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D. Jeni Seles Martina & G. Deepa

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RESEARCH ARTICLE

The energy of multi-valued neutrosophic matrix and neutrosophic hesitant matrix and relationship between them in multi-criteria decision-making

D. Jeni Seles Martina and G. Deepa

Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore, Tamil Nadu, India

ABSTRACT

The concept of energy in graphs and matrices is frequently used in all application fields. The energy of a matrix is an extended version of the energy of a graph. The neutrosophic matrix energy concept needs to be more noticeable in multi-criteria decision-making environments. This study presents the concept of the energy of a multi-valued neutrosophic matrix. The upper and lower bounds of the proposed energy were determined. The connection between the neutrosophic hesitant matrix and the multi-valued neutrosophic matrix is provided. Since the parameters of the two matrices differ in truth, indeterminacy, and false entries, the matrix is converted into a single-valued form, and then the energy was calculated. The MCDM problem was addressed with the proposed energy, which is solved by a novel decision-making method. The task is to select a machine from a group of repaired machines in a particular industry that is in good condition. The final ranking values are calculated by the ranking energy formula that decides which is preferred. Then the comparative results were given to demonstrate the accuracy of the proposed energy outcomes.

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1. Introduction

The neutrosophic set was first introduced by [1]. It deals with the issue of uncertainty. It was an extension of a fuzzy set and an intuitionistic fuzzy set [2]. Then they added further types of neutrosophic sets, including the single-valued neutrosophic set and the interval-valued neutrosophic set. It was applied in a number of decision-making circumstances. In the paper n-valued refined neutrosophic Logic, [3] presented the multi-valued neutrosophic set. It is an extension of the single-valued neutrosophic set.

Fuzzy matrix theory, which focuses on the convergence of fuzzy matrices' powers, was first presented by Michael G. Thomason in 1977. It can be applied in several circumstances. It is commonly known that the matrix representation offers an extra advantage in resolving the issue. Pal. et al. [4] were the first to introduce intuitionistic fuzzy matrices. It is difficult to determine the value of membership or non-membership as a point, though. Fuzzy relational maps and neutrosophic relational maps were proposed by [5]. In this, they included square neutrosophic matrices. The neutrosophic matrix and associated algebraic operations were invented by [6]. Graph spectra are one of the most fundamental concepts in graph theory. A graph's spectrum is related to the idea of graph energy. Gutman [7] was the first to bring up the idea of energy. It is described as the sum of the eigenvalues of the adjacency matrix of the graph. The graph energy was then given upper and lower boundaries. The energy of the graph is extended to uncertain surroundings. Christi DiStefano and colleagues introduced the idea of matrix energy in 2009. They generated the equation for the matrix's energy. An extension of the energy of a graph is the energy of a matrix. Bravo et al. [8] presented a study titled energy of matrixes. They produced a number of theorems on matrix energy as well as upper and lower bounds. In a recent article, [9] suggested the idea of neutrosophic matrix energy in rough sets. The rough neutrosophic matrix energy and its lower and upper boundaries were established. The suggested energy was used in the MCDM issue.

Based on similar research on hesitant fuzzy sets and intuitionistic fuzzy sets, [10] invented the multi-valued neutrosophic sets and their operations and devised a comparison approach. The multi-valued neutrosophic power-weighted average and multi-valued neutrosophic power-weighted geometric operators were subsequently built by [11]. The same authors [12, 13] looked into MCDM issues utilizing the qualitative flexible multiple criteria technique, where the criteria values are represented by multi-valued neutrosophic input, according to an ELECTRE method. As evidenced by the title N-valued refined neutrosophic soft sets and their

CONTACT G. Deepa deepa.g@vit.ac.in Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore 632014, Tamil Nadu, India

uses in decision-making problems and medical diagnosis, [14] have applied this technique for medical diagnoses. In the paper three-way n-valued neutrosophic idea lattice at different granulations by [15], a multi-valued neutrosophic set is afterward used for decision-making. Ye et al. [16] presented an article titled correlation coefficients of consistency in neutrosophic sets regarding neutrosophic multi-valued sets and their multi-attribute decision-making approach. Using the average values and stability degrees of the true, indeterminacy, and false of multi-valued sequences in neutrosophic multi-valued sets, they propose a method in this work for transforming neutrosophic multi-valued sets into consistent single-valued neutrosophic sets. The PROMETHEE technique was used by [17] to solve a multi-attribute decision-making problem where the choice information was given as multi-valued neutrosophic numbers. The notion of multi-valued neutrosophic matrices and its operations applied in the simplified neutrosophic TOPSIS approach was published by [18].

Ye [19] presented the idea of the MADM approach employing a single-valued neutrosophic hesitant fuzzy environment. As a further generalization of the ideas of fuzzy sets, intuitionistic fuzzy sets, single-valued neutrosophic sets, and hesitant fuzzy sets, this article suggests a single-valued neutrosophic hesitant fuzzy set. Liu and Shi [20] introduced the interval neutrosophic hesitant set. In order to make use of their advantages, we combine interval neutrosophic sets and interval-valued hesitant fuzzy sets in this study and propose the idea of the interval neutrosophic hesitant fuzzy set. The operations and comparison approach are then presented, and some novel aggregation operators are created for that. The term interval neutrosophic hesitant fuzzy set and its operational connections were proposed by [21]. Then, we create correlation coefficients for the proposed set and look into how the correlation coefficients and similarity measurements relate to one another. Liu and Zhang[22] proposed an expanded VIKOR technique for the multiple criteria decision-making issues with neutrosophic hesitant fuzzy information, extending the VIKOR method to process neutrosophic hesitant fuzzy information. Then, [23] introduced some fundamental operational guidelines, properties and the score, certainty, and accuracy functions for hesitant interval neutrosophic uncertain linguistic sets and linguistic elements. Pang and Yang [24] define a paper that extends a hesitant fuzzy set to include linguistic variables and neutrosophic fuzzy values in order to define the hesitant neutrosophic linguistic set. Some operational laws for linguistically imprecise information with hesitation are defined. To solve MADM problems where the attribute weight information is lacking and the decision information is expressed in streamlined neutrosophic hesitant fuzzy elements, [25] proposed a decision-making model based on the maximizing deviation method and TOPSIS. Biswas et al. [26] offer an NH-MADM approach based on expanded GRA for solving problems using neutrosophic hesitant fuzzy sets. Giri et al.[27] expand the TOPSIS approach for multi-attribute decision making based on single-valued and interval-valued neutrosophic hesitant fuzzy set. Sahin and Altun [28] define the modified probabilistic single-valued neutrosophic hesitant fuzzy set and suggest some changes to that theory. Additionally, they define a distance operator and enhance various algebraic characteristics of this set theory. Then, two aggregation operators are presented. Wang and Bao [29] investigate the geometric aggregate of single-valued neutrosophic hesitant fuzzy elements based on several normalized operations. Karaaslan et al. cite31 modified the ideas of complex neutrosophic set and hesitant fuzzy set, the concept of hesitant complex neutrosophic set (HCNS) is defined in this study. Additionally, Hausdorff, Hamming, and Euclidean distance measures based on distances between two HCNSs are introduced, and various correlations between them are investigated. The neutrosophic hesitant fuzzy multi-objective programming problems were created by [31] in a neutrosophic hesitant fuzzy environment. Then the authors applied matrix games with pay-offs to a variety of neutrosophic structures, such as single-valued neutrosophic numbers by [32], trapezoidal neutrosophic numbers by [33] and interval neutrosophic matrix by [34].

According to a review of the literature, there is more work being done on the MCDM environment for various kinds of neutrosophic sets and numbers, and the neutrosophic graph energy is widely used in many kinds of application fields. Particularly, multi-valued and hesitant neutrosophic sets are used in many problems. However, the neutrosophic decision field contributes less to the energy of the matrix structure. The energy of the matrix is calculated by the eigenvalues of the matrix. It can be used to determine the individual value of a matrix and the one which has the highest energy value is to be taken first in ranking order. So, the energy of the matrix is a unique value for each matrix. Therefore, our focus is on the neutrosophic matrix energy in a multi-valued structure, and it will be used to solve problems involving several factors for making decisions. By applying the neutrosophic matrix energy and its ranking algorithm, we will find a solution to the problem. Therefore, the main goal of this work is to create the idea of matrix energy in multi-valued neutrosophic structures.

