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LETTER TO THE EDITOR

A MODIFICATION OF THE METHOD WITH SHORT-RANGE ASYMPTOTIC WAVE FUNCTION IN THE MANY-BODY THEORY

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The method of short-range wave function analysis. The two-body interaction in the system represented by

$$V(r) = \frac{1}{2} \sum_{ij}{}' V(r_{ij}) \tag{1}$$

is divided in two parts

$$V(r) = V^1(r) + V^2(r), \tag{2}$$

where $V^1(r)$ describes the short-range interaction, while $V^2(r)$ describes the other of the interaction.

The wave function in the semi-free gas model[2] for short inter-particle distances is

$$\psi_{sr} = e^{-\frac{1}{2}\sum_{ij}' f(r_{ij})}, \tag{3}$$

where $f(r_{ij})$ is the two-particle asymptotic wave function for short inter-particle distances.

The Hamiltonian of the system observed is

$$H = \sum_i \left(-\frac{\hbar^2}{2m} \Delta_i \right) + \frac{1}{2} \sum_{ij}{}' V^1(r_{ij}) + \frac{1}{2} \sum_{ij}{}' V^2(r_{ij}). \tag{4}$$

The eigenvalue energy equation is

$$H\psi = E\psi \tag{5}$$

the solution of which is written as the product of wave functions describing the short-range motion and the remaining part of the motion

$$\psi = \psi_{sr} \cdot \Phi. \tag{6}$$

Introducing ψ from (6) and H from (4) into (5) we obtain.

$$\Phi\left[\sum_i \left(-\frac{\hbar^2}{2m}\Delta_i\right)\psi_{sr} + \frac{1}{2}\sum_{ij}{}' V_{ij}\psi_{sr}\right] + \psi_{sr}\left[\sum_i\left(-\frac{\hbar^2}{2m}\Delta_i\right)\Phi + \right.$$
$$\left. + \frac{1}{2}\sum_{ij}{}' V_{ij}\Phi\right] + \sum_i\left(-\frac{\hbar^2}{m}\right)\Delta_i\psi_{sr}\cdot\nabla\Phi = E\psi_{sr}\Phi. \tag{7}$$

Function $f(r_{ij})$ describing the short-range correlation is determined in such way so as to cancel the strong repulsive potential $V^1(r)$. This condition leads to equation

$$-\frac{\hbar^2}{m}(\Delta_i f_{ij})^2 + V_{ij}^1 = 0. \tag{8}$$

For the procedure given here, the following solution to this equation is selected

$$f(r) = \int_r^\infty \left[\frac{m}{\hbar^2}V^1(r')\right]^{1/2} \mathrm{d}r'. \tag{9}$$

Introducing $f(r)$, given with (9), into Equ. (7), we obtain

$$\psi_{sr}\left\{\left[\sum_i\left(-\frac{\hbar^2}{2m}\Delta_i\right)\Phi + \frac{1}{2}\sum_{ij}{}' W_{ij}\Phi\right] + \sum_{ijj'}{}''\left(-\frac{1}{2}\right)\left(V_{ij}^1 V_{ij'}^1\right)^{1/2}\cdot\right.$$
$$\left.\cdot\frac{\vec{r}_{ij}\cdot\vec{r}_{ij'}}{r_{ij}\cdot r_{ij'}}\cdot\Phi + \sum_{ij}{}'\left(\frac{\hbar^2}{m}\right)\cdot\left(\frac{m}{\hbar^2}V_{ij}^1\right)^{1/2}\cdot\frac{\vec{r}_{ij}}{r_{ij}}\cdot\nabla_i\Phi\right\} = E\psi_{sr}\Phi, \tag{10}$$

where

$$W_{ij} = V_{ij}^2 - \frac{1}{2}\left(\frac{m}{\hbar^2}V_{ij}^1\right)^{-1/2}\frac{\mathrm{d}V_{ij}^1}{\mathrm{d}r_{ij}} - \frac{\hbar^2}{m}\frac{2}{r_{ij}}\left(\frac{m}{\hbar^2}V_{ij}^1\right)^{1/2}. \tag{11}$$

