proportional to the square of initial amplitude of the launched wave f_1.

3. *Experiment and results*

The plasma source was simple ended circular waveguide which continuously coupled power (27 W, 9,3 GHz) into the plasma at ECR. The axis of the waveguide was parallel to the uniform magnetic field of 0.33 T. The plasma column, 70 cm long and 2 cm in diameter, was terminated by a cold plate. Argon gas was continuously leaked through the system at a pressure of 1.8 mtorr. The plasma density and temperature measured with Langmuir probe was $5 \cdot 10^9$ cm^{-3} and 1.7 eV, respectively.

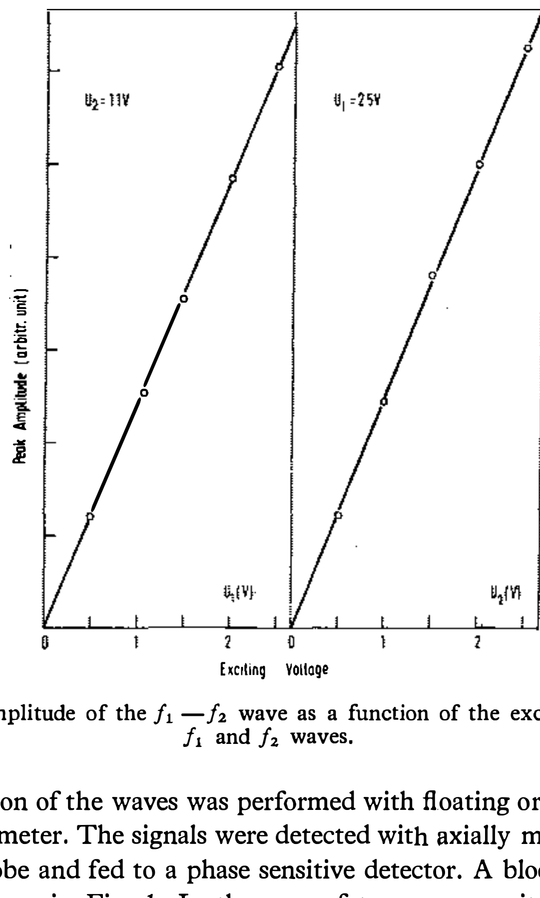

Fig. 3. The peak amplitude of the f_1 —f_2 wave as a function of the exciting voltages of the f_1 and f_2 waves.

The excitation of the waves was performed with floating or negatively biased grid 2.5 cm in diameter. The signals were detected with axially movable, negatively biased grid or probe and fed to a phase sensitive detector. A block diagram of the experiment is shown in Fig. 1. In the case of two wave excitations, two signal generators were used instead of one and in the reference path a mixer was introduced in order to obtain the $f_3 = f_1 - f_2$ and $f_2 = 2f_1$ reference frequencies. The measured phase velocity ($v = v_s + U_0 = 3.9 \cdot 10^3$ m/s) was constant in the frequency range $f = 100$ kHz — 2.1 MHz. It was also independently measured with the time of flight technique. From measured phase velocity, electron temperature and from the fact that there was no wave propagating upstream nor slow wave propagating downstream we concluded that the average drift velocity U_0 was nearly equal to the ion-acoustic velocity ($v_s = 2 \cdot 10^3$ m/s). The ratio of e-folding damping distance to the wavelength was measured as $\delta/\lambda = 0.8$— — 1.4 ($f = 100$ kHz — 2.1 MHz). The damping distance $\delta = 0.9$ cm — 9.3 cm.

In Fig. 2 it is shown that the waves driven by the exciting grid are damped exponentially with distance. In contrast, the nonlinearly generated $f_1 - f_2$ wave grows at first, reaches a maximum amplitude and damps monotonically (Fig. 2). It is

