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*LETTERS TO THE EDITOR*

# MOTION OF A FREE ELECTRON IN A UNIFORM MAGNETIC AND ELECTRIC FIELD

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In the present paper, the eigenfunctions and eigenvalues of a free-electron, moving in a uniform magnetic field H parallel to the z-axis, and a uniform electric field $\vec{E}(E_x, E_y, E_z)$, are found and expressed in a closed form.

The time-independed Schrödinger equation, in the presence of a uniform magnetic field $\vec{H}$, parallel to the z-axis and with the vector potential $\vec{A}$ in a symmetrical form $\vec{A}(-\frac{1}{2}Hy, \frac{1}{2}Hx, 0)$, and a uniform electric field $\vec{E}(E_x, E_y, E_z)$, has the following form

$$\Delta\Psi - iB\left(x\frac{\partial\Psi}{\partial y} - y\frac{\partial\Psi}{\partial x}\right) + \left[\frac{2m}{\hbar^2}E - \frac{B^2}{4}(x^2+y^2) - \frac{2mE}{\hbar^2}(E_x x + E_y y + E_z z)\right]\Psi = 0, \quad (1)$$

where it has been put

$$B = \frac{eH}{\hbar c}$$

If the electric field is equal to zero, $\vec{E} = 0$, the solution of the above equation is the following function (Schraubenfunction[1])

$$\Psi_{\vec{k},n}(\vec{r}) = e^{-\frac{1}{B}\{K_x^2 + K_y^2\} + i\vec{k}\vec{r}} (-K_y - iK_x)^n , \quad (2)$$

where the vector $\vec{K} = \vec{k} - \frac{e}{\hbar c} \vec{A}(\vec{r})$ (3)

and the eigenvalues of the energy are the known eigenvalues of Landau[2).

In case where $\vec{E} \neq 0$, the solution of equation (1) will be of the form

$$\Psi(\vec{r}) = F(x,y)\, U(z) \tag{4}$$

with

$$E = E_{\parallel} + E_{\perp} \tag{5}$$

The functions F(x,y) and U(z) fulfil the following equations

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} - iB\left(x \frac{\partial F}{\partial y} - y \frac{\partial F}{\partial x}\right) + \left[\frac{2m}{\hbar^2} E_{\perp} - \frac{B^2}{4}(x^2+y^2) - \frac{2me}{\hbar^2}(E_x x + E_y y)\right] F = 0 \tag{6}$$

$$\frac{d^2U}{dz^2} + \left[\frac{2m}{\hbar^2} E_{\parallel} - \frac{2me}{\hbar^2} E_z z\right] U = 0 . \tag{7}$$

The equation (7) is known[3) ; therefore we will not consider it here.

We will be mainly interested in equation (6), which has been recently studied by Chung and Mrowka[4) , with the help of the Schraubenfunctions. These authors have considered two systems of reference, one stationary and are moving, and they have used the Galileo transformations, so that the position vectors and the centers of the Schraubenfunctions depend on the time.

We will search for a direct solution of equation (6) by putting

$$F(x,y) = e^{ik_x x + ik_y y - \frac{1}{B}(K_x^2 + K_y^2)}\; e^{-\frac{1}{B}\frac{E_x - iE_y}{E_x + iE_y}(K_y - iK_x)^2 + \frac{2me}{\hbar^2 B^2}(E_x K_y - E_y K_x)} \cdot f(K_x,K_y) . \tag{8}$$

The new function $f(K_x,K_y)$ fulfils the following equation

$$\frac{B^2}{4}\left(\frac{\partial^2 f}{\partial K_x^2}+\frac{\partial^2 f}{\partial K_y^2}\right)+\frac{me}{\hbar^2}\left(E_x\frac{\partial f}{\partial K_y}-E_y\frac{\partial f}{\partial K_x}\right)-\frac{2B}{E_x-iE_y}\left(E_xK_y-E_yK_x\right)\left(\frac{\partial f}{\partial K_y}-i\frac{\partial f}{\partial K_x}\right)+$$

$$+\frac{2m}{\hbar^2}\left(\bar{E}_\perp+\frac{me^2(E_x^2+E_y^2)}{2\hbar^2B^2}\right)f=0\,, \qquad (9)$$

where it has been put

$$\bar{E}_\perp = E_\perp - \frac{\hbar^2 B}{2m} - \frac{2e}{B}\left(E_xk_y-E_yk_x\right). \qquad (10)$$

