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Yield Curve Estimation Based on Government Security Prices in the Croatian Financial Market

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Abstract: *This article investigates the estimation of the yield curve based on government security prices using the Nelson-Siegel model in the Croatian financial market. The yield curve was estimated for samples of government securities with and without currency clauses. Since the Croatian financial market is less developed characterized by limited trading activity in government bonds, Treasury bills were also included in the analysis. To examine the difference in the estimation of yield curve parameters between a less developed and a developed market, the U.S. sample was considered. The yield curve was estimated for the full US sample and for artificially created U.S. samples corresponding to the Croatian samples of government bonds with and without currency clauses. Despite the less developed Croatian financial market, it is possible to estimate the yield curve and derive meaningful economic interpretations from the estimates.*

Keywords: Nelson-Siegel model; nonlinear optimization; prices of government securities; U.S. government securities

JEL Classification: E43; G1; G12

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Introduction

The yield curve is frequently used in economic and financial analysis. It can be viewed from a macroeconomic perspective, as it has been shown to be an early indicator of recession in the economy. It is also viewed from an investor's perspective as it contains important information for the valuation of financial assets. Therefore, it is crucial to accurately estimate the yield curve and understand its key characteristics.

Yield curve estimation in developed markets is frequently implemented. Developed markets have a wide range of financial instruments. For bond markets, this means that there are various government bonds with different maturities. These instruments are frequently traded and have a high trading volume, which indicates good liquidity. Under these conditions, there are no major limitations in estimating the yield curve. Yield curve estimation using various models is a well-researched area supported by numerous academic papers in developed markets (Gürkaynak et al. (2007), Bowsher and Meeks (2008), Härdle and Majer (2016)). As this paper deals with the specific problems of less developed and less liquid financial markets in relation to the ability to estimate the yield curve, the literature for these markets is given more prominence and is presented below.

Less developed markets often have problems with infrequent trading and low trading volumes, resulting in a lack of data needed to estimate the yield curve. Despite these obstacles, there are many researchers trying to overcome these problems. For example, using bond yields for the Hungarian market, Reppa (2009) has shown that it is possible to use the Nelson-Siegel model even though it is a developing market. For the same market, Tran (2016) also used other models such as the Svensson model and the Bjork-Christensen-Legendre model in addition to the Nelson-Siegel model and proved that using the Svensson model is the best way to fit the yield curve. Nagy (2020) made a great contribution, not only for the Hungarian market but also for all other less developed markets, as she proved that the yield curve estimation should be based on government bond prices when there is a lack of observed data. For the Czech financial market, also an example of a less developed market, there are a number of papers that deal with the problem of estimating the yield curve. Kladívko (2010) and Hladíková i Radová (2012) have shown that it is possible to estimate the yield curve for the Czech market using the Nelson-Siegel model. Hanzal (2017) and Šedivá and Marek (2015) came to the same conclusion, but they included some additional models, such as the Svensson model and other types of parametric and non-parametric models. There are other papers which can be mentioned for other, less developed markets, such as the work of Baskot et al. (2018). For the financial market of Bosnia and Herzegovina, they estimated the yield curve using the Nelson-Siegel and Svensson models. They used monthly data on government bonds from 2014 to 2015 and showed that the Nelson-Siegel model fits the observations of the yield curve better than the Svensson model. Angelov (2002) and Ganchev (2009) investigated the estimation of the yield curve for the Bulgarian financial market. They used different models for the estimation, but the results showed that there is no single model that is superior compared to others. The application of the Nelson-Siegel model to the Bulgarian market can be found in the paper by Makariev (2021). He concludes that a less liquid and less developed financial market does not affect the ability to apply the Nelson-Siegel model. A similar work was conducted by Grum (2006) for the Slovenian financial market using different spline models, the Nelson-Siegel and the Svensson model. He showed that the Nelson-Siegel model fits the observations of the yield curve best for the less liquid Slovenian market. The Svensson model was tested on the Slovenian market in the work of Garcia and Carvalho (2019) and was shown to be suitable for the mentioned market.