The function Φ is further determined by $\Phi = \eta\, \varphi$ where φ is the solution to equation

$$\left[\sum_i \left(-\frac{\hbar^2}{2m} \Delta_i\right) + \frac{1}{2} \sum_{ij}{}' W_{ij}\right] \varphi = \varepsilon\, \varphi \tag{12}$$

and η is the solution of the remaining part of the equation. Where η is approximately constant, the problem of proper values is reduced to solution finding (12) so that the presented method acquires a practical value.

The improvement of the short-range wave function analysis method. The procedure presented above can be applied to systems which are in a fluid state. In such a state, systems will appear if they are composed of small mass particles and with weak attraction forces. When the particle mass increases the system tends to acquire crystal structure. In such a case, the term describing the three-particle interaction in Equ. (10) differs from zero for $j \neq j'$ so that V_{ij}^1 becomes significant because it contains a strong repulsive potential. In the cases like these, the Equ. (12) is not suitable any more for finding a solution to the problem observed.

To describe such systems it is necessary to introduce the three-particle term into W_{ij} as well. The equation (10) is written as

$$\psi_{sr}\left\{\left[\sum_i \left(-\frac{\hbar^2}{2m} \Delta_i\right) \Phi + \frac{1}{2} \sum_{ijj'}{}'' W_{ijj'}\right] + \right.$$
$$\left. + \sum_{ij}{}' \left(\frac{\hbar^2}{m}\right) \left(\frac{m}{\hbar^2} V_{ij}^1\right)^{1/2} \cdot \frac{\vec{r}_{ij}}{r_{ij}} \nabla_i\, \Phi\right\} = E\, \psi_{sr}\, \Phi, \tag{13}$$

where

$$W_{ijj'} = \left[V_{ij}^2 - \frac{1}{2} \left(\frac{m}{\hbar^2} V_{ij}^1\right)^{-1/2} \frac{\mathrm{d}V_{ij}^1}{\mathrm{d}r_{ij}} - \right.$$
$$\left. - \left(\frac{\hbar^2}{m}\right) \cdot \frac{2}{r_{ij}} \left(\frac{m}{\hbar^2} V_{ij}^1\right)^{1/2}\right] \cdot \delta_{jj'} - \left(V_{ij}^1\, V_{ij'}^1\right)^{1/2} \cdot \frac{\vec{r}_{ij} \cdot \vec{r}_{ij'}}{r_{ij} \cdot r_{ij'}}, \tag{14}$$

is a new reduced potential.

We assume that a section of equation (13) in the (square) brackets has a predominant role in describing the long-range correlative motion.

Therefore we write

$$\Phi = \varphi\, \eta \tag{15}$$

and determine function φ by equation

$$\left[\sum_i \left(-\frac{\hbar^2}{m}\Delta_i\right) + \frac{1}{2}\sum_{ijj'}'' W_{ijj'}\right]\varphi = \varepsilon\varphi. \tag{16}$$

Function η describes the other part of the motion.

Upon substitution of Φ from (15) into (13), taking into account (16), we obtain the equation for η

$$\psi_{sr}\Big\{\varepsilon\eta\varphi + \varphi\sum_i\left(-\frac{\hbar^2}{2m}\Delta_i\right)\eta + \sum_i\left(-\frac{\hbar^2}{m}\right)\nabla_i\varphi\cdot\nabla_i\eta + \\ + \sum_{ij}'\left(\frac{\hbar^2}{m}\right)\left(\frac{m}{\hbar^2}V_{ij}^1\right)^{1/2}\cdot\frac{\vec{r}_{ij}}{r_{ij}}\cdot\nabla_i(\varphi\eta)\Big\} = E\psi_{sr}\varphi\eta. \tag{17}$$

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