In this paper, the basic definitions are provided in Section 2, and the relationship between the multi-valued neutrosophic matrix and the neutrosophic hesitant matrix is covered in Section 3. We defined various theorems as well as the energy of those matrices.

We offered a novel approach to the problem of multi-criteria decision-making in Section 4, and its numerical example is shown in Section 5. In Section 6, it was discussed how the results of our suggested method and the TOPSIS method were compared, and the results were displayed in the figure. Then a conclusion was given.

2. Preliminaries

Definition 2.1 (Neutrosophic Set (NS) [1]): Let U be the universal set and every element a ∈ U has degree of True, Indeterminacy, False membership in neutrosophic set. It is denoted by S. Then it can be written as

$$S = \{\langle a, T_S(a), I_S(a), F_S(a)\rangle : a \in U\}$$

where, $0 \le T_S(a) + I_S(a) + F_S(a) \le 3$ and Truth Membership function $T_S : U \to [0,1]$, Indeterminacy Membership function $I_S : U \to [0,1]$ and False Membership function $F_S : U \to [0,1]$

Definition 2.2 (Multi-Valued Neutrosophic Set (MVNS) [3]): Let U be the universal set and a is an element of U. A MVNS $\tilde{S}$ in U is characterized by three functions $\tilde{T}_{\tilde{S}}(a), \tilde{I}_{\tilde{S}}(a)$ *and* $\tilde{F}_{\tilde{S}}(a)$ each ∈ [0, 1], which can be defined as follows:

$$\tilde{S} = \left\{\left\langle a, \tilde{T}_{\tilde{S}}(a), \tilde{I}_{\tilde{S}}(a), \tilde{F}_{\tilde{S}}(a)\right\rangle : a \in U\right\}$$

where, $\tilde{T}_{\tilde{S}}(a), \tilde{I}_{\tilde{S}}(a)$ *and* $\tilde{F}_{\tilde{S}}(a)$ are three sets denoting the truth, indeterminacy and false membership degree respectively and $0 \le \tilde{T}_{\tilde{S}}(a) + \tilde{I}_{\tilde{S}}(a) + \tilde{F}_{\tilde{S}}(a) \le 3$ with the conditions:

$$0 \le \alpha, \beta, \gamma \le 1 \qquad 0 \le \alpha^+ + \beta^+ + \gamma^+ \le 3$$

where, $\alpha \in \tilde{T}_{\tilde{S}}(a), \beta \in \tilde{I}_{\tilde{S}}(a), \gamma \in \tilde{F}_{\tilde{S}}(a)$ and $\alpha^+ = sup\tilde{T}_{\tilde{S}}(a), \beta^+ = sup\tilde{I}_{\tilde{S}}(a), \gamma^+ = sup\tilde{F}_{\tilde{S}}(a)$. $\tilde{T}_{\tilde{S}}(a), \tilde{I}_{\tilde{S}}(a)$ and $\tilde{F}_{\tilde{S}}(a)$ is set of crisp values between zero and one. Simply we write, $\tilde{S} = \{\langle \tilde{T}_{\tilde{S}}, \tilde{I}_{\tilde{S}}, \tilde{F}_{\tilde{S}}\rangle\}$ the multi valued neutrosophic number MVNN. Apparently, MVNSs are an extension of NSs.

Especially, If $\tilde{T}_{\tilde{S}}, \tilde{I}_{\tilde{S}}$ and $\tilde{F}_{\tilde{S}}$ have only one value α, β and γ respectively and $0 \le \alpha + \beta + \gamma \le 3$, then the MVNSs are reduced to single-valued neutrosophic sets. If $\tilde{I}_{\tilde{S}} = \phi$, then the MVNSs are reduced to dual hesitant fuzzy sets. If $\tilde{I}_{\tilde{S}} = \tilde{F}_{\tilde{S}} = \phi$, then the MVNSs are reduced to hesitant fuzzy sets. Thus the MVNSs are an extension of these sets above.

Definition 2.3 (Neutrosophic Hesitant Fuzzy Set (NHFS) [19]): Let U be a fixed set, a neutrosophic hesitant fuzzy set on U is expressed by

$$N = \{\left\langle a, \tilde{t}(a), \tilde{i}(a), \tilde{f}(a)\right\rangle : a \in U\}$$

where, $\tilde{t}(a), \tilde{i}(a)$, and $\tilde{f}(a)$ are three sets of some values in [0, 1], denoting the possible truth-membership hesitant degrees, indeterminacy-membership hesitant degrees, and falsity-membership hesitant degrees of the element $a \in U$ to the set N, respectively, with the conditions:

$$0 \le \delta, \xi, \psi \le 1$$
$$0 \le \delta^+ + \xi^+ + \psi^+ \le 3$$

where, $\delta \in \tilde{t}(a)$, $\xi \in \tilde{i}(a)$, $\psi \in \tilde{f}(a)$ and $\delta^+ \in \tilde{t}^+(a) = \cup_{\delta \in \tilde{t}(a)} max\{\delta\}$, $\xi^+ \in \tilde{i}^+(a) = \cup_{\xi \in \tilde{i}(a)} max\{\xi\}$, $\psi^+ \in \tilde{f}^+(a) = \cup_{\psi \in \tilde{f}(a)} max\{\psi\}$ for $a \in U$, and satisfies the limit $\delta \in [0,1], \xi \in [0,1], \psi \in [0,1]$

The $\tilde{n} = \{\tilde{t}(a), \tilde{i}(a), \tilde{f}(a)\}$ is called a neutrosophic hesitant fuzzy element (NHFE) which is the basic unit of the NHFS and is denoted by the symbol $\tilde{n} = \{\tilde{t}, \tilde{i}, \tilde{f}\}$

From this definition, we can see that the NHFS consists of three parts, which are the truth-membership hesitancy function, the indeterminacy-membership hesitancy function, and the falsity-membership hesitancy function, supporting exemplary and flexible access to assign values for each element in the domain and being able to handle three kinds of hesitancy in this situation. Thus, the existing sets, including fuzzy sets, intuitionistic fuzzy sets, SVNSs, HFSs, and DHFSs, can be regarded as special cases of NHFSs.

Definition 2.4 (Multi-Valued Neutrosophic Fuzzy Matrix (MVNFM) [18]): Multi-valued neutrosophic fuzzy matrix P of order $m \times n$ is defined as

$$P = [\left\langle \tilde{T}_{ijp}, \tilde{I}_{ijp}, \tilde{F}_{ijp}\right\rangle]_{m\times n}$$

where, $\tilde{T}_{ijp}, \tilde{I}_{ijp}, \tilde{F}_{ijp}$ are between [0,1] and satisfies the condition for $i = 1, 2, \ldots m$ and $j = 1, 2, \ldots n$

$$0 \le \alpha_{ij}, \beta_{ij}, \gamma_{ij} \le 1$$
$$0 \le \alpha_{ij}^+ + \beta_{ij}^+ + \gamma_{ij}^+ \le 3$$

where, $\alpha_{ij} \in \tilde{T}_{ijp}, \beta_{ij} \in \tilde{I}_{ijp}, \gamma_{ij} \in \tilde{F}_{ijp}$

$\alpha_{ij}^+ = sup\tilde{T}_{ijp}, \beta_{ij}^+ = sup\tilde{I}_{ijp}, \gamma_{ij}^+ = sup\tilde{F}_{ijp}$ and crisp values between 0 and 1.

For simplicity, $P = [\tilde{p}_{ij\alpha}, \tilde{p}_{ij\beta}, \tilde{p}_{ij\gamma}]_{n\times m}$ is called as Multi-Valued Neutrosophic Matrix. where $\tilde{p}_{ij\alpha} \in \tilde{T}_{ijp}, \tilde{p}_{ij\beta} \in \tilde{I}_{ijp}$ and $\tilde{p}_{ij\gamma} \in \tilde{F}_{ijp}$

If $\tilde{T}_{ijp}, \tilde{I}_{ijp}, \tilde{F}_{ijp}$ has only one value in each element of matrix and $0 \le \alpha_{ij} + \beta_{ij} + \gamma_{ij} \le 3$, then the MVNM are reduced to a single-valued neutrosophic matrix. If $\tilde{T}_{ijp}, \tilde{I}_{ijp}, \tilde{F}_{ijp}$ has interval values in each element of matrix, then the MVNM are reduced to an interval-valued neutrosophic matrix.