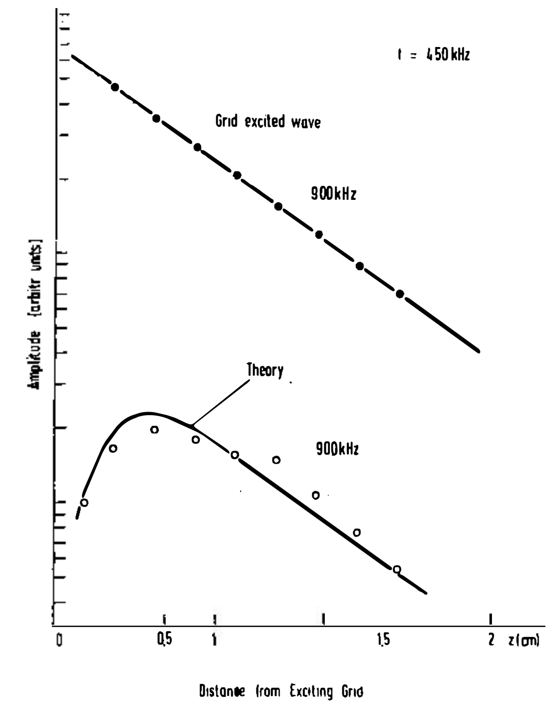

Fig. 4. The amplitude of the $2f_1$ (0) wave as a function of distance from the exciting grid The amplitude of the grid excited wave (●) is also shown.

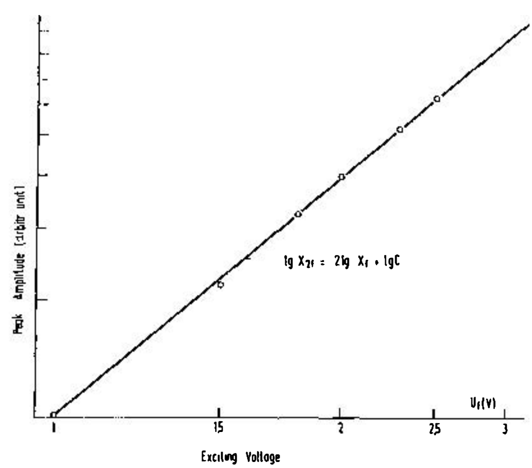

Fig. 5. The peak amplitude of the $2f_1$ wave as a function of the exciting voltage of the f_1 wave.

also confirmed that the amplitude of $f_1 - f_2$ wave is proportional to the initial amplitudes of f_1 and f_2 waves (Fig. 3), assuming that the amplitudes $X_1(0)$ and $X_2(0)$ are proportional to their exciting voltages.

In Fig. 4 the spatial evolution of second harmonic wave is shown. The grid excited wave with frequency $f_2 = 2f_1$ is linearly damped with distance. The nonlinearly generated wave first grows and then damps monotonically. The wave amplitude is proportional to te square of initial amplitude $X_1(0)$ (Fig. 5).

The measured spatial evolutions of nonlinearly generated ion-acoustic waves is seen to be qualitatively in accordance with that obtained from nonlinear fluid theory.

References

1) T. Ohnuma and Y. Hatta, J. Phys. Soc. Jap. **29** (1970) 1597;
2) N. Sato, Phys. Fluids **13** (1970) 2198;
3) L. P. Mix, L. N. Litzenberger and G. Bekefi, Phys. Fluids **15** (1972) 2020;
4) T. Ohnuma, Phys. Fluids **16** (1974, 1774;
5) T. Ohnuma, T. Fujita, S. Ohnuki, Y. Nakamura and S. Adachi, Phys. Rev. **8A** (1973) 1002;
6) M. Čerček and R. Tavzes, Proceedings of the Twelfth International Conference on Phenomena in Ionized Gases (North-Holland, Amsterdam, 1975) 318.

ŠIRJENJE NELINEARNO SKLOPLJENIH IONSKIH ZVOČNIH VALOVANJ

M. ČERČEK in R. TAVZES

Univerza v Ljubljani in Institut »J. Stefan«, Ljubljana

Vsebina

V članku so prikazani rezultati študija razsırjanja nelinearno sklopljenih ionskih zvočnih valovanj v ECR plazmi v magnetnem polju. V plazmi smo vzbudili dvoje ionskih zvočnih valovanj s frekvencama f_1 in f_2 tako, da smo na mrežico, postavljeno prečno na magnetno polje, priključili dvoje sinusnih napetosti z navedenima frekvencama. Zaradi nelinearnega značaja plazme se je pojavilo tretje valovanje s frekvenco $f_1 - f_2$. S pomično sondo vzdolž plazemskega stebra smo merili krajevno odvisnost amplitude tega valovanja.

Študirali smo tudi razširjanje valovanja s frekvenco $2f_1$, kadar smo z mrežico vzbudili le valovanje s frekvenco f_1. Izmerjeni rezultati se kvalitativno lepo ujemajo z napovedmi nelinearne perturbacijske teorije valovanja v plazmi.