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A HIGH-ENERGY REPRESENTATION IN RELATIVISTIC HAMILTONIAN THEORY

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Abstract: Representations for the scattering amplitude at high energies have been obtained by summing diagrams of a covariant Hamiltonian formulation of quantum field theory (QFT) and diagrams of a three-dimensional formulation of QFT on the light cone.

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1. Introduction

A high-energy representation for the scattering amplitude is a suitable tool for describing a number of properties of the hadron interaction at high energies (see reviews[1,2]). In order to obtain the so-called eikonal representation for two-particle amplitudes, perturbation-theory series were summed in the four-dimensional Feynman-Dyson formalism[3] by use of the quasipotential equation (QPE)[1] and by the functional-integration method in quantum field theory (QFT)[2].

In Ref.[4], a high-energy representation for the scattering amplitudes was derived in the framework of the QPE[5] using Fourier analysis on a three-parametric group of horospherical shifts embedded as a subgroup in the Lorentz group[6,7]. This representation has the form

$$T(s,t) = -2is\int e^{-i\tilde{\Delta}\tilde{\rho}} d^2\tilde{\rho}\{d^2\tilde{\rho}_1 \langle\tilde{\rho}_1|\hat{P}_z \exp\left[\frac{1}{2s}\int \hat{V}_s(z)dz\right]|\tilde{\rho}\rangle - 1\},$$

$$\tilde{\Delta}^2 = -t \,. \qquad (1.1)$$

Here $\hat{P}_z$ is an ordering operator, in which the step functions

$$\hat{\theta}(z) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} \frac{e^{iaz}}{e^{a}-1-i\varepsilon} da \,,$$

obtained by finite-difference analysis[7,8], are used instead of the usual θ functions; $\hat{V}_s(z)$ is a quasipotential operator in the space of "state vectors" $|\tilde{\rho}\rangle$, with the two-dimensional vector $\tilde{\rho} = (\rho_1,\rho_2)$ being an analogue of the impact parameter. Formula (1.1) is a direct relativistic generalization of the eikonal representation of nonrelativistic quantum mechanics[9].

In this paper, a generalized eikonal representation (1.1) for the scattering amplitude at high energies is derived by summing diagrams of a covariant Hamiltonian formulation of QFT[10] and diagrams of a three-dimensional formulation of QFT on the light cone[11]. We consider the interaction Lagrangian $\mathcal{L}(x) = g:\Psi^2(x)\Phi(x):$, where $\Psi(x)$ is a "scalar nucleon" with mass M and $\Phi(x)$ is a scalar meson with mass m.

2. A generalized eikonal representation in relativistic Hamiltonian theory

We demonstrate how the usual eikonal representation is obtained in the framework of a three-dimensional formulation of QFT by summing generalized ladder diagrams desdribing the scattering of two high-energy "nucleons". The summation is performed by the variational-derivative method (see Fig. 1). To this end let us recall the basic rules for construction of matrix elements. Suppose that all vertices of a given Feynman diagram are numbered. Then the continuous dotted line of quasiparticles must connect all vertices and be oriented along the increasing vertex number. The internal solid lines of physical particles are oriented in the opposite direction, along the decreasing vertex number. The factor $\frac{1}{2\pi}\frac{1}{\kappa-i\varepsilon}$ corresponds to each internal dotted line with the four-momentum $\lambda\kappa$ ($\lambda^2=\lambda_0^2-\vec{\lambda}^2=1$, $\lambda^0>0$) and the function $\Delta^{(+)}(k)=\theta(k^0)\delta(k^2-M^2)$ ($D^{(+)}(k)=\theta(k^0)\times\delta(k^2-m^2)$) corresponds to each internal "nucleon" (meson) line with the four-momentum k.

The sum of diagrams in Fig. 1 gives the following expression for the amplitude on the energy shell (i.e. when $p_1+p_2 = q_1+q_2$)

$$(2\pi)^4 T(p_1,p_2;q_1,q_2)\delta(p_1+p_2-q_1-q_2) = \tag{2.1}$$

$$= \lim_{p_i^2,q_i^2\to M^2} \prod_{i=1}^{2}(p_i^2-M^2)(q_i^2-M^2)\mathbb{K}_\phi \mathbb{K}_A G(p_2,q_2|\chi_2)G(p_1,q_1|\chi_1)|_{\chi_1=\chi_2=0},$$

where $G(p,q|\chi) = \int dx dx' e^{ipx-iqx'} G(x,x'|\chi)$ is the Fourier transform of the one-particle Green function of a nucleon in the external field $\chi(x)\equiv A(x)\phi(x)$; it satisfies the equation

