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A SEMICLASSICAL APPROACH TO THE PROBLEM OF NEUTRON-PROTON MASS DIFFERENCE*)

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Abstract: The neutron-proton mass difference is calculated in a model formulated in the framework of classical electrodynamics. The value thus obtained is consistent with the result which follows from the generalized Born approximation in quantum electrodynamics. As it is well known, this value is not satisfactory. Comparison of these approaches leads to the suggestion how to attack the problem, including more details of strong-interaction dynamics in the framework of quantum field theory.

1. Introduction

One of the most outstanding puzzles in hadron physics for decades has been the question why a neutron is heavier than a proton. This magnificent experimental fact, which ensures the stability of the world against the inverse β decay, contradicts naive expectation and has received no satisfactory theoretical explanation so far[1]. Calculations of $\Delta m = m_P - m_N$ along conventional lines have reached results which are, more or less, wrong even in sign.

If electromagnetism alone is able to distinguish these two otherwise identical particles, then quantum electrodynamics should explain this mass shift.

However, it has become obvious that it is the interplay of electromagnetic and strong interactions which is responsible for electromagnetic shifts of hadron

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masses. A full answer to the problem is certainly related to the fundamental questions concerning the origin of hadron masses themselves, and the presence of the strong interaction does not allow too much optimism at present.

Here we look at the problem from the semiclassical point of view, for it seems unlikely that its solution is exclusively a quantum-mechanical one. As a matter of fact, it was the classical electron where the trouble with an infinite self-energy appeared for the first time. The problem still remained in quantum electrodynamics, for there was no sucessful attempt how to abandon the concept of pointlike electron.

In the case of hadrons we are partly in a better position because they are not pointlike but smeared out by the strong interaction. However, our knowledge of their internal structure is not precise enough to calculate electromagnetic self--energies, usually identified with mass differences among members of the same isospin multiplet, in a satisfactory way. More details of strong-interaction dynamics have to be taken into account.

2. *The model*

Since a nucleon is, indeed, an extended object, visualized as a cloud or a bag of hadronic matter, we would naively expect that its electromagnetic self--energy might be calculated by using the charge density $\varrho_E(\vec{r})$ and the magnetic moment density $\vec{m}(\vec{r})$, as given by Fourier transforms of the corresponding electromagnetic form factors. Thus, the electrical energy is

$$U_{el} = \frac{e^2}{2} \int \frac{\varrho_E(\vec{r})\, \varrho_E(\vec{r}\,')\, d^3\vec{r}\, d^3 r'}{|\vec{r} - \vec{r}\,'|} =$$

$$= \frac{e^2}{2(2\pi)^6} \int d^3 r d^3 r' d^3 q d^3 q' \frac{e^{i(\vec{q}\vec{r} + \vec{q}\,'\vec{r}\,')}}{|\vec{r} - \vec{r}\,'|} G_E(-\vec{q}^{\,2})\, G_E(-\vec{q}\,'^2), \tag{1}$$

where the conventional definition of the spatial charge density[2)]

$$\varrho_E(\vec{r}) = \frac{1}{(2\pi)^3} \int d^3 r\, e^{i\vec{q}\vec{r}}\, G_E(-\vec{q}^{\,2}) \tag{2}$$

has been accepted. It should be emphasized that this definition depends on a particular choice of the Lorentz frame (the Breit system) in which the momentum transfer q has no time component and the nucleon is not stationary. The relation of the above density to any real physical extension of the nucleon is not clear at high momentum transfers. Fortunately, the form factors decrease so rapidly that the integral (2) is dominated by the contribution from low-momentum transfers.

