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THE ELECTRIC FIELD CONFIGURATION IN THE CYLINDRICAL MULTIWIRE PROPORTIONAL COUNTER

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Abstract: The multiwire proportional counter was built from a steel tube and two concentric series of the wire electrodes so that a central counter was surrounded by 15 counter elements. An electric potential configuration around the wire anode by means of the complex potential function and method of image was determined. The relation between electric field intensity, electrode high voltage and distance from the anode was derived and confirmed by the results of measurements.

1. Introduction

The multiwire proportional counter is a system of several concentric wire series situated in a conducting cylinder. An especial problem is to determine electric field intensity near such an anode and to find a relation between anode potentials of the counter elements of different geometry.

The electric field intensity E at a distance r from the long linear wire conductor, which has a linear charge per length unit q, is defined by the next relation (Smythe[1])

$$E = \frac{2q}{r}. \qquad (1)$$

In a cylindrical coordinate system the electric field vector has a radial orientation, e. g. $\vec{E} = \vec{E}_r$, $\vec{E}_\varphi = 0$.

Since E is equivalent to a potential gradient, from the relation (1) comes the electric potential V of the linear charge

$$V = -\,2q\ln(r). \tag{2}$$

In a plane of the complex variable z, a function of the complex potential w becomes for the linear charge

$$w = -2q\ln(z) = -2q\ln(r) - 2qi\varphi. \tag{3}$$

If the wire intersects the complex plane perpendicularly in the point $z_0 = r_0 e^{i\varphi_0}$, the complex potential in z is

$$w = -\,2q\ln(z - z_0). \tag{4}$$

The function of n linear charges, situated in $z_1, z_2, \ldots, z_n$, is addition of the several potentials

$$w = -2\sum_{s=1}^{n} q_s \ln(z - z_s). \tag{5}$$

For n the same wires situated at a circle in the points

$$z_s = r_0\, e^{i\varphi_s}, \qquad \varphi_s = \frac{2(s-1)\pi}{n},$$

with the total linear charge q, the complex potential becomes

$$w = -\frac{2q}{n}\ln\prod_{s=1}^{n}(z - z_s) = -\frac{2q}{n}\ln(z^n - r_0^n). \tag{6}$$

When a linear wire conductor is put in a metal cylinder coaxialy, the charge image method by means of the fictive charge q' situated in z' gives the complex potential[1)]

$$w = -\,2q\ln\left(\frac{z - z_0}{z - z'}\right). \tag{7}$$

For the cylinder with the radius r_3, linear charge situated in $z_0 = r_0 e^{i\varphi_0}$ and fictive charge in $z' = r' e^{i\varphi_0}$ is valued at the next relation[1)]

$$r' = \frac{r_3^2}{r_0}. \tag{8}$$

According to the relations (8), (7) and (6) the complex potential of n wires situated at a circle ($r_0 < r_3$) coaxially in the metal cylinder has the form in a point z (Morse and Feshbach[2])

$$w = -\frac{2q}{n} \ln \frac{z^n - r_0^n}{z^n - (r_3/r_0)^n}. \tag{9}$$

The wires have the total charge q, but the cylindrical conductor is at zero potential.

2. *Apparatus*

The multiwire proportional counter (MWPC) was built from a steel tube and sixty-one wire electrodes (Fig. 1.). The tube consisted of a central and fifteen counter elements around the central counter. The radii of the wires were: $r_K = 200\ \mu$m (cathode), $r_A = 25\ \mu$m (excentrical anodes) and $r_{A0} = 10\mu$m (central anode). The cylinder and wire cathodes were grounded but the central and excentrical anodes were at different positive potential. The pulses from the anodes were led to pre-amplifiers, amplifiers, anticoincidence circuit and scaler.

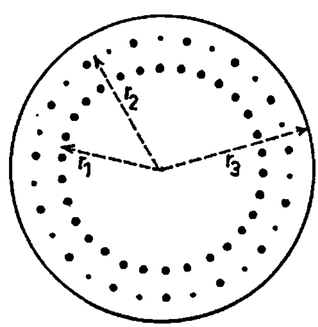

Fig. 1. The wire configuration in MWPC; $r_1 = 42{,}5$ mm, $r_2 = 53{,}7$ mm and $r_3 = 65$ mm.