Example 2.5: Let $\tilde{P} =$ be a 3 × 3 MVNFM.

$$\tilde{P} = \begin{pmatrix} \langle\{.1\}, \{.4, .5\}, \{.7\}\rangle & \langle\{.1, .2\}, \{.3\}, \{.6\}\rangle \\ \langle\{.3\}, \{.6, .7\}, \{.4, .5\}\rangle & \langle\{.4, .5\}, \{.8\}, \{.1\}\rangle \\ \langle\{.4\}, \{.7, .8\}, \{.1\}\rangle & \langle\{.1, .3\}, \{.6\}, \{.2\}\rangle \end{pmatrix}$$

$$\left.\begin{matrix}\langle\{.4\},\{.2,.3\},\{.8\}\rangle\\ \langle\{.2\},\{.7\},\{.5\}\rangle\\ \langle\{.1,.3\},\{.4,.5\},\{.8,.9\}\rangle\end{matrix}\right)$$

All the elements in $\tilde{P}$ are in the form of single-valued or interval-valued membership in each truth, indeterminacy and false value. This form of matrix is called multi-valued neutrosophic matrix.

Definition 2.6 (Neutrosophic Hesitant Fuzzy Matrix (NHFM)): Neutrosophic hesitant fuzzy matrix $\tilde{H}$ of order $m \times n$ is defined as

$$\tilde{H} = [\left\langle \tilde{t}_{ijh}, \tilde{i}_{ijh}, \tilde{f}_{ijh} \right\rangle]_{m\times n}$$

where, $\tilde{t}_{ijh}, \tilde{i}_{ijh}, \tilde{f}_{ijh}$ are between [0,1] and satisfies the condition for $i = 1, 2, \dots m$ and $j = 1, 2, \dots n$

$$0 \leq \upsilon_{ij}, \nu_{ij}, \omega_{ij} \leq 1$$
$$0 \leq \upsilon_{ij}^{+} + \nu_{ij}^{+} + \omega_{ij}^{+} \leq 3$$

where, $\upsilon_{ij} \in \tilde{t}_{ijh}, \nu_{ij} \in \tilde{i}_{ijh}, \omega_{ij} \in \tilde{f}_{ijh}$

$\upsilon_{ij}^{+} = sup\tilde{t}_{ijh}, \nu_{ij}^{+} = sup\tilde{i}_{ijh}, \omega_{ij}^{+} = sup\tilde{f}_{ijh}$ and crisp values between 0 and 1.

For simplicity, $H = [\tilde{h}_{ij\upsilon}, \tilde{h}_{ij\nu}, \tilde{h}_{ij\omega}]_{n\times m}$ is called as Neutrosophic Hesitant Fuzzy Matrix. where $\tilde{h}_{ij\upsilon} \in \tilde{t}_{ijh}$, $\tilde{h}_{ij\nu} \in \tilde{i}_{ijh}$ and $\tilde{h}_{ij\omega} \in \tilde{f}_{ijh}$. Every element of NHFM contains any kind of hesitant values.

If $\tilde{t}_{ijh}, \tilde{i}_{ijh}, \tilde{f}_{ijh}$ has only one value in each element of matrix and $0 \leq \upsilon_{ij} + \nu_{ij} + \omega_{ij} \leq 3$, then the NHFM are reduced to Single Valued Neutrosophic Hesitant Fuzzy Matrix (SVNHFM).

Example 2.7: Let $\tilde{H}$ be the NHFM with 2×2 matrix.

$$\tilde{H} = \begin{pmatrix} \langle\{0.5\},\{0.2,0.3,0.4\},\{0.6,0.7\}\rangle & \langle\{0.3,0.4\},\{0.5,0.6\},\{0.1,0.2,0.3\}\rangle \\ \langle\{0.2,0.3\},\{0.6,0.7\},\{0.1\}\rangle & \langle\{0.1,0.2,0.4,0.5\},\{0.8\},\{0.3,0.4\}\rangle \end{pmatrix}$$

All the elements of $\tilde{H}$ are the hesitant neutrosophic numbers.

Definition 2.8 (Energy of Matrix [8]): Let $M_n(\mathbb{C})$ denote the space of $n \times n$ matrices with entries in $\mathbb{C}$ and P be a matrix in $M_n(\mathbb{C})$. We define the energy of A as

$$E(P) = \sum_{i=1}^{n} |\lambda_i - \mu|$$

where, $\lambda_1, \lambda_2, \dots \lambda_n$ are the eigenvalues of P and μ is the mean of eigenvalues.

If $\mu = 0$ or P is the adjacency matrix of a graph G then E(P) is precisely the energy of the graph G.

Definition 2.9 (Energy of Neutrosophic Matrix): Let $P(N)$ be the neutrosophic matrix with the order of $n \times n$ (square matrix). It can be expressed as three matrices, the first matrix contains the entries a_{ij} as truth membership values, the second contains the entries b_{ij} as indeterminacy membership values and the third matrix contains the entries c_{ij} as false membership values. It is denoted as $P(N) = \langle P(T_{ij}), P(I_{ij}), P(F_{ij})\rangle_{n\times n}$ and $a_{ij} \in P(T_{ij})_{n\times n}$, $b_{ij} \in P(I_{ij})_{n\times n}$ and $c_{ij} \in P(F_{ij})_{n\times n}$

The energy of a neutrosophic matrix is defined as

$$\begin{aligned} &E[P(N)] \\ &= \left\langle E[P(T_{ij})], E[P(I_{ij})], E[P(F_{ij})]\right\rangle \\ &= \left\langle \sum_{i=1}^{n} |\lambda_i - \mu_\lambda|, \sum_{i=1}^{n} |\zeta_i - \mu_\zeta|, \sum_{i=1}^{n} |\eta_i - \mu_\eta| \right\rangle \end{aligned}$$

where, λ_i, ζ_i and η_i $(i = 1, 2, \dots n)$ are the eigenvalues of Truth, Indeterminacy, and False membership values respectively and μ_λ, μ_ζ, and μ_η are the mean values of λ_i, ζ_i and η_i respectively.

3. The energy of multi-valued neutrosophic fuzzy matrix and neutrosophic hesitant fuzzy matrix

In this section, we discussed the difference and relationship between the multi-valued neutrosophic fuzzy matrix and the neutrosophic hesitant fuzzy matrix. Then we defined the energy of both matrices and derived some theorems.

3.1. Difference between multi-valued and hesitant neutrosophic fuzzy matrices

The multi-valued neutrosophic fuzzy matrix contains the elements of multi-valued neutrosophic numbers. In Example 2.5, every element of the matrix contains single-valued or interval-valued values. This form of matrix is called a multi-valued neutrosophic fuzzy matrix. The neutrosophic hesitant fuzzy matrix contains the elements of hesitant neutrosophic numbers. In Example 2.7, every element of the matrix contains hesitant values. Each membership contains one or more parameters. This form of a matrix is called a neutrosophic hesitant fuzzy matrix.

From this, we can say the multi-valued neutrosophic fuzzy set is a subset of the neutrosophic hesitant fuzzy set.

$$MVNFS \subset NHFS$$

A multi-valued neutrosophic fuzzy matrix is also called a neutrosophic hesitant fuzzy matrix. But the converse need not be true.

In this study, we defined the energy of MVNFM and HNFM. However, because the size of the elements is not the same to form the same order square matrix, we are unable to separate it into truth, indeterminacy, and false matrices. Therefore, we use a maximum minimal procedure to transform these matrices into single-valued neutrosophic fuzzy matrices. For each element in the matrix, we take maximum values for truth and minimum values for indeterminacy and falsehood. The truth energy plays an important role in neutrosophic matrix energy, so we take the maximum value for the truth.

3.2. Energy of MVNFM and NHFM

Let $\tilde{H}$ be a multi-valued neutrosophic matrix or neutrosophic hesitant fuzzy matrix with the order of $n \times n$ which is a square matrix. By using a max of truth values, min of indeterminacy, and false values, we convert it into a single-valued neutrosophic matrix H.

$$NHFM\ \tilde{H} = [\tilde{t}_{ij}, \tilde{i}_{ij}, \tilde{f}_{ij}]_{n\times n}$$
$$SVNFM\ H = [t_{ij}, i_{ij}, f_{ij}]_{n\times n}$$

The first matrix contains truth membership values (a_{ij}), the second matrix contains indeterminacy membership values (b_{ij}) and the third matrix contains the entries of false membership values (c_{ij})

where, $a_{ij} \in H(T)$, $b_{ij} \in H(I)$ and $c_{ij} \in H(F)$

$$E[H] = \langle E[H(T)], E[H(I)], E[H(F)]\rangle = \left\langle \sum_{i=1}^{n} |\delta_i - \mu_\delta|, \sum_{i=1}^{n} |\xi_i - \mu_\xi|, \sum_{i=1}^{n} |\psi_i - \mu_\psi| \right\rangle$$

where, δ_i, ξ_i, and ψ_i (i = 1,2,…, n) are the eigenvalues of truth, indeterminacy, and false membership values respectively. μ_δ, μ_ξ, and μ_ψ are the mean values of δ_i, ξ_i and ψ_i respectively.