$$[\partial_x^2-M^2-g\chi(x)]G(x,x',|\chi) = \delta(x-x') .$$

The operators $\mathbb{K}_A$ and $\mathbb{K}_\phi$, which involve the derivative operators over the external fields $A_i(i=1,2)$ and ϕ, have the form

$$\mathbb{K}_A = \exp\{-ig^2\int du dv[\theta(\lambda u-\lambda v)D^{(-)}(u-v)-\theta(\lambda v-\lambda u)D^{(+)}(u-v)]\frac{\delta^2}{\delta A_1(u)\delta A_2(v)}\}$$

$$\mathbb{K}_\phi = \int dz \frac{\delta}{\delta\phi(z)} \exp\{\int\theta(\lambda z-\lambda z_1)\frac{\delta}{\delta\phi(z_1)} dz_1\} ,$$

where $D^{(\mp)}(x) = \pm i(2\pi)^{-3}\int e^{\mp ikx} D^{(+)}(k)dk$ is the negative- (positive-) frequency part of the Pauli-Jordan commutator function. The operator $\mathbb{K}_\phi$ appears owing to the presence of quasiparticles in the theory.

The validity of formula (2.1) can be verified by perturbation expansion using the following formal properties of the θ function:

$$\text{(i)}\quad \theta(x-x_1)\theta(x-x_2) = \theta(x-x_1)\theta(x_1-x_2)+\theta(x-x_2)\theta(x_2-x_1)\ ;$$

$$\text{(ii)}\quad \theta(x_1-x_2)\theta(x_2-x_3)\dots\theta(x_n-x_1)=0,\qquad n\geq 2\ ;$$

$$\text{(iii)}\quad \theta^n(x) = \theta(x),\quad n\geq 1; \tag{2.2}$$

$$\text{(iv)}\quad \sum_{\substack{\text{over all } n!\\ \text{permutations}}} \theta(x_{i_1}-x_{i_2})\theta(x_{i_2}-x_{i_3})\dots\theta(x_{i_{n-1}}-x_{i_n}) = 1\ .$$

Let us compare (2.1) with the analogous formula of Refs.[3] obtained by summing generalized ladder diagrams of the four-dimensional Feynman-Dyson formalism (see Fig. 2):

$$(2\pi)^4 T(p_1,p_2;q_1,q_2)\delta(p_1+p_2-q_1-q_2) = \tag{2.3}$$

$$= \lim_{p_1^2,q_1^2\to M^2} \prod_{i=1}^{2} (p_i^2-M^2)(q_i^2-M^2)\mathbb{K}G(p_2,q_2|A_2)G(p_1,q_1|A_1)\Big|_{A_1=A_2=0}\ ,$$

where

$$\mathbb{K} = \exp\{-ig^2\int D^c(u-v)\frac{\delta^2}{\delta A_1(u)\delta A_2(v)}\,du\,dv\}$$

For the physical external lines, the relativistic Hamiltonian scheme coincides with the Feynman one, the action of the operator $\mathbb{K}_\phi$ leads to the multiplication by unity, and the chain of diagrams of Fig. 1 coincides with the one given in Fig. 2, i.e. the relation (2.1) gives the same result as (2.3).

In $k_i k_j = 0$ approximation[2,3] for the amplitude $T(p_1,p_2;q_1,q_2)\equiv T(s,t)$, we obtain the eikonal representation

$$T(s,t) = -2is\int d^2b\, e^{-i\vec{\Delta}\vec{b}}\left(\exp\left[\frac{ig^2}{2s}\int\frac{d\vec{k}_\perp}{(2\pi)^2}\frac{e^{-i\vec{k}_\perp\vec{b}}}{m^2+\vec{k}_\perp^2}\right]-1\right),\quad \vec{\Delta}^2=-t. \tag{2.4}$$

In order to derive the generalized eikonal representation (1.1), let us examine, in the ladder approximation, completely

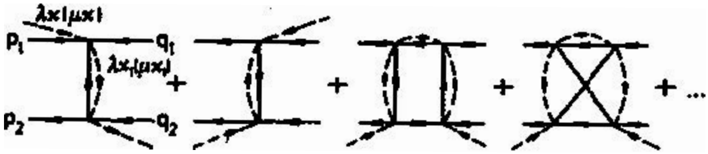

Fig. 1

Fig. 2

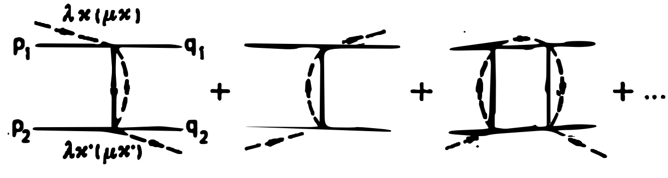

Fig. 3

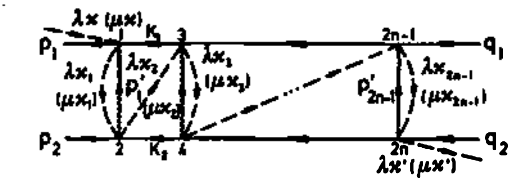
$\lambda\varkappa\,(\mu\varkappa)$
K_1
p_1
q_1
p_2
q_2
K_2
$\lambda\varkappa'(\mu\varkappa')$