In the Croatian capital market, several studies related to yield curve estimation have been conducted, with two approaches: fitting a yield curve and interest rate modeling. The first scientific papers by Aljinović and Šego (2003), Aljinović et al. (2009), Ercegovac and Kundid (2010), which dealt with the topic of estimating the yield curve on the Croatian market, referred to interest rate models such as Vasicek and Cox-Ingersoll-Ross (CIR). These papers have shown that Vasicek and CIR models can be applied to the Croatian financial market, a less developed financial market characterized by a relatively small number of different financial instruments. Since the Vasicek and CIR models are single-factor models that can identify only a few shapes of the yield curve, Aljinović et al. (2009) included the Nelson-Siegel model as a form of yield curve fitting model that can take different shapes of the yield curve. They showed that for the Croatian financial market, the Nelson-Siegel model should be used rather than the Vasicek and CIR models. Later papers by Aljinović et al. (2012), Zoričić and Orsag (2013a), Zoričić and Orsag (2013b) and Zoričić and Badurina (2013) also used yield curve fitting models and concluded that it is possible to apply the Nelson-Siegel and Svensson models to the Croatian financial market. The main feature of all the above papers is that the yield curve estimation is based on zero-coupon yields of government securities derived using the bootstrapping method.

Zero-coupon yields in developed financial markets, which are extremely liquid and in which many government bonds with different maturities are traded, can be observed directly on the market. If there are no observable zero-coupon yields, it is possible to derive them from observations of yields to maturity of Treasury bills and coupon government bonds using the bootstrapping method (Zoričić, 2012). Since less developed financial markets, such as the Croatian market, do not have a sufficient number of securities with different maturities, it is not possible to apply the bootstrapping method correctly. The Croatian financial market is characterized by few government bonds, even if the Treasury bills, which belong to the money market, are included in the sample. For this reason, it is difficult to estimate the zero-coupon yield using the bootstrapping method. Therefore, if the financial market is less developed and less liquid as Croatian one, it is better to estimate the yield curve based on government bond prices (Nagy, 2020) instead of using the bootstrapping method. For this reason, the purpose of this paper is to investigate whether it is possible to estimate the yield curve using the Nelson-Siegel model based on the prices of Croatian government bonds and Treasury bills. The idea of estimating the yield curve based on prices comes from the paper by Nagy (2020), from which part of the methodology was adopted in this paper.

This article makes several contributions in the context of yield curve estimation in less developed and less liquid markets. First, it contributes to the existing literature on the mentioned topic, which is currently quite scarce. This impact is particularly visible in the papers for the Croatian financial market. Not only are there few papers available, but the latest research for this market was published in 2013. Secondly, the estimation of the yield curve for such a market is based on government bond prices in order to overcome problems arising from the lack of market observation data. Finally, to prove the validity of estimating the yield curve in a less developed and less liquid market, the yield curve was estimated for the entire sample of the developed U.S. financial market. Furthermore, for the artificially generated U.S. sample, which was created based on Croatian government bonds.

Theoretical background

Nelson and Siegel (1987) have made a significant contribution to yield curve models by developing a basic model that is simple, parametric, parsimonious, and flexible enough to effectively capture various shapes observed in yield curves. It is simplest for the usual monotonic shapes, which can be either rising or falling. The Nelson-Siegel model is also suitable for humped curves as well as for the rare occurrence of the S-shaped curve, which is rarely observed. Due to these properties, the Nelson-Siegel model is often used to estimate the yield curve.

In developed markets with a high volume of frequently traded government bonds, it makes little difference to estimate the yield curve using either yields or prices of government bonds. However, in less developed markets with limited observations, it is better to use government bond prices to estimate the yield curve. Following Nagy (2020), government bond and Treasury bill prices were used to estimate the parameters of the Nelson-Siegel model. Using the estimated parameters, it is possible to estimate the zero-coupon yield using the following expression:

$$\hat{r}_m = \hat{\beta}_0 + \hat{\beta}_1 \left[\frac{1 - e^{-\hat{\lambda} m}}{\hat{\lambda} m}\right] + \hat{\beta}_2 \left(\frac{1 - e^{-\hat{\lambda} m}}{\hat{\lambda} m} - e^{-\hat{\lambda} m}\right), \tag{1}$$