Example 3.1 (The energy of multi-valued neutrosophic fuzzy matrix): From Example 2.5, we find the energy of matrix $\tilde{P}$. Here we separate the matrix into 3 matrices which are truth, indeterminacy, and false matrices. It is different in the number of elements in each matrix. So we convert the MVNFM into SVNFM by max min process. For the truth matrix we take the max value of each element, for indeterminacy, and false we take the min value of each element. Then we find the energy of the matrix.

$$SVNFM P = \begin{pmatrix} \langle\{0.1\},\{0.4\},\{0.7\}\rangle & \langle\{0.2\},\{0.3\},\{0.6\}\rangle & \langle\{0.4\},\{0.2\},\{0.8\}\rangle \\ \langle\{0.3\},\{0.6\},\{0.4\}\rangle & \langle\{0.5\},\{0.8\},\{0.1\}\rangle & \langle\{0.2\},\{0.7\},\{0.5\}\rangle \\ \langle\{0.4\},\{0.7\},\{0.1\}\rangle & \langle\{0.3\},\{0.6\},\{0.2\}\rangle\rangle & \langle\{0.3\},\{0.4\},\{0.8\} \end{pmatrix}$$

$$P(T) = \begin{pmatrix} 0.1 & 0.2 & 0.4 \\ 0.3 & 0.5 & 0.2 \\ 0.4 & 0.5 & 0.3 \end{pmatrix}$$

$$P(I) = \begin{pmatrix} 0.4 & 0.3 & 0.2 \\ 0.6 & 0.8 & 0.7 \\ 0.7 & 0.6 & 0.4 \end{pmatrix} P(F) = \begin{pmatrix} 0.7 & 0.6 & 0.8 \\ 0.4 & 0.1 & 0.5 \\ 0.1 & 0.2 & 0.8 \end{pmatrix}$$

Energy of Matrix P = [1.3489, 2.0523, 1.5497]

Example 3.2 (The energy of neutrosophic hesitant fuzzy matrix): From Example 2.7, we find the energy of matrix $\tilde{H}$. We separate the matrix into 3 matrices which are truth, indeterminacy and false matrices. It is different in the number of elements in each matrix with hesitant values. So we convert the NHFM into SVNFM by max min process. For the truth matrix we take the max value of each element, for indeterminacy, and false we take the min value of each element. Then we find the energy of the matrix.

$$SVNFM H = \begin{pmatrix} \langle\{0.5\},\{0.2\},\{0.6\}\rangle & \langle\{0.4\},\{0.5\},\{0.1\}\rangle \\ \langle\{0.3\},\{0.6\},\{0.1\}\rangle & \langle\{0.5\},\{0.8\},\{0.3\}\rangle \end{pmatrix}$$

$$H(T) = \begin{pmatrix} 0.5 & 0.4 \\ 0.3 & 0.5 \end{pmatrix} \quad H(I) = \begin{pmatrix} 0.2 & 0.5 \\ 0.6 & 0.8 \end{pmatrix}$$

$$H(F) = \begin{pmatrix} 0.6 & 0.1 \\ 0.1 & 0.3 \end{pmatrix}$$

Energy of Matrix H = [0.6928, 1.2490, 0.3606]

Theorem 3.3: *Let $\tilde{H}$ be a multi-valued neutrosophic fuzzy matrix or neutrosophic hesitant fuzzy matrix with the order of $n \times n$. If δ_i, ξ_i and ψ_i $(i = 1, 2, \ldots n)$ are the eigenvalues of Truth, Indeterminacy, and False membership matrices respectively. then,*

$$(i) \sum_{i=1}^{n}(\delta_i - \mu_\delta) = \sum_{i=1}^{n}(a_{ii} - \mu_\delta) = 0$$
$$\sum_{i=1}^{n}(\xi_i - \mu_\xi) = \sum_{i=1}^{n}(b_{ii} - \mu_\xi) = 0$$
$$\sum_{i=1}^{n}(\psi_i - \mu_\psi) = \sum_{i=1}^{n}(c_{ii} - \mu_\psi) = 0$$
$$(ii) \sum_{i=1}^{n}(\delta_i - \mu_\delta)^2 = \sum_{i=1}^{n} a_{ii}^2 + 2\sum_{1\le i<j\le n} a_{ij}a_{ji} - n\mu_\delta^2$$
$$\sum_{i=1}^{n}(\xi_i - \mu_\xi)^2 = \sum_{i=1}^{n} b_{ii}^2 + 2\sum_{1\le i<j\le n} b_{ij}b_{ji} - n\mu_\xi^2$$
$$\sum_{i=1}^{n}(\eta_i - \mu_\psi)^2 = \sum_{i=1}^{n} c_{ii}^2 + 2\sum_{1\le i<j\le n} c_{ij}c_{ji} - n\mu_\psi^2$$

where, a_{ij}, b_{ij}, and c_{ij} are the elements of truth, indeterminacy, and false matrices respectively.

Theorem 3.4: *Let $\tilde{H} = \langle\tilde{H}(T), \tilde{H}(I), \tilde{H}(F)\rangle$ be the neutrosophic matrix then, the lower bound and upper bound* *of truth, indeterminacy, and false matrix energy are given below.*

$$(i)\ \sqrt{\begin{array}{c}\left(\sum_{i=1}^{n} \mid \delta_i - \mu_\delta \mid\right)^2 \\ -2\sum_{1\le i<j\le n} \mid \delta_i - \mu_\delta \mid\mid \delta_j - \mu_\delta \mid \\ +n(n-1)\left[\mid \tilde{H}(T) - \mu_\delta \mid\right]^{\frac{2}{n}}\end{array}} \le E[\tilde{H}(T)] \le \sqrt{\begin{array}{c} n\left[\left(\sum_{i=1}^{n} \mid \delta_i - \mu_\delta \mid\right)^2 \right.\\ \left. -2\sum_{1\le i<j\le n} \mid \delta_i - \mu_\delta \mid\mid \delta_j - \mu_\delta \mid\right]\end{array}}$$

$$(ii)\ \sqrt{\begin{array}{c}\left(\sum_{i=1}^{n} \mid \xi_i - \mu_\xi \mid\right)^2 \\ -2\sum_{1\le i<j\le n} \mid \xi_i - \mu_\xi \mid\mid \xi_j - \mu_\xi \mid \\ +n(n-1)\left[\mid \tilde{H}(I) - \mu_\xi \mid\right]^{\frac{2}{n}}\end{array}} \le E[\tilde{H}(I)] \le \sqrt{\begin{array}{c} n\left[\left(\sum_{i=1}^{n} \mid \xi_i - \mu_\xi \mid\right)^2 \right.\\ \left. -2\sum_{1\le i<j\le n} \mid \xi_i - \mu_\xi \mid\mid \xi_j - \mu_\xi \mid\right]\end{array}}$$

$$(iii)\ \sqrt{\begin{array}{c}\left(\sum_{i=1}^{n} \mid \psi_i - \mu_\psi \mid\right)^2 \\ -2\sum_{1\le i<j\le n} \mid \psi_i - \mu_\psi \mid\mid \psi_j - \mu_\psi \mid \\ +n(n-1)\left[\mid \tilde{H}(F) - \mu_\psi \mid\right]^{\frac{2}{n}}\end{array}} \le E[\tilde{H}(F)] \le \sqrt{\begin{array}{c} n\left[\left(\sum_{i=1}^{n} \mid \psi_i - \mu_\psi \mid\right)^2 \right.\\ \left. -2\sum_{1\le i<j\le n} \mid \psi_i - \mu_\psi \mid\mid \psi_j - \mu_\psi \mid\right]\end{array}}$$

4. The neutrosophic energy method for multi-valued neutrosophic fuzzy matrix and neutrosophic hesitant fuzzy matrix

This section introduces a novel MCDM technique that makes use of multi-valued or hesitant neutrosophic matrix energy. We take into account a set of l alternatives, a set of m criteria, and a group of n decision-makers when solving this approach.