Fig. 4

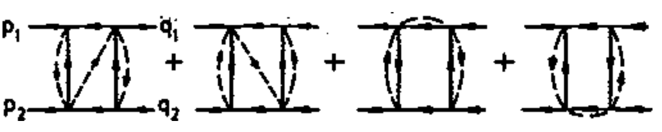
p_1
q_1
p_2
q_2

Fig. 5

reducible (CR) diagrams describing the process under consideration (Fig. 3). In the ladder approximation, we call a diagram a CR diagram if its irreducible components correspond to one-meson exchange. (In what follows we omit the words "in the ladder approximation" for brevity.)

For one of the CR diagrams of the $2n^{th}$ order in g, depicted in Fig. 4 (the number of all CR diagrams that differ in the way the vertices are connected by dotted lines, is equal to 2^n), one obtains the following expression

$$\tilde{T}_n(s,t) = \frac{g^{2n}}{(2\pi)^{3(n-1)}} \int \prod_{j=o}^{2n-2} dk_j \Delta^+(k_j) \cdot \prod_{j=o}^{2n-1} \frac{d\kappa_j}{\kappa_j - i\varepsilon} \times$$

$$\times \prod_{j=o}^{n-1} D^{(+)}(\Delta'_{2j+1} + \lambda\kappa_{2j+1} - \lambda\kappa_{2j}) \tag{2.5}$$

$$\delta(\Delta'_{2j+1} - \Delta'_{2j+2} + \lambda\kappa_{2j+2} - \lambda\kappa_{2j}) ,$$

where $k_{-1} \equiv p_1$, $k_o \equiv p_2$, $k_{2n-1} \equiv q_1$, $k_{2n} \equiv q_2$, $\kappa_o \equiv \kappa$, $\kappa_{2n} \equiv \kappa'$, $\Delta'_j = k_{j-2} - k_j$.

When the integration is performed over k_{2j+2} ($j=0,1,\ldots n-2$), the expression (2.5) takes the form

$$\tilde{T}_n(s,t) = \frac{g^{2n}}{(2\pi)^{3(n-1)}} \int \prod_{j=o}^{n-2} \Big[dk_{2j+1} \Delta^{(+)}(k_{2j+1}) \times$$

$$\times \Delta^{(+)}(\Delta_{2j+1} + \lambda\kappa_{2j+2}) \frac{d\kappa_{2j+2}}{\kappa_{2j+2} - i\varepsilon}\Big] \prod_{j=o}^{n-1} \Big[D^{(+)}(\Delta'_{2j+1} + \lambda\kappa_{2j+1} - \lambda\kappa_{2j}) \times$$

$$\times \frac{d\kappa_{2j+1}}{\kappa_{2j+1} - i\varepsilon}\Big], \qquad \Delta_{2j+1} = P - k_{2j+1}, \; P = q_1 + q_2 . \tag{2.6}$$

Now, integrating over κ_{2j+1} ($j=0,1,\ldots,n-1$), we obtain:

$$\tilde{T}_n(s,t) = \frac{g^{2n}}{(2\pi)^{3(n-1)}} \int \prod_{j=o}^{n-2} [dk_{2j+1}\Delta^{(+)}(k_{2j+1})$$

$$\Delta^{(+)}(\Delta_{2j+1} + \lambda\kappa_{2j+2}) \frac{d\kappa_{2j+2}}{\kappa_{2j+2} - i\varepsilon}] \times$$

$$\prod_{j=o}^{n-1} [\frac{1}{2\omega'_{2j+1}} \cdot \frac{1}{\kappa_{2j} - \lambda\Delta'_{2j+1} + \omega'_{2j+1} - i\varepsilon}] ,$$

$$\omega'_{2j+1} = ((\lambda\Delta'_{2j+1})^2 - \Delta'^2_{2j+1} + m^2)^{1/2} . \qquad (2.7)$$

All subsequent calculations are carried out on the energy shell ($\kappa = \kappa' = 0$).