After slight modifications, we first perform the integration over $\vec{r}$ and $\vec{r}'$ in (1)

$$\int d^3 r' \, e^{i\vec{r}' (\vec{q}+\vec{q}')} \int d^3 r \frac{e^{i\vec{q}(\vec{r}-\vec{r}')}}{|\vec{r}-\vec{r}'|} = \frac{4\pi}{q^2} (2\pi)^3 \, \delta(\vec{q}+\vec{q}'),$$

so we finally obtain

$$U_{el} = \frac{a}{\pi} \int_0^\infty dq \, G_E^2(-\vec{q}^2), \qquad a \equiv e^2 = 1/137. \tag{3}$$

On the other hand, we define the density of the magnetic moment as

$$\vec{m}(\vec{r}) = \mu \frac{e}{2m} \vec{k}_0 \, \varrho_M(\vec{r}), \tag{4}$$

where μ denotes the total magnetic moment of the nucleon, $\vec{k}_0$ is a unit vector of no importance here, and ϱ_M is given by

$$\mu \varrho_M(\vec{r}) = \frac{1}{(2\pi)^3} \int d^3 q \, e^{i\vec{q}\vec{r}} G_M(-\vec{q}^2). \tag{5}$$

The magnetic moment density (4) generates the vector of magnetic induction

$$\vec{B}(\vec{r}) = \int d^3 r' \, \vec{\nabla}_r \times \frac{\vec{m}(\vec{r}') \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} =$$

$$= -\frac{2}{3} \int d^3 r' \, \vec{m}(\vec{r}') \, \nabla^2 \left(\frac{1}{|\vec{r}-\vec{r}'|} \right) = \frac{2}{3} 4\pi \, \vec{m}(\vec{r}). \tag{6}$$

The total magnetic self-energy is then

$$U_{mag} = -\int \vec{m}(\vec{r}) \, \vec{B}(\vec{r}) \, d^3 r = -\frac{a}{\pi} \frac{1}{3m^2} \int_0^\infty dq \, q^2 G_M^2(-\vec{q}^2). \tag{7}$$

As the form factor decreases rapidly from its value at $q^2 = 0$, the magnetic self--energy is strongly damped by the factor q^2 in the integrand. This factor is not present in the electric charge contribution (3). A moderate decrease of form factors may, nevertheless, cause that

$$\Delta m \equiv \Delta m_{el} + \Delta m_{mag}$$

becomes negative due to the dominance of the magnetic contribution (7). The existing experimental situation does not indicate that this might happen. More precisely, using the experimental form factors[3]

$$G_{E,P}(q^2) \cong \frac{G_{M,P}(q^2)}{\mu_P} \cong \frac{G_{M,N}(q^2)}{\mu_N} \cong \frac{m_D^4}{(m_D^2 - q^2)^2} \equiv G_D(q^2), \tag{8}$$

$$G_{E,N}(q^2) \cong 0, \quad m_D^2 = 0.71 \text{ GeV}^2, \quad q^2 \to -\vec{q}^{\,2}, \tag{8'}$$

we obtain

$$\Delta m = \frac{\alpha}{\pi} \int_0^\infty \mathrm{d}q \left[G_{E,P}^2 - G_{E,N}^2 - \frac{\vec{q}^{\,2}}{3m^2} (G_{M,P}^2 - G_{M,N}^2) \right] \tag{9}$$

$$= \frac{\alpha}{\mu} \int_0^\infty \mathrm{d}q \left[1 - \frac{\mu_P^2 - \mu_N^2}{3m^2} \vec{q}^{\,2} \right] G_D^2 (-\vec{q}^{\,2}) = 0.73 \text{ MeV}. \tag{10}$$

If the universal function G_D instead of a dipole has been given the form of a simple ϱ pole

$$G_D \to G_\varrho = \frac{m_\varrho^2}{m_\varrho^2 - q^2}, \tag{11}$$

then a small negative value is obtained

$$\Delta m = -0.11 \text{ MeV}. \tag{12}$$

This value is still far from the observed value[4]

$$\Delta m_{\text{exp}} = (-1.29344 \pm 0.00007) \text{ MeV}. \tag{13}$$

It is, however, possible to choose the magnetic form factors $(G_{M,N}^2 - G_{M,P}^2)$ in a form not too much different from the dipole behaviour, so that the dominance of the magnetic mass would be strong enough to compensate the charge mass and reproduce the observed value. However, it is not justified to make such a choice at present.