where $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2 \; i \; \hat{\lambda}$ denote the estimated parameters of the yield curve and refer to the estimated values of the level, slope, curvature, and time component lambda, while $\hat{r}_m$ is the estimated zero-coupon yield for maturity m. Using the estimated values of the zero-coupon yield for different maturities, it is possible to calculate the estimated price for each bond analyzed:

$$\hat{B}_{\mathrm{m}} = \frac{I_t}{(1+\hat{r}_1)} + \frac{I_t}{(1+\hat{r}_2)^2} + \cdots + \frac{I_t + N}{(1+\hat{r}_m)^{\mathrm{m}}} \tag{2}$$

where $\hat{B}_{\mathrm{m}}$ s the estimated price of a security with maturity m, I_t is the interest in a given currency, while N is the nominal value in a given currency. The rate at which the interest and the nominal value must be discounted is the estimated zero-coupon yield, which is the rate denoted by $\hat{r}_m$.

Data

For the empirical analysis, the daily prices of Croatian government bonds and Treasury bills from April 2006 to December 2022 were collected and downloaded from the Bloomberg information system[1]. For each month and for each bond that was available in the analyzed month, the average monthly price was calculated and used to estimate the yield curve. In the Republic of Croatia, government bonds were issued as bonds without currency clauses and bonds with currency clauses until the end of 2022. Until December 2022, government securities without a currency clause were issued in the local currency (kuna). As Croatia is closely related to the European markets, government securities were also issued with a currency clause, which means that they were issued in a foreign currency (euro). Different currencies carry different risks, which is why it is important to analyze these two samples separately. Since the samples of government bonds with and without currency clauses have relatively few observations per month, observations on Treasury bills with and without currency clauses are included in the analysis to increase the number of observations. Accordingly, two additional samples were observed, comprising bonds without currency clauses with corresponding Treasury bills and bonds with currency clauses with corresponding Treasury bills. The following data were used to estimate the yield curve using the Nelson-Siegel model for the above four samples: observation date (April 2006 - December 2022), maturity date, monthly prices and coupon rates of government securities.

The Nelson-Siegel model described above was estimated using the *fitNelsonSiegel*[2] function from the MATLAB software package. The *fitNelsonSiegel* function enabled the estimation of parameters of the level, slope, curvature and lambda parameters based on the prices of government securities used to describe the movement of the yield curve. The results of the initial tests using the *fitNelsonSiegel* function showed that the estimated parameters can only lead to a meaningful economic interpretation if there are at least 3 prices of government securities and these are relatively evenly distributed in terms of maturity. For the purposes of this study, in addition to estimating the level, slope and curvature, the time component lambda was also estimated, which is why the non-linear optimization method is used in this study. In order to obtain the estimated values of the parameters of the Nelson-Siegel model based on the prices and the estimated prices of the government securities, the deviations were minimized using the *fmincom*[3] function, which is used to minimize deviations when performing non-linear optimization. The yield curve was estimated for four samples. The quality of the estimates for each sample was evaluated separately to show which sample the Nelson-Siegel model best fit the observations of the yield curve. For this purpose, the root mean square error (RMSE) was calculated. To calculate the mentioned measures, the value of the residual sum of squares between the prices estimated by the model and the real price of the security was obtained directly from MATLAB using the *ResNorm*[4] command.

Results & Discussion

This section presents the results for the Croatian financial market as an example of a less developed market. It also includes a comparison of the Croatian financial market with the developed market in the U.S. market.