Suppose that at high energies (i.e. when $s = P^2 \to \infty$, $t = (p_1-q_1)^2 = \text{const.}$), the terms κ_{2j} in the denominators of the type $\kappa_{2j} - \lambda\Delta'_{2j+1} + \omega'_{2j+1}$ of the integrand (2.7) may be neglected (for a discussion of this approximation, see the Appendix), i.e.

$$\kappa_{2j} - \lambda\Delta'_{2j+1} + \omega'_{2j+1} \to -\lambda\Delta'_{2j+1} + \omega'_{2j+1} . \qquad (2.8)$$

In this approximation the sum of all terms corresponding to 2^n CB diagrams is equal to

$$T_n(s,t) = \frac{1}{(2\pi)^{3(n-1)}} \int \prod_{j=o}^{n-2} dk_{2j+1} \; 2\Delta^{(+)}(k_{2j+1})\Delta^{(+)}(\Delta_{2j+1} + \lambda\kappa_{2j+2})$$

$$\frac{d\kappa_{2j+2}}{\kappa_{2j+2} - i\varepsilon} \prod_{j=o}^{n-1} V[(\vec{k}_{2j-1}(-)\vec{k}_{2j+1})^2] , \qquad (2.9)$$

where $V[(\vec{k}_1(-)\vec{k}_2)^2] = \frac{g^2}{m^2 - (k_1-k_2)^2 - i\varepsilon}$, $\vec{k}_1(-)\vec{k}_2 = \vec{k}_1 - \frac{\vec{k}_2}{M}[k_1^o - \frac{\vec{k}_1\vec{k}_2}{M+k_2^o}]$.

We choose the four-vector λ in the form

$$\lambda = \frac{P}{(P^2)^{1/2}} = \frac{k_{2j+1} + k_{2j}}{((k_{2j+1} + k_{2j})^2)^{1/2}} . \qquad (2.10)$$

Then expression (2.9) in the centre-of-mass system (i.e. $\vec{q}_1=-\vec{q}_2=\vec{q}$, $\vec{p}_1=-\vec{p}_2=\vec{p}$) can be written as follows

$$T_n(s,t) = \frac{1}{(2\pi)^{3(n-1)}} \int \prod_{j=o}^{n-2} \left[\frac{d\Omega_{2j+1}}{8E_{2j+1}(E_{2j+1} - E_q - i\varepsilon)}\right] \times$$

$$\times \prod_{j=o}^{n-1} V[(\vec{k}_{2j-1}(-)\vec{k}_{2j+1})^2] \quad , \qquad (2.11)$$

where $E_{2j+1} = (M^2+\vec{k}^2_{2j+1})^{1/2}$, $E_p = E_q = (M^2+\vec{q}^2)^{1/2}$, $d\Omega_{2j+1} = \frac{d\vec{k}_{2j+1}}{E_{2j+1}}$.

Since high energies are carried by nucleon lines, (i.e. the essential contribution in (2.9) comes from the regions $E_{2j+1} \sim E_q$), . $E^{-1}_{2j+1}(E_{2j+1}-E_q-i\varepsilon)^{-1}$ can be substituted by $E_q^{-1}(E_{2j+1}-E_q-i\varepsilon)$, so

$$T_n(s,t) = (2\pi)^{-3(n-1)} \left(\frac{1}{8E_q}\right)^{n-1} \int \prod_{n=o}^{n-2} \frac{d\Omega_{2j+1}}{E_{2j+1} - E_q - i\varepsilon} \times$$

$$\times \prod_{j=o}^{n-1} V[(\vec{k}_{2j-1}(-)\vec{k}_{2j+1})^2] \ . \qquad (2.12)$$

In order to find the asymptotics of (2.12), we apply the technique developed in Ref.[4].

$$\vec{p}(-)\vec{q} = \vec{\Delta}, \quad \vec{k}_{2j+1}(-)\vec{q} = \vec{\chi}_j \ (\vec{p} = \vec{\Delta}(+)\vec{q}, \ \vec{k}_{2j+1} = \vec{\chi}_j(+)\vec{q}) \qquad (2.13)$$

and taking into account relations $d\Omega_{2j+1} = d\Omega_{\vec{\chi}_j}$, we can write (2.12) in the form

$$T_n(s,t) = (2\pi)^{-3(n-1)} (8E_q)^{-(n-1)} \int \prod_{j=1}^{n-1} \left[\frac{d\Omega_{\vec{\chi}_j}}{E_{\vec{\chi}_j(+)\vec{q}} - E_q - i\varepsilon}\right] \times$$

$$\times V[(\vec{\Delta}(-)\vec{\chi}_1)^2] V[(\vec{\chi}_1(-)\vec{\chi}_2)^2] \ldots V(\vec{\chi}^2_{n-1}) \ . \qquad (2.14)$$