$$W = \begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_m \end{array} \begin{array}{c} \begin{array}{cccc} DM_1 & DM_2 & \dots & DM_n \end{array} \\ \begin{pmatrix} \langle \tilde{\upsilon}_{11}, \tilde{\nu}_{11}, \tilde{\omega}_{11}\rangle & \langle \tilde{\upsilon}_{12}, \tilde{\nu}_{12}, \tilde{\omega}_{12}\rangle & \dots & \langle \tilde{\upsilon}_{1n}, \tilde{\nu}_{1n}, \tilde{\omega}_{1n}\rangle \\ \langle \tilde{\upsilon}_{21}, \tilde{\nu}_{21}, \tilde{\omega}_{21}\rangle & \langle \tilde{\upsilon}_{22}, \tilde{\nu}_{22}, \tilde{\omega}_{22}\rangle & \dots & \langle \tilde{\upsilon}_{2n}, \tilde{\nu}_{2n}, \tilde{\omega}_{2n}\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle \tilde{\upsilon}_{m1}, \tilde{\nu}_{m1}, \tilde{\omega}_{m1}\rangle & \langle \tilde{\upsilon}_{m2}, \tilde{\nu}_{m2}, \tilde{\omega}_{m2}\rangle & \dots & \langle \tilde{\upsilon}_{mn}, \tilde{\nu}_{mn}, \tilde{\omega}_{mn}\rangle \end{pmatrix} \end{array}$$

Step 1: Build a decision matrix

Construct a decision matrix for each alternative and criteria. The decision matrix for weights of criteria is taken as $m \times n$ matrix. The decision matrix of the alternative is taken as $n \times m$ matrix. where, $k = 1, 2, \dots l$.

$$A_k = \begin{array}{c} \\ DM_1 \\ DM_2 \\ \vdots \\ DM_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \dots & C_m \end{array} \\ \begin{pmatrix} \langle x_{11}, y_{11}, z_{11}\rangle & \langle x_{12}, y_{12}, z_{12}\rangle & \dots & \langle x_{1m}, y_{1m}, z_{1m}\rangle \\ \langle x_{21}, y_{21}, z_{21}\rangle & \langle x_{22}, y_{22}, z_{22}\rangle & \dots & \langle x_{2m}, y_{2m}, z_{2m}\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle x_{n1}, y_{n1}, z_{n1}\rangle & \langle x_{n2}, y_{n2}, z_{n2}\rangle & \dots & \langle x_{nm}, y_{nm}, z_{nm}\rangle \end{pmatrix} \end{array}$$

Step 2: Weights of decision makers [35]

To determine the weights of individual decision-makers we use the following formula.

The weights of jth decision maker is

$$w_j = \bigcup_{\tilde{t},\tilde{i},\tilde{f}} \frac{1 - \sqrt{\{(1-\tilde{t}(a))^2 + (\tilde{i}(a))^2 + (\tilde{f}(a))^2\}/3}}{\sum_{j=1}^{n}(1 - \sqrt{\{(1-\tilde{t}(a))^2 + (\tilde{i}(a))^2 + (\tilde{f}(a))^2\}/3})} \quad (1)$$

where, $\sum_{j=1}^{n} w_j = 1$

Step 3: Weighted decision matrix

From the decision makers we separate the matrix into 3 individual matrices, which as truth, indeterminacy, and false matrices. When we separate the matrix the order of each matrix is the same, but the number of elements in the matrix is different from one other. So we convert the unformed matrix to a single-valued matrix. We will use the max min process for converting this matrix. For the truth matrix, we take the max values of each element, for indeterminacy and the false matrix we take the min values of each element. For every $\tilde{\upsilon} \in \tilde{t}_{ij}$, $\tilde{\nu} \in \tilde{i}_{ij}$ and $\tilde{\omega} \in \tilde{f}_{ij}$

$$\text{Truth matrix} = \begin{pmatrix} max(\tilde{\upsilon}_{11}) & max(\tilde{\upsilon}_{12}) & \dots & max(\tilde{\upsilon}_{1n}) \\ max(\tilde{\upsilon}_{21}) & max(\tilde{\upsilon}_{22}) & \dots & max(\tilde{\upsilon}_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ max(\tilde{\upsilon}_{m1}) & max(\tilde{\upsilon}_{m2}) & \dots & max(\tilde{\upsilon}_{mn}) \end{pmatrix}$$

$$\text{Indeterminacy matrix} = \begin{pmatrix} min(\tilde{\nu}_{11}) & min(\tilde{\nu}_{12}) & \dots & min(\tilde{\nu}_{1n}) \\ min(\tilde{\nu}_{21}) & min(\tilde{\nu}_{22}) & \dots & min(\tilde{\nu}_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ min(\tilde{\nu}_{m1}) & min(\tilde{\nu}_{m2}) & \dots & min(\tilde{\nu}_{mn}) \end{pmatrix}$$

$$\text{False matrix} = \begin{pmatrix} min(\tilde{\omega}_{11}) & min(\tilde{\omega}_{12}) & \dots & min(\tilde{\omega}_{1n}) \\ min(\tilde{\omega}_{21}) & min(\tilde{\omega}_{22}) & \dots & min(\tilde{\omega}_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ min(\tilde{\omega}_{m1}) & min(\tilde{\omega}_{m2}) & \dots & min(\tilde{\omega}_{mn}) \end{pmatrix}$$

Then we will use the following operation for determining the weighted decision matrix. For every matrix, each element is multiplied by the weights of respective decision makers.

$$wW = (W(T), W(I), W(F))$$

$$= (1-(1-T)^w, (I)^w, (F)^w) \quad (2)$$

$$wA_k = (wA_k(T), wA_k(I), wA_k(F)) = (1-(1-T)^w, (I)^w, (F)^w) \quad (3)$$

Truth-weighted decision matrix for criteria weights

$$W(T) = \begin{pmatrix} 1-(1-\upsilon_{11})^{w_1} & 1-(1-\upsilon_{12})^{w_2} & \dots & 1-(1-\upsilon_{1n})^{w_n} \\ 1-(1-\upsilon_{21})^{w_1} & 1-(1-\upsilon_{22})^{w_2} & \dots & 1-(1-\upsilon_{2n})^{w_n} \\ \vdots & \vdots & \ddots & \vdots \\ 1-(1-\upsilon_{m1})^{w_1} & 1-(1-\upsilon_{m2})^{w_2} & \dots & 1-(1-\upsilon_{mn})^{w_n} \end{pmatrix}$$

Indeterminacy weighted decision matrix for criteria weights

$$W(I) = \begin{pmatrix} (\nu_{11})^{w_1} & (\nu_{12})^{w_2} & \dots & (\nu_{1n})^{w_n} \\ (\nu_{21})^{w_1} & (\nu_{22})^{w_2} & \dots & (\nu_{2n})^{w_n} \\ \vdots & \vdots & \ddots & \vdots \\ (\nu_{m1})^{w_1} & (\nu_{m2})^{w_2} & \dots & (\nu_{mn})^{w_n} \end{pmatrix}$$

False weighted decision matrix for criteria weights

$$W(F) = \begin{pmatrix} (\omega_{11})^{w_1} & (\omega_{12})^{w_2} & \dots & (\omega_{1n})^{w_n} \\ (\omega_{21})^{w_1} & (\omega_{22})^{w_2} & \dots & (\omega_{2n})^{w_n} \\ \vdots & \vdots & \ddots & \vdots \\ (\omega_{m1})^{w_1} & (\omega_{m2})^{w_2} & \dots & (\omega_{mn})^{w_n} \end{pmatrix}$$

Smilarly for Truth weighted decision matrix for alternatives

$$wA_k(T) = \begin{pmatrix} 1-(1-x_{11})^{w_1} & 1-(1-x_{12})^{w_1} & \dots & 1-(1-x_{1m})^{w_1} \\ 1-(1-x_{21})^{w_2} & 1-(1-x_{22})^{w_2} & \dots & 1-(1-x_{2m})^{w_2} \\ \vdots & \vdots & \ddots & \vdots \\ 1-(1-x_{n1})^{w_m} & 1-(1-x_{n2})^{w_m} & \dots & 1-(1-x_{nm})^{w_n} \end{pmatrix}$$