Analysis of Croatian government securities

The yield curve was estimated for four samples of government securities with and without a currency clause. This resulted in values for the parameters level, slope, curvature and time component lambda. For the sample of bonds without a currency clause (first sample), there are consistently 3 or more observations on bond prices in the observed period. Within some months, the observations are extremely clustered or there is an isolated observation within a month to which the parameter estimation of the Nelson-Siegel model reacts sensitively, so that such estimated parameters are atypical. For such parameters, the zero-coupon yields estimated by the Nelson-Siegel model are therefore extremely small, sometimes negative or even extremely high. Such atypical parameter values are considered outliers and are therefore excluded from further analysis. The yield curve was also estimated for a sample of bonds and Treasury bills without currency clauses (second sample), for which the same procedure as mentioned above was used to remove outliers. Based on the estimated parameters for the two samples analyzed, descriptive statistics were generated as shown in Table 1. The same procedure was performed for the samples of government securities with currency clauses (third and fourth samples) and the descriptive statistics can be found in Table 2.

Table 1: Descriptive statistics for estimated yield curve parameters for samples without currency clauses (April 2006 – December 2022)

	Panel A: Government coupon bonds without currency clause				**Panel B:** Government coupon bonds and Treasury bills without currency clause			
	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\lambda}$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\lambda}$
Mean	5.13	-0.44	-5.26	2.68	5.17	-3.32	-1.89	2.37
σ	2.01	8.27	11.57	4.07	1.91	2.37	4.47	1.76
Min.	1.04	-8.27	-57.78	0.01	1.03	-9.36	-26.36	0.03
Max.	9.84	44.26	20.41	44.01	9.97	12.26	18.51	13.51
n	138	138	138	138	163	163	163	163

Source: author's calculation

Table 2: Descriptive statistics for estimated yield curve parameters for a currency clause samples (April 2006 – December 2022)

	Panel C: Government coupon bonds with currency clause				**Panel D:** Government coupon bonds and Treasury bills with currency clause			
	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\lambda}$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\lambda}$
Mean	4.65	-1.77	-4.37	1.93	5.01	-0.01	-8.48	1.45
σ	1.71	4.07	7.77	1.59	1.71	5.12	7.80	1.48
Min.	1.53	-7.56	-31.70	0.25	1.53	-7.56	-37.09	0.30
Max.	8.08	17.73	13.97	11.26	8.26	16.58	12.01	11.26
n	117	117	117	117	141	141	141	141

Source: author's calculation

Panels A and B in Table 1 show that the signs of all estimated parameters are equal. The values for level and lambda are similar, while a larger difference in value can be observed for slope and curvature. The inclusion of Treasury bills increases the number of observations at the short end of the yield curve. Looking at the absolute value of the slope parameter, it takes on a higher value for the sample of bonds and Treasury bills without currency clauses than for the sample of bonds without currency clauses. As a result, the yield curve for the sample of bonds and Treasury bills without currency clauses is still upward sloping, but has a steeper slope. The addition of Treasury bills also reduces the average values of the standard deviations of the estimated parameters, making the estimate more stable. The results presented in Panel C and Panel D show that the signs of the estimated parameters are the same as in the samples without the currency clause. Similar to the sample without currency clause, the values of level and lambda are similar, while the largest deviations are visible in the values of the estimated parameters for slope and curvature. The inclusion of Treasury bills in the analysis has reduced the number of outliers. This increased the number of estimated parameters with economic interpretation from 117 to 141, for which the descriptive statistics are shown in Panel C and Panel D. The inclusion of Treasury bills had a small effect on the standard deviation values for the estimated parameters, which is not consistent with the previous two samples. Thus, although a larger number of estimates were obtained, the volatility of the estimated parameters is greater in such a sample.

Since the period from April 2006 to December 2022 is quite long and the number of outliers varies in each sample, the period from January 2012 to December 2015, which covers 48 months, was chosen for the final comparison of the four observed samples. The reason for choosing the above-mentioned period is the fact that at least one observation of the price of Treasury bills is known almost every month in this shorter period. In this respect, it is possible to compare the estimated yield curves for samples without and with Treasury bills, i.e. to conclude that the short-term prices of the securities influence the final estimate.

Based on the results shown in Table 3 for all four samples tested, the signs of all estimated parameters are the same, and the inclusion of Treasury bills in the analysis increases the number of estimates that have an economically meaningful interpretation. In addition, the comparison of the results in panels E and F again shows that the addition of Treasury bills significantly reduces the standard deviations of the estimated parameters. This is not the case for the sample of bonds with currency clauses. As shown in panels G and H, the inclusion of Treasury bills increases the standard deviation for level and slope, while it decreases slightly for curvature and lambda.