Using the relations $(\vec{\lambda}_i(-)\vec{\lambda}_j)^2 = (\vec{\lambda}_i \oplus \vec{\lambda}_j^{-1})^2$ (see Ref.[4]), we pass to holospherical coordinates $\vec{\Delta} = (a,\tilde{\gamma})$, $\vec{\lambda}_j = (a_j,\tilde{\gamma}_j)$ in (2.14) by use of the formulas $E_{\lambda_j}+\lambda_{j_3} = Me^{a_j}$,
$E_{\lambda_j}-\lambda_{j_3} = Me^{-a_j} + \frac{1}{M}\tilde{\gamma}_j^2 e^{a_j}$, $\tilde{\lambda}_j = (\lambda_{j_1},\lambda_{j_2})$, $d\Omega_{\lambda_j} = e^{2a_j}da_j d^2\tilde{\gamma}_j$.

Then we obtain

$$T_n(s,t) = (2\pi)^{-3(n-1)}(8E_q^2)^{-(n-1)} \int \prod_{j=1}^{n-1} \left[\frac{e^{2a_j}\, da_j d^2\tilde{\gamma}_j}{e^{a_j} - 1 - i\varepsilon}\right] \prod_{j=1}^{n} V(\vec{\Delta}_j'^2), \tag{2.15}$$

where $\vec{\Delta}' = \vec{\Delta} \oplus \vec{\lambda}_1^{-1}$, $\vec{\Delta}_2' = \vec{\lambda}_1 \oplus \vec{\lambda}_2^{-1}, \ldots, \vec{\Delta}'_{n-1} = \vec{\lambda}_{n-2} \oplus \vec{\lambda}_{n-1}^{-1}$,
$\vec{\Delta}_n' = \vec{\lambda}_{n-1}$.

The definitions of the operations (+) and $\oplus$ are given in Ref.[7], for example. We have chosen $\vec{q}$ in the form $\vec{q} = (0,0,q)$ and have taken into account the fact that the following approximation holds for $S \gg M^2, |t|$

$$E_{\vec{\lambda}_j(+)\vec{q}} - E_q = (E_{\lambda_j}E_q + \lambda_{j_3}q)/M - E_q \simeq E_q\left(\frac{E_{\lambda_j}+\lambda_{j_3}}{M} - 1\right) = E_q(e^{a_j} - 1) .$$

Since $\Delta' = (a_k',\tilde{\gamma}_k')$ $(k=1,2,\ldots,n)$, then $a_k' = a_{k-1} - a_k$, $\tilde{\gamma}_k' = e^{a_k}(\tilde{\gamma}_{k-1} - \tilde{\gamma}_k)$ when $1 \le k \le n-1$ and $a_k' = a_{n-1}$, $\tilde{\gamma}_k' = \tilde{\gamma}_{n-1}$ when $k=n$, where by definition $a_0 \equiv a$, $\tilde{\gamma}_0 \equiv \tilde{\gamma}$. Using in (2.15) the operator Fourier transformation on the group T(3)

$$V(\tilde{\Delta}_j') \equiv V(a_j',\tilde{\gamma}_j') = \int dz_j d^2\tilde{\rho}_1^{(j)} d^2\tilde{\rho}^{(j)} \langle\tilde{\rho}_1^{(j)}|\hat{V}(z_j)|\tilde{\rho}^{(j)}\rangle \times$$
$$\times\, e^{-ia_j'z_j - i\tilde{\gamma}_j\tilde{\rho}^{(j)} - a_j'}$$

and performing the integration over all $\tilde{\gamma}_j$, we arrive at the expression

$$T_n(s,t) = (16\pi E_q^2)^{-(n-1)} e^{-a} \int e^{-i\tilde{\gamma}\tilde{\rho}} d^2\rho \times$$

$$\times \int \prod_{j=1}^{n-1} \left[\frac{e^{i\sum_{j=1}^{n-1} a_j(z_j - z_{j+1}) - iaz_1} da_j}{e^{a_j} - 1 - i\varepsilon} \right]$$

$$\times \int \prod_{j=1}^{n} dz_j d^2\tilde{\rho}_1^{(j)} \langle \tilde{\rho}_1^{(j)} | \hat{V}(z_j) | \tilde{\rho} e^{-a_j} \rangle . \qquad (2.16)$$

Taking into account that in the high-energy regime the relations $a \sim 0$, $\tilde{\gamma} \sim \tilde{\Delta}$, $|t| \sim \tilde{\gamma}^2$ hold and performing the integration over a_j $(j=1,2,\ldots,n-1)$, we obtain the representation

$$T_n(s,t) = -2is \int e^{-i\tilde{\Delta}\tilde{\rho}} d^2\tilde{\rho} d^2\tilde{\rho}_1 \left(\frac{i}{2s}\right)^n \int \hat{\theta}(z_1 - z_2) \ldots \hat{\theta}(z_{n-1} - z_n) \times$$

$$\langle \tilde{\rho}_1 | \hat{V}(z_1) \ldots \hat{V}(z_n) | \tilde{\rho} \rangle dz_1 \ldots dz_n \qquad (2.17)$$

Thus, we have demonstrated that the summation of the chain of diagrams (Fig. 3) at high energies leads to the representation (1.1) for the amplitude $T(s,t) = \sum_{n=1}^{\infty} T_n(s,t)$, which was derived in Ref.[4] from the QPE.