3. *The electric field intensity near the MWPC anode*

Within the MWPC (Fig. 1.), according to the formula (9), the function of the complex potential is formed as the potential addition of central anode, 30 cathodes at radius r_1, 15 cathodes and 15 anodes at radius r_2

$$w = -2q_{A0} \ln\left(\frac{z}{r_3}\right) - \frac{2}{15} q_{A2} \ln \frac{z^{15} - r_2^{15}}{z^{15} - (r_3^2/r_2)^{15}} + \frac{2}{30} q_{K1} \ln \frac{z^{30} - r_1^{30}}{z^{30} - (r_3^2/r_1)^{30}} + $$
$$+ \frac{2}{15} q_{K2} \ln \frac{z^{15} - r_2^{15}}{z^{15} - (r_3^2/r_2)^{15}}. \tag{10}$$

The central anode potential V_{A0} is the real part of the complex function (10). If we choose a point at the anode surface $z_{A0} = r_A, (\varphi_0 = 0)$, with the potential V_{A0} the relation (10) gives

$$V_{A0} = -2q_{A0} \ln \left(\frac{r_A}{r_3}\right) - \frac{2}{15} q_{A2} \ln \frac{r_A^{15} - r_2^{15}}{r_A^{15} - (r_3^2/r_2)^{15}} +$$

$$+ \frac{2}{30} q_{K1} \ln \frac{r_A^{30} - r_1^{30}}{r_A^{30} - (r_3^2/r_1)^{30}} + \frac{2}{15} q_{K2} \ln \frac{r_A^{15} - r_2^{15}}{r_A^{15} - (r_3^2/r_2)^{15}}. \quad (11)$$

After the quotient division by r_A and approximations $r_1/r_A, r_2/r_A, r_3^2/r_1 r_A, r_3^2/r_2 r_A \gg 1$, the relation (11) becomes

$$V_{A0} = 2q_{A0} \ln \frac{r_3}{r_A} + 4q_{A2} \ln \frac{r_3}{r_2} - 4q_{K1} \ln \frac{r_3}{r_1} - 4q_{K2} \ln \frac{r_3}{r_2}. \quad (12)$$

We wish now to express q_{A0} explicitly wirth V_{A0} and to introduce it to the relation (1). The linear charges q_{A2}, q_{K1} and q_{K2} can be eliminated by means of the formation of next equations: the potential definition of the excentrical anode V_{A2} and cathode V_{K1} in the points $z_2 = (r_2 + r_A)$ and $z_1 = (r_1 + r_K)$, then the treatment of these potential equations and approximation as with the equation (11) give

$$V_{A2} = 2q_{A0} \ln \frac{r_3}{r_2} + \frac{2}{15} q_{A2} \ln \left(\frac{r_3}{r_2}\right)^{30} \frac{r_2}{15 r_A} - \frac{2}{30} q_{K1} \ln \left(\frac{r_3^2}{r_1 r_2}\right)^{30} -$$

$$- \frac{2}{15} q_{K2} \ln \left(\frac{r_3}{r_2}\right)^{30} \frac{r_2}{15 r_A}, \quad (13)$$

$$V_{K1} = 2q_{A0} \ln \frac{r_3}{r_1} + \frac{2}{15} q_{A2} \ln \left(\frac{r_3}{r_2}\right)^{30} - \frac{2}{30} q_{K1} \ln \left(\frac{r_3}{r_1}\right)^{60} \frac{r_1}{r_K} \frac{1}{30} -$$

$$- \frac{2}{15} q_{K2} \ln \left(\frac{r_3}{r_2}\right)^{30}$$

Instead the equation (12) and (13) we can write

$$V_{A0} = a_1 q_{A0} + a_2 q - a_3 q_{K1},$$
$$V_{A2} = a_4 q_{A0} + a_5 q - a_6 q_{K1}, \quad (14)$$
$$V_{K1} = a_7 q_{A0} + a_7 q - a_8 q_{K1},$$

where $q = q_{A2} - q_{K2}$, and a_i $(i = 1, 2, \dots, 8)$ are the coefficients determined in equations (12) and (13) (for instance, $a_1 = 2 \ln(r_3/r_A)$, $a_2 = 4 \ln(r_3/r_2)$, etc.).