Table 3: Descriptive statistics for the estimated yield curve (January 2012 - December 2015)

Panel E: Government coupon bonds without currency clause				
	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\lambda}$
Mean	6.53	-1.51	-3.93	3.52
σ	1.62	8.68	12.30	7.78
Min.	3.70	-8.27	-41.51	0.38
Max.	9.78	28.99	20.41	44.01
n	30	30	30	30

Panel F: Government coupon bonds and Treasury bills without currency clause				
	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\lambda}$
Mean	6.47	-4.92	-0.10	2.12
σ	1.61	1.30	1.91	1.26
Min.	3.55	-7.92	-5.44	0.65
Max.	9.97	-2.24	5.35	5.64
n	38	38	38	38

Panel G: Government coupon bonds with currency clause				
	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\lambda}$
Mean	5.13	-3.69	-1.74	1.71
σ	0.98	3.04	8.92	1.12
Min.	3.15	-6.54	-21.20	0.36
Max.	6.64	9.70	12.22	4.08
n	27	27	27	27

Panel H: Government coupon bonds and Treasury bills with currency clause				
	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\lambda}$
Mean	5.74	-1.19	-9.31	0.92
σ	1.04	5.10	8.76	0.78
Min.	2.82	-5.28	-43.30	0.29
Max.	7.69	22.13	12.01	4.62
n	44	44	44	44

Source: author's calculation

To examine the quality of the estimation of the yield curve parameters for the same observation period, the relative root mean square error was calculated and the results are presented below.

Table 4: Average values of the root mean square error for the Croatian samples

	$RMSE_{st.dev.}$	$RMSE_{max-min}$
Bonds without currency clause	0.083	0.031
Bonds and Treasury bills without currency clause	0.068	0.022
Bonds with currency clause	0.059	0.025
Bonds and Treasury bills with currency clause	0.074	0.028

Source: author's calculation

The sample of bonds without currency clauses has a higher number of observations per month than the sample of bonds with currency clauses, and therefore there are more observations on short-term securities, which is why the relative root mean square errors are higher for the sample of bonds without currency clauses. When observations of Treasury bill prices are added to the sample of bonds without currency clauses, data on short-term securities are almost consistently available. Therefore, fitting the observations with the Nelson-Siegel model is easier for a sample of bonds without currency clauses with Treasury bills included than for such a sample without Treasury bills, resulting in the lower values of the relative root mean square errors. The sample of bonds and Treasury bills without currency clauses has on average 1.5 more observations per month for the observed period than the sample of bonds without currency clauses.

This increase in the number of observations reduces the relative RMSE, expressed in standard deviations, by 23% (decrease from 0.083 to 0.068) and, expressed in maximum and minimum price differences, by 40% (decrease from 0.031 to 0.022). It can be concluded that adding only a few observations per month significantly improves the fit of the observations to the yield curves. This is not the case for the sample of government securities with a currency clause. The addition of Treasury bills to the sample of bonds with a currency clause leads to larger deviations in the fit of the observations. One reason for this is that there is only a continuous set of observations for Treasury bills with a maturity of 6 months, while there are significantly fewer observations for a maturity of 3 months and few observations with a maturity of 12 months. The inclusion of Treasury bills increases the total number of observations, but there are still an insufficient number of observations for medium-term maturities, which makes fitting the long and short ends of the yield curve even more difficult and leads to higher values of the relative RMSE compared to the sample of bonds with currency clauses. Although the number of observations per month increases for the sample of bonds and Treasury bills with currency clauses, the deviations are larger, which is why this part of the study has shown that the volatility of the estimated parameters is higher for such a sample.