Indeterminacy weighted decision matrix for alternatives

$$wA_k(I) = \begin{pmatrix} (y_{11})^{w_1} & (y_{12})^{w_1} & \dots & (y_{1m})^{w_1} \\ (y_{21})^{w_2} & (y_{22})^{w_2} & \dots & (y_{2m})^{w_2} \\ \vdots & \vdots & \ddots & \vdots \\ (y_{n1})^{w_m} & (y_{n2})^{w_m} & \dots & (y_{nm})^{w_m} \end{pmatrix}$$

False weighted decision matrix for alternatives

$$wA_k(F) = \begin{pmatrix} (z_{11})^{w_1} & (z_{12})^{w_1} & \dots & (z_{1m})^{w_1} \\ (z_{21})^{w_2} & (z_{22})^{w_2} & \dots & (z_{2m})^{w_2} \\ \vdots & \vdots & \ddots & \vdots \\ (z_{n1})^{w_m} & (z_{n2})^{w_m} & \dots & (z_{nm})^{w_m} \end{pmatrix}$$

Step 4: Converting the matrices into square matrix

In this step, we form a square matrix for calculating energy. For converting the square matrix from the above-weighted decision matrices, we will use the following process.

$$wA_k(T)_{n\times m} * W(T)_{m\times n} = \begin{pmatrix} \upsilon x_{11} & \upsilon x_{12} & \dots & \upsilon x_{1n} \\ \upsilon x_{21} & \upsilon x_{22} & \dots & \upsilon x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \upsilon x_{n1} & \upsilon x_{n2} & \dots & \upsilon x_{nn} \end{pmatrix}_{n\times n}$$

Step 5: Determine the energy

From the above matrix find the energy of the neutrosophic matrix by definition 2.9. We got 3 energies for truth, indeterminacy, and false matrices respectively.

$$E[A_k] = (E[A_k(T)], E[A_k(I)], E[A_k(F)])$$

Continue steps 3 to step 5 for every alternative For each alternative, neutrosophic matrix energy occurs as $E[A_1], E[A_2], \dots, E[A_l]$

Step 6: Ranking of alternatives

Finally, the following formula determines the rank of the alternatives. It states that truth energy plays an essential part; hence, it will be taken by double, added to the indeterminacy energy value, and the false energy value subtracted from them.

$$R = 2 \times E[A_l(T)] + E[A_l(I)] - E[A_l(F)] \quad (4)$$

By the final ranking values, the one that gets the highest will be the best one.

5. Numerical example

In this section, our proposed method is applied to the MCDM problem. The problem is about selecting the good condition of the machine. There are 4 repair machines that are taken as an alternative (A_1, A_2, A_3, A_4). The 4 operators are considered as decision makers (DM_1, DM_2, DM_3, DM_4). There are some quality checking of machines which will be considered as criteria for this problem. Testing frequency, Reliable, Safety to handle, Machine temperature, and User-friendly $(c_1, c_2, c_3, c_4, c_5)$.

Step 1: Decision matrix for weights of criteria and alternative

Table 1 shows the values of weights of criteria and Table 2 shows the values of alternatives values given by 4 decision-makers over each criterion.

Step 2: Weights of decision makers

Weights of each decision maker in terms of hesitant values.

$DM_1 = \langle\{0.8, 0.9\}, \{0.4, 0.5\}, \{0.2\}\rangle$, $DM_2 = \langle\{0.9\}, \{0.3, 0.5, 0.6\}, \{0.1, 0.2\}\rangle$,$DM_3 = \langle\{0.7, 0.8\}, \{0.3\}, \{0.5\}\rangle$ and $DM_4 = \langle\{0.8\}, \{0.4\}, \{0.2\}\rangle$.

Table 1. Decision matrix for weights of alternatives.

A_k	DM_n	Criteria	
A_1	DM_1	c_1 ⟨{0.5}, {0.2}, {0.6, 0.7}⟩	c_2 ⟨{0.6, 0.7}, {0.3}, {0.5}⟩
		c_3 ⟨{0.6}, {0.2, 0.3}, {0.5}⟩	c_4 ⟨{0.5, 0.6, 0.7}, {0.2}, {0.4}⟩
		c_5 ⟨{0.8}, {0.5}, {0.2}⟩	
	DM_2	c_1 ⟨{0.7}, {0.3, 0.4}, {0.2}⟩	c_2 ⟨{0.5}, {0.3}, {0.1}⟩
		c_3 ⟨{0.8}, {0.1, 0.2, 0.3}, {0.5}⟩	c_4 ⟨{0.4, 0.5}, {0.1, 0.2}, {0.3, 0.4}⟩
		c_5 ⟨{0.7, 0.8}, {0.3}, {0.1}⟩	
	DM_3	c_1 ⟨{0.7}, {0.3}, {0.2, 0.4}⟩	c_2 ⟨{0.7}, {0.2, 0.3}, {0.4}⟩
		c_3 ⟨{0.7, 0.8}, {0.1}, {0.3}⟩	c_4 ⟨{0.6}, {0.3}, {0.5}⟩
		c_5 ⟨{0.6, 0.7}, {0.3, 0.4}, {0.2}⟩	
	DM_4	c_1 ⟨{0.7, 0.8}, {0.5}, {0.4}⟩	c_2 ⟨{0.6}, {0.5}, {0.3, 0.4}⟩
		c_3 ⟨{0.8}, {0.2}, {0.4}⟩	c_4 ⟨{0.5}, {0.2}, {0.1}⟩
		c_5 ⟨{0.6}, {0.3}, {0.2, 0.4}⟩	
A_2	DM_1	c_1 ⟨{0.9}, {0.3}, {0.1, 0.2}⟩	c_2 ⟨{0.8}, {0.2, 0.3}, {0.4}⟩
		c_3 ⟨{0.7, 0.8}, {0.5}, {0.2}⟩	c_4 ⟨{0.8, 0.9}, {0.2}, {0.3}⟩
		c_5 ⟨{0.9}, {0.2}, {0.1}⟩	
	DM_2	c_1 ⟨{0.8, 0.9, 1}, {0.5}, {0.2}⟩	c_2 ⟨{0.7, 0.8}, {0.3}, {0.3}⟩
		c_3 ⟨{0.8}, {0.4}, {0.1}⟩	c_4 ⟨{0.7}, {0.6}, {0.2, 0.3}⟩
		c_5 ⟨{0.8}, {0.5}, {0.2, 0.4}⟩	
	DM_3	c_1 ⟨{0.7, 0.8}, {0.4}, {0.6}⟩	c_2 ⟨{0.7}, {0.4}, {0.2}⟩
		c_3 ⟨{0.9, 1}, {0.3}, {0.1, 0.2}⟩	c_4 ⟨{0.6, 0.7}, {0.4, 0.5}, {0.2}⟩
		c_5 ⟨{0.7}, {0.5}, {0.4}⟩	
	DM_4	c_1 ⟨{0.6, 0.7}, {0.3, 0.4}, {0.1, 0.2}⟩	c_2 ⟨{0.7}, {0.5}, {0.2}⟩
		c_3 ⟨{0.6}, {04}, {0.2}⟩	c_4 ⟨{0.7}, {0.5}, {0.2, 0.3}⟩
		c_5 ⟨{0.7, 0.8}, {0.1}, {0.3, 0.4}⟩	
A_3	DM_1	c_1 ⟨{0.4}, {0.7}, {0.6, 0.8}⟩	c_2 ⟨{0.5, 0.6}, {0.4}, {0.2, 0.3}⟩
		c_3 ⟨{0.7}, {0.3}, {0.1}⟩	c_4 ⟨{0.5}, {0.5}, {0.6, 0.7}⟩
		c_5 ⟨{0.4}, {0.3}, {0.8}⟩	
	DM_2	c_1 ⟨{0.3, 0.4}, {0.5}, {0.7}⟩	c_2 ⟨{0.5}, {0.1, 0.2}, {0.3, 0.4}⟩
		c_3 ⟨{0.6}, {0.2}, {0.4, 0.5}⟩	c_4 ⟨{0.6, 0.7}, {0.5}, {0.4, 0.5}⟩
		c_5 ⟨{0.4}, {0.3}, {0.8}⟩	
	DM_3	c_1 ⟨{0.6}, {0.8}, {0.2}⟩	c_2 ⟨{0.5, 0.6}, {0.1}, {0.2}⟩
		c_3 ⟨{0.7}, {0.1, 0.3}, {0.5}⟩	c_4 ⟨{0.8}, {0.4}, {0.3}⟩
		c_5 ⟨{0.2}, {0.1}, {0.5}⟩	
	DM_4	c_1 ⟨{0.5, 0.6}, {0.6, 0.7}, {0.2}⟩	c_2 ⟨{0.4}, {0.3}, {0.4, 0.6}⟩
		c_3 ⟨{0.6}, {0.5}, {0.3, 0.4}⟩	c_4 ⟨{0.7}, {0.5}, {0.5}⟩
		c_5 ⟨{0.5, 0.6}, {0.2}, {0.4}⟩	
A_4	DM_1	c_1 ⟨{0.6}, {0.2}, {0.3, 0.4}⟩	c_2 ⟨{0.7}, {0.1}, {0.4, 0.5}⟩
		c_3 ⟨{0.5, 0.6}, {0.1, 0.2}, {0.54}⟩	c_4 ⟨{0.5}, {0.3, 0.4}, {0.7}⟩
		c_5 ⟨{0.3}, {0.5, 0.6}, {0.8}⟩	
	DM_2	c_1 ⟨{0.5}, {0.1, 0.2}, {0.7}⟩	c_2 ⟨{0.6, 0.7}, {0.2}, {0.3}⟩
		c_3 ⟨{0.6}, {0.3, 0.4}, {0.5.0.7}⟩	c_4 ⟨{0.5}, {0.4}, {0.3}⟩
		c_5 ⟨{0.4}, {0.5}, {0.6, 0.7}⟩	
	DM_3	c_1 ⟨{0.2}, {0.5}, {0.8}⟩	c_2 ⟨{0.3, 0.5}, {0.7}, {0.8}⟩
		c_3 ⟨{0.5}, {0.1}, {0.3, 0.4}⟩	c_4 ⟨{0.7, 0.8}, {0.1, 0.2}, {0.3, 0.4}⟩
		c_5 ⟨{0.4}, {0.1}, {0.5}⟩	
	DM_4	c_1 ⟨{0.5}, {0.6}, {0.7}⟩	c_2 ⟨{0.5, 0.7}, {0.1, 0.2, 0.3}, {0.4}⟩
		c_3 ⟨{0.6, 0.7}, {0.2}, {0.4}⟩	c_4 ⟨{0.7}, {0.3, 0.4}, {0.5}⟩
		c_5 ⟨{0.6, 0.7}, {0.2}, {0.4}⟩	