The described MWPC has the coefficients

$$a_1 = 17.520,\quad a_3 = 1.704,\quad a_5 = 1.533,\quad a_7 = 0.852,$$
$$a_2 = 0.760,\quad a_4 = 0.380,\quad a_6 = 1.232,\quad a_8 = 1.882. \tag{15}$$

The main determinant of the system (14) is

$$D = a_2 (a_1 a_6 + a_4 a_8 - a_6 a_7 - a_3 a_4) + a_5 (a_3 a_7 - a_1 a_8)$$

and the other determinant of q_{A0}, after the practical condition $V_{K1} = 0$

$$D_1 = a_2 (a_8 - a_3) V_{A2} + (a_2 a_6 - a_5 a_8) V_{A0}.$$

The variable $q_{A0} = D_1/D$ is now

$$q_{A0} = \frac{a_2(a_8 - a_3)}{D} V_{A2} + \frac{(a_2 a_6 - a_5 a_8)}{D} V_{A0}. \tag{16}$$

The electron multiplication starts near the proportional counter anode (one or several anode radii from the anode surface) and in this region the electric field intensity is well described by the relation (1). From the relations (1) and (16) follows the electric fiield intensity near the MWPC central anode

$$E_{A0} = \frac{2(a_2 a_6 - a_5 a_8)}{D} \frac{V_{A0}}{r} + \frac{2a_2(a_8 - a_3)}{D} \frac{V_{A2}}{r}. \tag{17}$$

According to the relation (17) the MWPC with the coefficients (15) has the electric field intensity at distance r from the central anode

$$E_{A0} = \frac{V_{A0}}{8.34\, r} - \frac{V_{A2}}{250\, r}. \tag{18}$$

The classical proportional counter with an anode which has a cylindrical cathode instead of thirty wire cathodes of MWPC has the electric field intensity, Snell[3)]

$$E = \frac{V_0}{\ln (r_3/r_A)\, r},$$

or

$$E = \frac{V_0}{8.34\, r}. \tag{19}$$

The comparison of the relations (18) and (19) leads to the conclusion that a charge wire series can be changed correctly by the charge surface.

The electric field intensity around excentrical anode can be determined in the same way as for the central anode. But now, the excentrical anode with charge q_A is situated in the coordinate origin of the cylindrical system. A counter element with wires 0,1,2,3,4,5 (Fig. 2.), (an influence of the other electrodes is neglected) according to the relations (6) and (7) has the complex potential

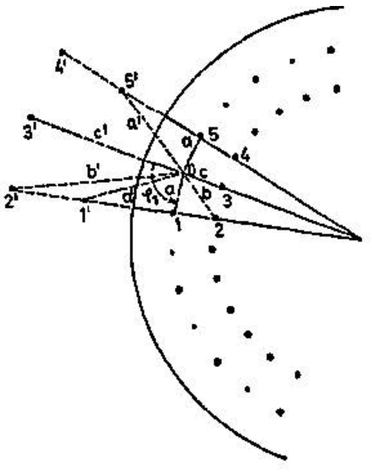

Fig. 2. An excentrical counter element.

$$w_e = -2q_A \ln \frac{z}{z - z_0'} + \frac{2}{5} q_K \ln \frac{\prod_{i=1}^{5} (z - z_i)}{\prod_{i=1}^{5} (z - z_i')}, \tag{20}$$

which gives the electric potential in the point $z_A = r_A$

$$V_{A2} = -2q_A \ln \frac{r_A}{(r_3^2/r_2) - r_A} - \frac{2}{5} q_K L(r_A), \tag{21}$$

where

$$L(r_A) = \ln \frac{(r_A - a' \cos \varphi_1')^2 (r_A - b' \cos \varphi_2')^2 (r_A - c' \cos \varphi_3')}{(r_A - a \cos \varphi_1)^2 (r_A - b \cos \varphi_2)^2 (r_A - c \cos \varphi_3)} \tag{21a}$$

Five roundabout cathodes with total charge q_K have the position parameters in MWPC

$a = 11.2$ mm,	$a' = 28$ mm,	$\varphi_1 = 97°30'$	$\varphi_1' = 23°10'$,
$b = 15.3$ mm,	$b' = 48$ mm,	$\varphi_2 = 143°12'$,	$\varphi_2' = 14°10'$,
$c = 11.2$ mm,	$c' = 45.8$ mm,	$\varphi_3 = 180°$,	$\varphi_3' = 0°$,

and $L(r_A) = 9.79$.