Comparison with the U.S. developed financial market

In this article, the focus is on the less developed and less liquid Croatian financial market. In order to prove that it is possible to estimate the parameters with a reasonable economic interpretation of the yield curve for less developed financial markets, the U.S. financial market was included in the analysis. The aim was to examine the differences in the value of parameter estimation between a less developed and a developed financial market. Daily data for the U.S. Treasury and government bonds from 2015 to 2019 was used to make the aforementioned comparison. The period from 2015 to 2019 was chosen because it represents a relatively calm period without significant external events that could affect the yield curve estimation, such as the financial crisis. Therefore, the period can be used to investigate whether the problem of estimating the yield curve for a less developed financial market exists due to the quantity and structure of observations. The U.S. financial market was chosen for comparison because it is one of the most liquid and efficient markets in the world. In such a market, there are bonds with different maturities that are traded regularly and the data is publicly available. For this reason, the U.S. market is a good choice to analyze the quality of the estimation of yield curves for samples that have a significantly lower number of observations.

The U.S. sample was reduced to the Croatian market, which is also done in a similar way in Nagy's work (2020) for the Hungarian market. The government security price observations available for the Croatian and U.S. samples were compared for each month, and bonds with comparable maturities in the U.S. sample were retained to match the U.S. sample to the Croatian sample. For example, in January 2015, there were six government bonds without currency clauses in the Croatian sample. For the same period, there were 299 government bonds in the U.S. sample. In order to obtain two comparable samples, six U.S. government bonds were included in the further analysis based on the maturity dates, which means that 293 government bonds were excluded from the U.S. sample. This procedure was applied to the entire period under observation. In this way, two samples were created for the U.S. market, one corresponding to the U.S. sample with the Croatian government bond sample without a currency clause and the other corresponding to the Croatian government bond sample with a currency clause. It should be emphasised that the U.S. sample contains an average of 310 observable government bond prices with different maturities in the observed period, while the Croatian samples without and with currency clause have 7.5 and 5.7 observations per month.

The yield curve was estimated using the Nelson-Siegel model, based on prices for the entire U.S. data sample and two U.S. data samples that were replications of the Croatian samples. Descriptive statistics were performed and are presented in the table below.

Table 5: Descriptive statistics of the estimated parameters of the analyzed samples (January 2015-December 2019)

	Panel A: United States, full sample				**Panel B:** United States sample based on artificially generated missing observations of Croatian government bonds without currency clause				**Panel C:** United States sample based on artificially generated missing observations of Croatian government bonds with currency clause			
	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\lambda}$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\lambda}$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\lambda}$
Mean	3.33	-2.13	-0.45	5.13	2.74	-1.43	-1.49	2.71	2.71	-0.99	-1.99	2.55
σ	0.29	1.00	0.91	1.99	0.90	1.39	2.06	2.57	0.46	2.06	3.17	1.83
Min.	2.55	-3.75	-2.62	1.99	0.00	-7.06	-4.54	0.79	1.31	-3.26	-15.08	0.88
Max.	3.75	-0.63	1.03	11.13	7.06	1.16	7.21	13.31	3.42	8.04	3.85	8.30
n	60	60	60	60	54	54	54	54	47	47	47	47

Source: author's calculation

The descriptive statistics for the entire U.S. sample are based on 60 estimates, which means that there is not a single outlier in the observed period. This is not the case for the artificially generated U.S. samples, which are a replication of the Croatian samples, as there are several outliers. The estimation of the yield curve by the price-based Nelson-Siegel model proved to be more appropriate for the U.S. sample, which is based on artificially generated missing observations without a currency clause, than the artificially generated U.S. sample of Croatian bonds with a currency clause. As a result, for the same observation period, the artificially generated U.S. sample without a currency clause has 54 estimates and the artificially generated U.S. sample with a currency clause has only 47 estimates after removing the outliers. The estimated parameters for the artificially generated U.S. sample without currency clause also show smaller standard deviations compared to the artificially generated U.S. sample with currency clause, with the exception of the parameter level. Since the artificially generated U.S samples have similar values and the same sign of the estimated parameters as the full U.S. sample, it can be concluded that it is possible to estimate the yield curve for a less developed and less liquid market with a significantly smaller number of government bond prices.