Table 2. Decision matrix for weights of criteria.

Criteria	DM_1	DM_2
c_1	⟨{0.3}, {0.1, 0.2}, {0.4, 0.5}⟩	⟨{0.2, 0.3, 0.4}, {0.1}, {0.6}⟩
c_2	⟨{0.7, 0.8}, {0.5}, {0.3, 0.4}⟩	⟨{0.6, 0.7}, {0.5, 0.6}, {0.3}⟩
c_3	⟨{0.7}, {0.2, 0.3, 0.5}, {0.6, 0.7}⟩	⟨{0.5}, {0.4}, {0.6}⟩
c_4	⟨{0.5, 0.6}, {0.2, 0.3}, {0.1, 0.2}⟩	⟨{0.5, 0.6, 0.7}, {0.4}, {0.1}⟩
c_5	⟨{0.8}, {0.5}, {0.2, 0.3}⟩	⟨{0.8, 0.9}, {0.4, 0.5}, {0.2}⟩
Criteria	DM_3	DM_4
c_1	⟨{0.3}, {0.2}, {0.5}⟩	⟨{0.3, 0.4}, {0.2}, {0.7}⟩
c_2	⟨{0.6, 0.7}, {0.4}, {0.3, 0.5}⟩	⟨{0.5}, {0.2}, {0.7}⟩
c_3	⟨{0.6, 0.7}, {0.3}, {0.5}⟩	⟨{0.5}, {0.3}, {0.4}⟩
c_4	⟨{0.6}, {0.3}, {0.2}⟩	⟨{0.7}, {0.4}, {0.1}⟩
c_5	⟨{0.9}, {0.1}, {0.1}⟩	⟨{0.8, 0.9, 1}, {0.2, 0.3}, {0.4}⟩

By using Equation (1), we determine the weights of decision makers. $w_1 = 0.264$, $w_2 = 0.081$, $w_3 = 0.182$, $w_4 = 0.473$

Step 3: Weighted decision matrix

From decision matrices, we take max values of truth and min values of indeterminacy and false matrices. Table 3 shows the weighted decision matrix of weights of criteria.

The weighted decision matrices are separated into 3 matrices which as truth, indeterminacy, and false matrices. By Equation (2), we determine the weighted decision matrix.

$$(C_1)(DM_1) = 1 - (1 - 0.3)^{0.264}, (0.1)^{0.264}, (0.4)^{0.264} = (0.0899, 0.5445, 0.7851)$$

$$(C_1)(DM_2) = 1 - (1 - 0.4)^{0.081}, (0.1)^{0.081}, (0.6)^{0.081} = (0.0405, 0.8299, 0.9595)$$

$$(C_1)(DM_3) = 1 - (1 - 0.3)^{0.182}, (0.2)^{0.182}, (0.5)^{0.182} = (0.0629, 0.7461, 0.8815)$$

$$(C_1)(DM_4) = 1 - (1 - 0.4)^{0.473}, (0.2)^{0.473}, (0.7)^{0.473} = (0.2146, 0.4671, 0.8448)$$

Table 3. Max min values of decision matrix of weights of criteria.

Criteria	DM_1	DM_2	DM_3	DM_4
c_1	⟨{.3}, {.1}, {.4}⟩	⟨{.4}, {.1}, {.6}⟩	⟨{.3}, {.2}, {.5}⟩	⟨{.4}, {.2}, {.7}⟩
c_2	⟨{.8}, {.5}, {.3}⟩	⟨{.7}, {.5}, {.3}⟩	⟨{.7}, {.4}, {.3}⟩	⟨{.5}, {.2}, {.7}⟩
c_3	⟨{.7}, {.2}, {.6}⟩	⟨{.5}, {.4}, {.6}⟩	⟨{.7}, {.3}, {.5}⟩	⟨{.5}, {.3}, {.4}⟩
c_4	⟨{.6}, {.2}, {.1}⟩	⟨{.7}, {.4}, {.1}⟩	⟨{.6}, {.3}, {.2}⟩	⟨{.7}, {.4}, {.1}⟩
c_5	⟨{.8}, {.5}, {.2}⟩	⟨{.9}, {.4}, {.2}⟩	⟨{.9}, {.1}, {.1}⟩	⟨{1}, {.2}, {.4}⟩

Similarly, we calculate for every criterion. The following W(T), W(I), and W(F) are the truth, indeterminacy, and false weighted decision matrix of weights of criteria.

Truth-weighted decision matrix for criteria weights

$$W(T) = \begin{pmatrix} 0.0899 & 0.0405 & 0.0629 & 0.2146 \\ 0.3462 & 0.0929 & 0.1968 & 0.2795 \\ 0.2723 & 0.0546 & 0.1968 & 0.2795 \\ 0.2149 & 0.0929 & 0.1536 & 0.4342 \\ 0.3462 & 0.1701 & 0.3423 & 1.0000 \end{pmatrix}$$

Indeterminacy and False weighted decision matrix for criteria weights

$$W(I) = \begin{pmatrix} 0.5445 & 0.8299 & 0.7461 & 0.4671 \\ 0.8328 & 0.9454 & 0.8464 & 0.4671 \\ 0.6538 & 0.9285 & 0.8032 & 0.5658 \\ 0.6538 & 0.9285 & 0.8032 & 0.6483 \\ 0.8328 & 0.9285 & 0.6577 & 0.4671 \end{pmatrix}$$

$$W(F) = \begin{pmatrix} 0.7851 & 0.9595 & 0.8815 & 0.8448 \\ 0.7277 & 0.9071 & 0.8032 & 0.8448 \\ 0.8738 & 0.9595 & 0.8815 & 0.6483 \\ 0.5445 & 0.8299 & 0.7461 & 0.3365 \\ 0.6538 & 0.8778 & 0.6577 & 0.6483 \end{pmatrix}$$

The same process is continued for each alternative to determine the weighted decision matrix. The following $wA_1(T), wA_1(I), andwA_1(F)$ are the truth, indeterminacy, and false weighted decision matrix of alternative A_1.