After approximation $r_A \ll r_3^2/r_2$ the potential (21) becomes

$$V_{A2} = 2q_A \ln \frac{r_3^2}{r_2 r_A} - \frac{2}{5} q_K L(r_A). \tag{22}$$

The potential at the anode surface in the point $z_K = -(a + r_K)$, considering the relation (20), comes to

$$V_K = 2q_A \ln \left(\frac{r_3^2}{r_2 (a + r_K)} - 1 \right) - \frac{2}{5} q_K L[-(a + r_K)]. \tag{23}$$

The described MWPC has $L[-(a + r_K)] = 11.48$.

If $V_K = 0$ the equation (22) and (23) can be written

$$V_{A2} = a_{10} q_A - a_{11} q_K,$$
$$0 = a_{12} q_K - a_{13} q_K,$$

with a solution

$$q_A = \frac{a_{13}}{a_{10} a_{13} - a_{11} a_{12}} V_{A2}, \tag{24}$$

where

$$a_{10} = 2 \ln (r_3^2/r_2 r_A), \quad a_{11} = \frac{2}{5} L(r_A),$$

$$a_{12} = 2 \ln \left(\frac{r_3^2}{r_2 (a + r_K)} - 1 \right), \quad a_{13} = \frac{2}{5} L[-(a + r_K)].$$

According to the relation (1) and (24) the electric field intensity near the excentrical anode is

$$E_{A2} = \frac{2 a_{13}}{(a_{10} a_{13} - a_{11} a_{12})} V_{A2}. \tag{25}$$

The MWPC has

$$E_{A2} = \frac{V_{A2}}{7.44 r}.$$

The condition of equal gas amplification in central and excentrical counter element demands the equality of the electric field intensity in all counter elements, e. g. $E_{A0} = E_{A2}$, giving from the relations (17) and (25)

$$V_{A0} = \frac{\dfrac{a_{13} D}{a_{10} a_{13} - a_{11} a_{12}} - a_2 (a_8 - a_3)}{a_2 a_6 - a_5 a_8} V_{A2}. \tag{26}$$

The MWPC has

$$V_{A0} = 1.150\ V_{A2}. \qquad (26a)$$

The formula (26) is important for the multiwire proportional counting because it shows the relation between high voltages of the counter elements of different geometry; the parameter values depend on the counter geometry only.

After the measurements of the count number vs. voltage for central and excentrical counter element, the work voltages U_c and U_e were chosen in the conditions of the same gas amplifications. The MWPC filled by methan at pressure of 1.15 atm had the voltages $U_c = 2600$ V and $U_e = 2300$ V. The quotient $U_c/U_e = 1.13$ is very near the value 1.15 from the relation (26a), which confirms applicability and validity of the formula (26).

4. *Conclusion*

Electric potential configuration within the multiwire proportional counter can be determined by means of the complex potential function formed for sixty-one wire electrodes, cylindrical conductor and corresponding charge images. The relation between electric field intensity, electrode high voltage and distance from the central or excentrical anode was derived; the formula was valuable in the gas amplification region. The estimated theoretical value of the quotient of central and excentrical anode potentials was confirmed by the measurements of count number vs. voltage and choosing the counter working voltages.

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KONFIGURACIJA ELEKTRIČNOG POLJA U CILINDRIČNOM VIŠEŽIČANOM PROPORCIONALNOM BROJAČU

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Sadržaj

Izgrađen je višežičani proporcionalni brojač za mjerenje niskih beta aktivnosti od čelične cijevi i dva koncentrična niza žičanih elektroda, tako da je središnji brojač okružen s 15 brojačkih elemenata. Konfiguracija električnog potencijala oko žičane anode određena je pomoću funkcije kompleksne varijable i metode slike naboja. Izvedena relacija između jakosti električnog polja, napona elektroda i udaljenosti od anode potvrđena je rezultatima mjerenja.