Table 6: Average values of root mean square error

	$RMSE_{st.dev.}$	**$RMSE_{max\text{-}min}$**
US, full sample	0.011	0.002
US_bonds without currency clause	0.044	0.016
US_bonds with currency clause	0.031	0.013

Source: author's calculation

The lowest values of the relative root mean square error suggest that it is best to estimate the yield curve using the Nelson-Siegel model, starting with prices for the entire U.S. data sample. Since the U.S. sample contains many observations per month, estimating the yield curve is less difficult than for samples with an average of several observations per month, such as the artificially generated U.S. samples. Therefore, the lowest values of relative root mean square error obtained for the full U.S. sample are not surprising. The value of the relative RMSE, expressed in standard deviations, is about three times higher for the artificial samples compared to the full U.S. sample and six times higher when expressed as the difference between the highest and lowest prices. This increase is not significantly higher when one considers that the full U.S. sample is on average 50 times larger than the artificial U.S. samples.

Conclusion

Instead of using the well-known bootstrapping method, where the yield curve is estimated based on the zero-coupon yields of government securities, this paper estimates the yield curve based on government prices. Previous research has shown that it is better to estimate the yield curve based on government bond prices when dealing with a less developed market, such as the Croatian market. The Nelson-Siegel model was used for this purpose.

The results of the conducted analysis confirm that the Nelson-Siegel model is an appropriate model for estimating the yield curve based on Croatian government bond prices. For the entire observed sample from April 2006 to December 2022, the estimated values for the level and lambda parameters are similar for all four samples, while there are larger differences in the values for slope and curvature. In addition, the signs are the same for all estimated parameters. For the final comparison, the shorter observation period from January 2012 to December 2015 was chosen. The mentioned period represents a relatively quiet period on the Croatian financial market, and at least one Treasury bill price is known almost every month. The Treasury bill samples have shown that by adding just one observation, it is possible to obtain more estimated values of parameters that have a meaningful economic interpretation. It also reduces the standard deviation and the average value of the relative root mean square error.

Due to all the limitations of a less developed market, the U.S. sample was selected to refine this analysis. In this way, the difference in parameter estimates between a less developed and a developed financial market could be assessed. Since the U.S. sample is on average 50 times larger than the artificially generated U.S. samples, a three time increase in the relative RMSE expressed in standard deviations and a six time increase in the relative RMSE expressed in the difference between the highest and lowest price is not a significantly large increase.

Given the lack of previous research dealing with the estimation of the yield curve in the Croatian financial market, this paper can contribute to this area and stimulate further research, not only for Croatia, but also for other similar, less developed markets. For the future, the possibility of forecasting the yield curve on the Croatian market and testing the use of the yield curve in a macroeconomic context can also be explored.

Declarations

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Conflicts of interest/Competing interests

There is no conflict of interest/Competing interests

Availability of data and material

The data supporting the findings of this study are available from Bloomberg (https://www.bloomberg.com/professional/solution/bloomberg-terminal/)

Code Availability

The results from the computer program are presented in the tables in the manuscript. The functions used in MATLAB are described in the manuscript.

Authors' Contributions

Zrinka Orlović: Formal analysis, Data curation, Investigation, Visualization, Writing-Original draft preparation. Davor Zoričić: Supervision, Validation, Conceptualization. Zrinka Lovretin Golubić: Writing- Reviewing and Editing

NOTES

1 https://www.bloomberg.com/professional/solution/bloomberg-terminal/

2 The *fitNelsonSiegel* function fits a Nelson-Siegel model to bond data. To use this function, data about maturity, the coupon rate, the settlement date, and the bond prices should be inserted. A detailed description of the function can be found at the following link: https://www.mathworks.com/help/fininst/fitnelsonsiegelirfunctioncurve.html

3 The *fmincon* function is generally used to find the minimum of a particular problem. A detailed description of the function can be found at the following link: https://www.mathworks.com/help/optim/ug/fmincon.html

4 *ResNorm* is a measure of the difference between the actual and predicted values in an optimization context. It is a component of the *lsqcurvefit* function. A detailed description of the function and the *ResNorm* feature, can be found at following link: https://www.mathworks.com/help/optim/ug/lsqcurvefit.html

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