Truth weighted decision matrix for alternative A_1

$$wA_1(T) = \begin{pmatrix} 0.1672 & 0.2723 & 0.2149 & 0.2723 & 0.3462 \\ 0.0929 & 0.0546 & 0.1222 & 0.0546 & 0.1222 \\ 0.1968 & 0.1968 & 0.2539 & 0.1536 & 0.1968 \\ 0.5329 & 0.3517 & 0.5329 & 0.2795 & 0.3517 \end{pmatrix}$$

Indeterminacy weighted decision matrix for alternative A_1

$$wA_1(I) = \begin{pmatrix} 0.0572 & 0.0899 & 0.0572 & 0.0572 & 0.1672 \\ 0.0285 & 0.0285 & 0.0085 & 0.0085 & 0.0285 \\ 0.0629 & 0.0398 & 0.0190 & 0.0629 & 0.0629 \\ 0.2795 & 0.2795 & 0.1002 & 0.1002 & 0.1552 \end{pmatrix}$$

False weighted decision matrix for alternative A_1

$$wA_1(F) = \begin{pmatrix} 0.2149 & 0.1672 & 0.1672 & 0.1262 & 0.0572 \\ 0.0179 & 0.0085 & 0.0546 & 0.0285 & 0.0085 \\ 0.0398 & 0.0888 & 0.0629 & 0.1185 & 0.0398 \\ 0.2146 & 0.1552 & 0.2146 & 0.0486 & 0.1002 \end{pmatrix}$$

Step 4: Converting the matrices into square matrix

By multiplying respected Weighted decision matrices of alternatives and criteria, we get the square matrix. Here we got the Truth square matrix of Alternative A_1.

$$wA_1(T)_{n\times m} * W(T)_{m\times n} = \begin{pmatrix} 0.3461 & 0.1280 & 0.2667 & 0.6365 \\ 0.1146 & 0.0414 & 0.0909 & 0.2153 \\ 0.2561 & 0.0879 & 0.1920 & 0.4317 \\ 0.4966 & 0.1692 & 0.3709 & 0.8347 \end{pmatrix}_{n\times n}$$

Similarly, the indeterminacy and false square matrices of A_1 will be calculated.

Step 5: Determine the energy

By definition (2.9) the energy of the above matrix is calculated. We got 3 energies for truth, indeterminacy, and false matrices respectively. The energy of the Truth matrix of $A_1 = 2.1647$, Energy of Indeterminacy matrix of $A_1 = 1.5716$ and Energy of False matrix of $A_1 = 2.1475$. Determine the energy of each alternative in the same way. The energy of each alternative is as follows

$$E[A_1] = [2.1647, 1.5716, 2.1475]$$
$$E[A_2] = [2.6992, 2.0964, 1.3898]$$
$$E[A_3] = [1.7092, 2.5609, 2.6289]$$
$$E[A_4] = [1.9050, 1.5764, 3.6858]$$

Step 6: Ranking of alternatives

Calculate the final ranking values by Equation (4), Table 4 represents the final values of each alternative and raking order.

The ranking order is $A_2 > A_1 > A_3 > A_4$. By the result Machine 2 is selected as the best one.

Table 4. Ranking of alternatives.

Alternatives	Truth energy	Ranking order
A_1	3.7535	II
A_2	6.105	I
A_3	3.3504	III
A_4	1.7006	IV

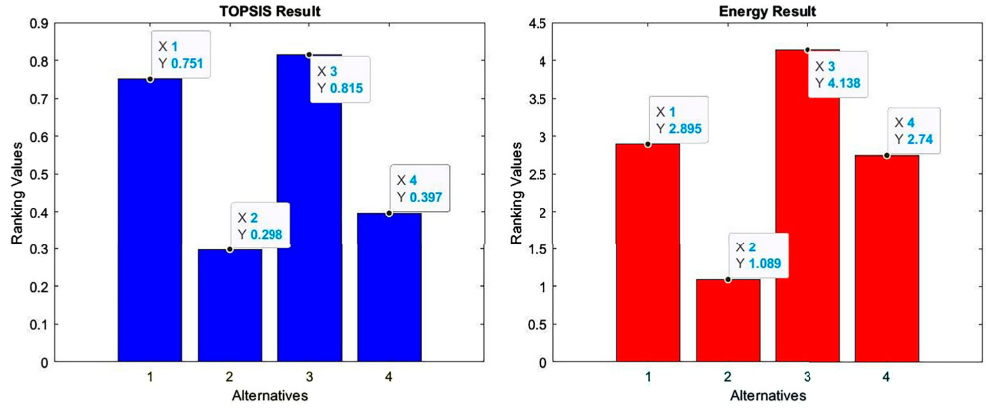

Figure 1. Camparison of TOPSIS result and energy result.

6. Comparison of our proposed method

In this section, we compare our proposed method to the TOPSIS method. In 2022, Deepa et al. presented a paper titled "Operations on Multi-Valued Neutrosophic Matrices and Its Application to Neutrosophic Simplified-TOPSIS Method". In this paper, they use a multi-valued neutrosophic set for solving a decision-making problem. The problem is about selecting the best sim card for the phone. In this problem, there are 4 alternatives, 3 decision-makers, and 4 criteria. They solved this issue well with the TOPSIS method. Here we solve this same problem with our proposed multi-valued neutrosophic energy method. In this problem, they use the linguistic variable for multi-valued neutrosophic numbers. So directly we take the max value of truth and the min value of indeterminacy, false values for the matrix. Then all other steps are done with the same procedure for this problem. We get the square matrix of the first alternative truth matrix.

$$wA_1(T)_{3\times4} * W(T)_{4\times3}$$
$$= A_1(T) = \begin{pmatrix} 0.5059 & 0.4307 & 0.4540 \\ 0.2389 & 0.2321 & 0.2088 \\ 0.2628 & 0.2400 & 0.2286 \end{pmatrix}_{4\times4}$$

The energy of truth matrix of Alternative $A_1 = 1.2597$

The remaining energies are calculated for each alternative.

$$E[A_1] = [1.2597, 2.1013, 1.7260]$$
$$E[A_2] = [0.8416, 3.5908, 4.1851]$$
$$E[A_3] = [1.6347, 1.4042, 0.5354]$$
$$E[A_4] = [1.1104, 2.1104, 1.5895]$$

By Equation (4), calculate the final ranking values. $R_1 = 2.8947$, $R_2 = 1.0889$, $R_3 = 4.1382$, $R_4 = 2.7405$.

6.1. Results and discussion

We compare the neutrosophic TOPSIS result with our proposed neutrosophic energy result. It is shown in Table 5.

Table 5. Comparison and results.

Alternative	TOPSIS result	Rank	Enery result	Rank
A_1	0.751	II	2.8947	II
A_2	0.298	IV	1.0889	IV
A_3	0.815	I	4.1382	I
A_4	0.397	III	2.7405	III

The order of ranking for the multi-valued neutrosophic TOPSIS method and multi-valued neutrosophic energy method is as follows $A_3 > A_1 > A_4 > A_2$. It was shown in Figure 1. The ranking order is the same and the process is easy to compare our method to the neutrosophic TOPSIS method. Our proposed method simplifies our work and solves the problem in an easy way. These comparative results were given to demonstrate the truthfullness of the proposed energy outcomes and existing method's outcome. The method made the new concept of neutrosophic matrix energy in decision-making problems. So it was clear our suggested method is applicable to solving the MCDM problem, further, it will develop in all application areas.

7. Conclusion

In a multi-valued structure, the new concept of the neutrosophic matrix's energy was applied. In this case, the hesitant neutrosophic set was closely connected to the multi-valued neutrosophic set. So we provided the relationship between the multi-valued and hesitant neutrosophic matrices, and by the max-min process, they will convert it into single-valued neutrosophic matrices to find the matrix energy. We solved the multi-criteria decision-making problem using a unique multi-valued neutrosophic matrix energy technique. The problem is to deciding which machine is in good condition at a particular company, and as a result, machine 2 is in better condition than the other alternatives. The proposed energy plays an important role in calculating the final ranking. Numerous methods exist in the neutrosophic multi-criteria decision-making field for selecting the best alternative. However, the suggested energy approach produced a highly effective and perfect result, and it also made the work easier and emphasized the value of the matrix. The results of the proposed method were then compared to the existing MCDM method's results to prove the correctness of the study. The comparative results are displayed in the figure for clarity. In the future, the proposed method can be applied to other extensions of fuzzy sets, such as picture fuzzy and quasirung ortho-pair fuzzy sets [36, 37]. Which are newly developing sets in the MCDM environment. Further, it will be applied to the various types of neutrosophic matrices.

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