The solution of equation (9) is obtained by the integral-transformation[5]

$$f(K_x,K_y) = \frac{1}{2\pi i}\int_C e^{s(E_xK_y-E_yK_x)}\sigma(s)ds\,, \qquad (11)$$

where the solution $\sigma(s)$ verifies the first-order differential equation

$$2Bs\frac{d\sigma}{ds} = -\left[\frac{B^2}{4}(E_x^2+E_y^2)s^2+\frac{me}{\hbar^2}(E_x^2+E_y^2)s+\frac{2m}{\hbar^2}\left(\bar{E}_\perp+\frac{2\hbar^2 B}{2m}+\frac{me^2(E_x^2+E_y^2)}{2\hbar^2B^2}\right)\right]\sigma \qquad (12)$$

from which we have

$$\sigma(s)\sim e^{-\frac{B}{16}(E_x^2+E_y^2)s^2-\frac{me}{2\hbar^2 B}(E_x^2+E_y^2)s-\frac{1}{2B}\frac{2m}{\hbar^2}\left[\bar{E}_\perp+\frac{2\hbar^2B}{2m}+\frac{me^2}{2\hbar^2B^2}(E_x^2+E_y^2)\right]}\cdot s \qquad (13)$$

The function under the integral has a pole and the following condition must hold

$$\bar{E}_\perp = \frac{\hbar^2 B}{2m}2n - \frac{me^2}{2\hbar^2B^2}(E_x^2+E_y^2) \qquad n = 0,1,2,\ldots \qquad (14)$$

from which we obtain the eigenvalues

$$E_\perp = \hbar\omega(n+\frac{1}{2}) + \frac{2e}{B}(E_x k_y - E_y k_x) - \frac{me^2}{2\hbar^2 B^2}(E_x^2 + E_y^2) , \qquad (15).$$

where $\omega = \frac{eH}{mc}$

and the eigenfunctions (8) take the form

$$F(x,y) \sim e^{ik_x x + ik_y y - \frac{1}{B}(K_x^2 + K_y^2) - \frac{1}{B}\frac{E_x - iE_y}{E_x + iE_y}(K_y - iK_x)^2 + \frac{2me}{\hbar^2 B^2}(E_x K_y - E_y K_x)} \times$$

$$\times \frac{1}{2\pi i} \int_c e^{-\frac{B}{16}(E_x^2 + E_y^2)s^2 + \{E_x K_y - E_y K_x - \frac{me}{2B\hbar^2}(E_x^2 + E_y^2)\} s} \cdot s^{-(n+1)} ds , \qquad (16)$$

Using the transformation

$$\sqrt{\frac{B}{8}}\,(E_x^2 + E_y^2)s = t \qquad (17)$$

and after integration /6/, we obtain the eigenfunctions

$$F(x,y) \sim \exp\{ik_x x + ik_y y - \frac{1}{B}(K_x^2 + K_y^2) - \frac{1}{B}\frac{E_x - iE_y}{E_x + iE_y}(K_y - iK_x)^2 + \frac{2me}{\hbar^2 B^2}(E_x K_y - E_y K_x)\}$$

$$\cdot \frac{1}{n!}\left(\frac{1}{2}\sqrt{\frac{B}{2}}(E_x^2 + E_y^2)\right)^n He_n\left(\frac{2}{\sqrt{\frac{B}{2}(E_x^2 + E_y^2)}}\{E_x K_y - E_y K_x - \frac{me}{2B\hbar^2}(E_x^2 + E_y^2)\}\right), \qquad (18)$$

where $He_n(u)$ are the Hermite polynomials.

Of interest is the case where $E_z = 0$, i.e the case of a completely free electron in the direction of the magnetic field. Then the eigenvalues E of the energy will be given by the relation

$$E = \hbar\omega(n + \frac{1}{2}) + \frac{2e}{B}(E_x k_y - E_y k_x) - \frac{me^2(E_x^2 + E_y^2)}{2\hbar^2 B^2} + \frac{\hbar^2 k_z^2}{2m} . \qquad (19)$$

In the above eigenvalues of the energy appear the quantum numbers of Landau n and $k_z$, as well as the linear combination

$$E_x k_y - E_y k_x = E k_o = \sqrt{E_x^2 + E_y^2} \cdot k_o \ . \tag{20}$$

Consequently, the problem remains still degenerate in a high degree, namely infinite. If we denote by $E$ the magnitude of the intensity of the electric field the eigenvalues of the energy (20) are now written

$$E = \hbar\omega(n + \frac{1}{2}) + \frac{2e}{B} E k_o - \frac{mc^2 E^2}{2 H^2} + \frac{\hbar^2 k_z^2}{2m} \ . \tag{21}$$

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