3. *Conclusion and discussion*

It is interesting that our semiclassical result (10) is consistent with the generalized Born approximation in quantum electrodynamics, represented by the diagram in Fig. 1a. Using the same experimental form factors (8) and (8′) one obtains Δm_{Born} of typically about $+1$ MeV[1]. It is believed that the inclusion of all possible hadronic intermediate states, graphically represented in Fig. 1b, should reverse the sign. This has led to the study of a four-dimensional integral over the virtual foton-nucleon forward scattering amplitude, which, by itself, represents another difficult problem. No satisfactory results have been obtained so far, and the main achievement consists in searching for possible leading terms which should contribute to Δm with the negative sign.

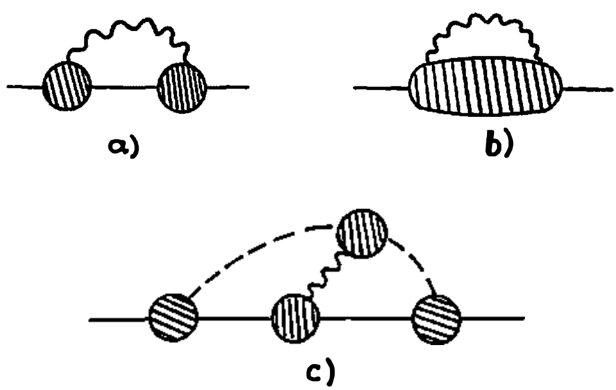

Fig. 1. a) The generalized Born approximation for the electromagnetic mass shift of the nucleon,
b) The most general diagram for the mass shift to the second order in the electromagnetic coupling,
c) A particular diagram which is contained in (b) and represents a large contribution to the mass shift.
(wavy line — photon, dashed line — pion, solid line — nucleon)

How can we understand the fact that the generalized Born approximation and our classical model lead to practically the same result. A natural possibility is that both approaches somehow take into account the same part of the nucleon self-energy. It is a crucial point to observe that uncertainties in our form factors at high momentum transfers cause uncertainties in densities $\varrho_E(\vec{r})$ and $\varrho_M(\vec{r})$ at small space distances inside nucleons. This is a property of the Fourier transform (2). If $\vec{q}$ is large, then the exponential factor oscillates strongly and averages the contributions to zero unless $\vec{r}$ is very small. Thus, form factors at high momentum transfers contribute essentially to the charge and the magnetic moment densitites only in the region deeply inside the nucleon. This is just the source of quantum-mechanical fluctuations which are responsible for the nucleon structure.

Formula (9) rather accurately takes into account the outer part of the nucleon's bag. The same seems to be true of the generalized Born approximation (Fig. 1a). In this approximation the lightest components of the nucleon structure emit and reabsorb a photon. Short-range components, localized at small distances, do not get pronounced by replacing pointlike vertices by form factors only. More structure has to be included.

In particular, the physical nucleon structure may be viewed in the sense of a hadronic bootstrap

$$|N> = |N\pi> + |N\pi\pi> + \ldots + |\Sigma K> + \ldots \tag{14}$$

where the right-hand side represents all possible hadronic states (components) with quantum numbers of a nucleon. Of course, the life-time of the different components is not the same. Arguing along the line of the uncertainty principle, we may conclude that the πN component should be the dominant one. This standpoint has also been recently advocated by Granovskii[6], who suggests that merely radiative corrections to diagrams representing the strong self-mass are responsible for electromagnetic mass shifts.

There is now a variety of possibilities how to couple an intermediate-photon line when electromagnetism is switched on. The internal structure of all components in (14) enters the game. Besides the generalized Born term the next important contribution, relevant for $m_P - m_N$, is represented by the diagram in Fig. 1c. This calculation is in progress.

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