3. A generalized eikonal representation in a three-dimensional formulation of QFT on the light cone

According to the diagram technique of a three-dimensional formulation of QFT on the light cone[11], the dotted line corresponds to the four-momentum $\mu\kappa$; here the four-vector μ, in contrast to the four-vector λ, is light-like: $\mu^2 = \mu_0^2 - \vec{\mu}^2 = 0$, $\mu_0 > 0$. In this case the CR diagram of the $2n^{th}$ order in g, depicted in Fig. 4, can be expressed by the expression

$$\tilde{T}_n(s,t) = \frac{g^{2n}}{(2\pi)^{3(n-1)}} \int \prod_{j=o}^{n-2} dk_{2j+1}\Delta^{(+)}(k_{2j+1})\Delta^{(+)}(\Delta_{2j+1}+\mu\kappa_{2j+2}) \times$$

$$\times \frac{d\kappa_{2j+2}}{\kappa_{2j+2}-i\varepsilon} \prod_{j=o}^{n-1} D^{(+)}(\Delta'_{2j+1}+\mu\kappa_{2j+1}-\mu\kappa_{2j+2}) \frac{d\kappa_{2j+1}}{\kappa_{2j+1}-i\varepsilon} . \quad (3.1)$$

When we perform the integration over κ_j $(j=1,2,\ldots 2n-1)$, instead of (3.1) we obtain *)

$$\tilde{T}_n(s,t) = \frac{g^{2n}}{(2\pi)^{3(n-1)}} \int \prod_{j=o}^{n-2} dk_{2j+1}\Delta^{(+)}(k_{2j+1}) \frac{\theta(\mu\Delta_{2j+1})}{M^2-\Delta^2_{2j+1}-i\varepsilon} \times$$

$$\times \prod_{j=o}^{n-1} \frac{\theta(\mu\Delta'_{2j+1})}{\kappa_{2j} + \dfrac{M^2-\Delta'^2_{2j+1}}{2\mu\Delta'_{2j+1}} - i\varepsilon} \frac{1}{|2\mu\Delta'_{2j+1}|} , \quad (3.2)$$

where $\kappa_{2j} = (M^2-\Delta^2_{2j+1})/|2\mu\Delta_{2j+1}|$, $j=1,2,\ldots n-1$.

Suppose that at high energies the terms κ_{2j} in the denominators of the form $\kappa_{2j}+(M^2-\ '^2_{2j+1})/|2\mu\Delta'_{2j+1}|$ may be neglected (cf. with (2.8)). Then the sum of all terms corresponding to 2^n CR diagrams is equal to (in the following $\kappa = \kappa' = 0$)

$$T_n(s,t) = (2\pi)^{-3(n-1)} \int \prod_{j=o}^{n-2} dk_{2j+1}\Delta^{(+)}(k_{2j+1}) \frac{\theta(\mu\Delta_{2j+1})}{M^2-\Delta^2_{2j+1}- i\varepsilon} \times$$

$$\times \prod_{j=o}^{n-1} V[(k_{2j-1}(-)k_{2j+1})^2] . \quad (3.3)$$

In (3.3) it is convenient to pass to the centre-of-mass system ($\vec{P}=0$), where

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*) As it is known, the points $\mu\Delta_{2j+1} = \mu\Delta'_{2j+1} = 0$ do not contribute in (3.1) (see Ref.[11]).

$$T_n(s,t) = (2\pi)^{-3(n-1)}(8E_q)^{-(n-1)} \int \prod_{j=o}^{n-2} \frac{d\Omega_{2j+1}}{E_{2j+1} - E_q - i\varepsilon} \times$$

$$\times\ \theta(2E_q - E_{2j+1} + \vec{n}\vec{k}_{2j+1}) \prod_{j=o}^{n-1} V[(\vec{k}_{2j-1}(-)\vec{k}_{2j+1})^2], \qquad (3.4)$$

$$\vec{n} = \frac{\vec{\mu}}{\mu_o} .$$

In the high-energy limit, i.e. when $2E_q = \sqrt{s} \to \infty$, the θ functions in (3.4) can be replaced by unity. As a result, the expression (3.4) for T_n takes the form

$$T_n(s,t) = (2\pi)^{-3(n-1)}(8E_q)^{-(n-1)} \int \prod_{j=o}^{n-2} \frac{d\Omega_{2j+1}}{E_{2j+1} - E_q - i\varepsilon} \times$$

$$\times \prod_{j=o}^{n-1} V[(\vec{k}_{2j-1}(-)k_{2j+1})^2], \qquad (3.5)$$

which coincides exactly with expression (2.12) of the preceding section.

Thus, we again obtain the representation (1.1) for the scattering amplitude; it differs from the usual eikonal representation (2.4) due to a more complicated dependence of the phase function on energy and potential. For example, in the case when the matrix $\langle\tilde{\rho}_1|\hat{V}(z)|\tilde{\rho}\rangle = \delta(\tilde{\rho}_1 - \tilde{\rho})V_s(z,\tilde{\rho})$ is diagonal, the logarithmic dependence arises[4,7]

$$T(s,t) = -4\pi i s \int_0^\infty \rho d\rho J_o(\sqrt{-t}\,\rho)\{e^{i\int_{-\infty}^{\infty} \ln(1 + \frac{V_s(z,\tilde{\rho})}{2s})dz} - 1\} . \qquad (3.6)$$

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Appendix

In deriving expression (2.9) we used the approximation (2.8), which is analogous to the approximation $\sum k_i k_j = 0$ [2,3]. It can be shown that in this approximation the asymptotics of the amplitude is conserved. The conservation of asymptotics consists in the following[12]. Let $\tilde{T}_n(s,t)$ be the exact contribution of CR diagrams of the 2nth order in g to the amplitude, and let $T_n(s,t)$ be in the approximation (2.8). Then, as $s \to \infty$, $t = \text{const}$.

$$\tilde{T}_n(s,t) \to \tilde{\alpha}(t)\tilde{\beta}(s),$$
$$T_n(s,t) \to \alpha(t)\beta(s) \; . \tag{A.1}$$

If $\beta(s) = \tilde{\beta}(s)$, then the asymptotics does not change, though $\alpha(t)$ and $\tilde{\alpha}(t)$ are different.

For example, let us consider CR diagrams of the fourth order (Fig. 5); they give the following contribution to the amplitude:

$$\begin{aligned} T_2(s,t) = \text{const} \int & \Delta^{(+)}(k)\Delta^{(+)}(k')D^{(+)}(p')D^{(+)}(q') \times \\ & \times \prod_{j=1}^{3} \frac{d\kappa_j}{\kappa_j - i\varepsilon} [\delta(q_1-k+q'+\lambda\kappa_2-\lambda\kappa_3) \times \\ & \times \delta(k-p_1+p'-\lambda\kappa_1)\delta(q_2-k'-q'+\lambda\kappa_3)+\delta(q_1-k-q'+\lambda\kappa_3) \times \\ & \times \delta(k-p_1-p'+\lambda\kappa_1-\lambda\kappa_2)\delta(q_2-k'+q'+\lambda\kappa_2-\lambda\kappa_3) + \\ & + \delta(q_1-k+q'+\lambda\kappa_2-\lambda\kappa_3)\delta(k-p_1-p'+\lambda\kappa_1-\lambda\kappa_2)\delta(q_2-k'-q'+\lambda\kappa_3) + \\ & + \delta(q_1-k-q'+\lambda\kappa_3)\delta(k-p_1+p'-\lambda\kappa_1)\delta(q_2-k'+q'+\lambda\kappa_2-\lambda\kappa_3)] \times \\ & \times dk dk' dp' dq' \; . \end{aligned} \tag{A.2}$$

We write the relation (A.2) in the form

$$\tilde{T}_2(s,t) = T_2(s,t) + A_2(s,t) \quad . \qquad (A.3)$$

Here $\tilde{T}_2$ is defined by formula (2.9) with n=2, and for the case when λ is taken in the form (2.10), A_2 in the centre-of-mass system has the form ($E'_k = (m^2+k^2)^{1/2}$)

$$A_2(s,t) = -\text{const} \int \frac{d\Omega_k}{E_k} \left\{ \frac{1}{E'_{p-k}[m^2-(k-q_1^2)-i\varepsilon][E'^2_{p-k} - (E_q-E_k)^2-i\varepsilon]} + \right.$$

$$+ \frac{1}{E'_{q-k}[m^2-(k-p_1)^2-i\varepsilon][E'^2_{q-k}-(E_q-E_k)^2-i\varepsilon]} +$$

$$\left. + \frac{1}{E'_{p-k}E'_{q-k}[E'^2_{q-k}-(E_q-E_k)^2-i\varepsilon][E'^2_{p-k}-(E_q-E_k)^2-i\varepsilon]} \right\} \quad .$$

The approximation (2.8) implies that the terms $A_n(s,t) = T_n(s,t)-T_n(s,t)$, $n\geq 2$ (see (A.1)) are neglected. Here we demonstrate that the asymptotics of $A_2(s,t)$ is of the form 1/s. Because of (2.13) and the relation

$$E_{\vec{p}(\pm)\vec{k}} \equiv [M^2 + (\vec{p}(\pm)\vec{k})^2]^{1/2} = \frac{E_pE_k \pm \vec{p}\vec{k}}{M} \quad , \qquad (p^2=k^2=M^2) \quad ,$$

we have

$$m^2-(k-q_1)^2 = E'^2_{q-k} - (E_q-E_k)^2 = f(\vec{\lambda}) ,$$

$$m^2-(k-p_1)^2 = E'^2_{p-k} - (E_q-E_k)^2 = f(\vec{\Delta}(-)\vec{\lambda}) ,$$

where $f(\vec{k}) = 2ME_k+m^2-2M^2$ and when $s \gg m^2, |t|, M^2$

$$E'^2_{p-k} \simeq E'^2_{q-k} \simeq E_q \left|1 - \frac{E_\lambda+\lambda_3}{M}\right| .$$

Consequently,

$$A_2(s,t) = -\frac{const}{s}\int d\Omega_{\vec{\lambda}} \frac{(1-\frac{E_\lambda+\lambda_3}{M})^{-2}}{f(\vec{\lambda})f(\vec{\Lambda}(-)\vec{\lambda})}\left[2+\mathrm{sign}(1-\frac{E_\lambda+\lambda_3}{M})\right], \quad (A.4)$$

which is proved.

In the same way we can prove that omission of κ_{2j} in the denominators of the integrand in the expression (3.2) does not change the asymptotics of this expression. Indeed, confining ourselves to the CR diagram of the fourth perturbation-expansion order (see Fig. 5), we obtain the following expression

$$A_2(s,t) = -const\int \frac{\theta(\mu P-\mu k)}{2\mu(P-k)}\,\frac{d\Omega_{\vec{k}}}{[m^2-(k-q_1)^2-i\varepsilon][m^2-(k-p_1)^2-i\varepsilon]} \times$$

$$\times\{\frac{\theta(\mu p_1-\mu k)\theta(\mu k-\mu q_1)}{\kappa_1+\kappa_2-i\varepsilon}+\frac{\theta(\mu q_1-\mu k)\theta(\mu k-\mu p_1)}{\kappa_2+\kappa_3-i\varepsilon}+$$

$$+\theta(\mu k-\mu q_1)\theta(\mu k-\mu p_1)\left[\frac{1}{\kappa_1+\kappa_2-i\varepsilon}+\frac{1}{\kappa_2+\kappa_3-i\varepsilon}-\right.$$

$$\left.-\frac{\kappa_2}{(\kappa_1+\kappa_2-i\varepsilon)(\kappa_2+\kappa_3-i\varepsilon)}\right]\} \quad (A.5)$$

where $\kappa_1 = \frac{m^2-(k-q_1)^2}{2\mu(k-q_1)}$, $\kappa_2 = \frac{m^2-(P-k)^2}{2\mu(P-k)}$, $\kappa_3 = \frac{m^2-(k-p_1)^2}{2\mu(k-p_1)}$.

As above, we orient the vector $\vec{q}$ along the z axis and the vector $\vec{n}$ against this axis. Then, at high energies in the centre-of-mass system, we obtain the relations $\mu q_1 \simeq \mu p_1 \simeq 2E_q = \mu P$ and the integrand in (A.5) vanishes. It means that with $E_q \to \infty$, the asymptotics of $A_2(s,t)$ is smaller than that of $T_2(s.t)$.

We may draw the following conclusion. In deriving the representation (1.1) we took into account only some generalized ladder diagrams (in contrast to the eikonal representation where all generalized ladder diagrams are used). In spite of this, the asymptotics of the amplitude is not changed since the sum of all omitted diagrams in each 2nth order tends to zero as $1/s^{n-1}$.

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VISOKOENERGETSKA REPREZENTACIJA U TEORIJI RELATIVISTIČKOG HAMILTONIJANA

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Sadržaj

Dobivene su reprezentacije amplitude raspršenja kod visokih energija, sumiranjem dijagrama u formulaciji kovarijantnog Hamiltonijana u kvantnoj teoriji polja te dijagrama trodimenzionalne formulacije kvantne teorije polja na svjetlosnom konusu.

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