Enhanced Hybrid Detection Technique for Minimum Mean Square Equalizer in Uplink Massive MIMO Systems

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Original scientific article

***Abstract*—Uplink massive Multiple-Input Multiple-Output (MIMO) systems include huge antennas in the base station (BS) that simultaneously serve fewer Users with single-antenna, making signal detection a major issue Due to the huge matrix inversion requirement. We propose an iterative detection technique based on the enhanced Alternating Direction Method of Multipliers–Conjugate Gradients (ADMM-CG) to avoid direct matrix inversion. The ADMM is first applied as an initial vector. Then, the CG iteration algorithm terminates the calculations for the rest of the iterations. A low-complexity initial method based on trace tridiagonal has been proposed to improve the suggested technique's performance. This integration is crucial for optimizing the tradeoff between performance and complexity. The proposed technique outperforms traditional iterative approaches regarding signal detection performance. Monte Carlo simulations show that the proposed detector performs near optimally, decreases complexity, and necessitates fewer iterations. Furthermore, it outperforms existing solutions, which are sensitive at high modulation orders and when the number of users approaches the entire number of antennas in the base station.**

***Index terms*—Alternating Direction Method of Multipliers, Conjugate Gradients, Hybrid detection, Massive MIMO.**

I. INTRODUCTION

The next-generation mobile (NGM) system is being developed to meet the growing demand for wireless data flow and services. The fifth generation of cellular networks (5G) is designed to deliver improved spectrum and energy efficiency, addressing the need for higher data throughput and reliable connectivity. This advancement is crucial for supporting the exponential growth in wireless data traffic and enabling new services requiring high-performance communication capabilities [1]. Massive Multiple-Input Multiple-Output (MIMO) exhibits considerable promise as a pivotal technology for wireless communications in the post-5th generation and 6th generation (6G) due to their remarkable energy efficiency and spectrum coverage [2, 3]. These systems have hundreds of detecting antennas at each base station (BS) to serve dozens of users simultaneously [4, 5]. Massive MIMO systems are used by high-speed millimeter wave (mm-Wave) communications, which are critical for 5G/6G systems, to overcome signal attenuation issues [6]. The detection methods in the development of mm-wave technology include multi-carrier modulation techniques and MIMO concepts that affect the growth of detection methods for massive MIMO systems. High-traffic mm-wave massive MIMO uplink communication has unique challenges like hardware constraints and channel estimation complexity, hence necessitating efficient hybrid detection methods to unlock these next-generation wireless networks full potential. However, a detector for massive MIMO signals is hard to build because it deals with highly complex computations caused by a large antenna number of users and related BSs. Maximum likelihood (ML) detectors may work perfectly, but as the number of received signals grows, it becomes exponentially more complicated ($\mathcal{O}^{N_T}$). Hence, ML detection is not feasible for this scenario [7, 8]. Linear detectors, such as Minimum Mean-Squared Equalizer (MMSE) and Zero-Forcing (ZF) detectors, can achieve performance that is approximately optimum in massive MIMO systems [2]. However, these detectors require directly inverting the matrix, which increases the complexity of the number of transmitting antennas $\mathcal{O}(N_T{}^3)$ [9]. Therefore, calculating the direct inverse becomes incredibly costly in the context of massive MIMO detection. Recently, to simplify the matrix inversion, iterative algorithms such as Jacobi (JI), Successive Over-Relaxation (SOR), Gauss-Seidel (GS), and Richardson Iteration (RI) have been used to recover the transmitted vector iteratively without any division operator [9]. In contrast, the overall complexity can be reduced by one magnitude order. These techniques begin by choosing a starting vector and then enhance the performance through a series of iterations. However, increasing the iteration number requires more computation. Moreover, choosing an appropriate initial vector can accelerate the convergence, which enhances the performance and reduces the required iteration. Due to the crucial starting vector for achieving accuracy and convergence in the completed vector solution, an iterative detection technique based on the enhanced Alternating Direction Method of Multipliers (ADMM) is presented as an initial stage, followed by Conjugate Gradients (CG) for the reminder iterations. The suggested technique combines the hybrid iteration method and scaled-diagonal initialization to enhance the performance and decrease the computational complexity.

Manuscript received July 4, 2024; revised August 2, 2024. Date of publication November 14, 2024. Date of current version November 14, 2024. The associate editor prof. Gordan Šišul has been coordinating the review of this manuscript and approved it for publication.

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Digital Object Identifier (DOI): 10.24138/jcomss-2024-0060

The following are the key contributions of this paper:

• Based on the ADMM and CG iteration, we provide a new near-optimum detector for massive MIMO uplink systems that are computationally efficient. The ADMM offers an efficient search direction for the CG algorithm in the suggested scheme, leading to a faster convergence rate.

• Since the MMSE filtering matrix dominates diagonally for massive MIMO wireless systems, we suggest using its tridiagonal component to initialize the proposed to reduce the complexity.

• Different detectors are compared regarding the complexity of the calculation, and numerical simulations are used to show how different they perform. According to simulation results, with growing system dimensions such as BS antennas, users, or modulation orders, the suggested detection strategy works better than the previously suggested methods with less computing complexity.

Iterative massive MIMO detection algorithms have found their way into various corners of modern wireless communication systems, revolutionizing how we connect and communicate. In the realm of 5G and future cellular networks, these algorithms are a cornerstone technology, dramatically boosting both capacity and coverage. They're also proving invaluable in millimeter-wave systems, where large antenna arrays help overcome the inherent challenges of signal path loss. The Internet of Things benefits greatly from this technology too, as massive MIMO can handle the vast number of connected devices that define IoT networks. In urban landscapes, these algorithms facilitate wireless backhaul for dense small cell deployments, ensuring seamless connectivity in complex environments. Crowded spaces like stadiums and airports now enjoy high-speed WLANs thanks to this technology, keeping thousands of users connected simultaneously. Even beyond our planet, satellite communications are leveraging large antenna arrays empowered by these algorithms to enhance link reliability and boost data throughput. From the ground to space, iterative massive MIMO detection is quietly powering the wireless revolution across a diverse range of applications.

The remainder of this paper is organized as follows: section II gives the related work. Section III describes the system model and massive MIMO channel model. Section IV gives an overview of MIMO detection methods. Section V describes the new proposed matrix inversion method. Section VI presents computational complexity analysis. Section VII introduces simulation results. Finally, Section VIII concludes the work.

Notation: The symbols $\Re(z)$ and $\Im(z)$ represent the real and imaginary components of the complex number *z*. Vectors and matrices are denoted by lowercase and uppercase boldface letters. Furthermore, $\boldsymbol{I}_{\boldsymbol{N_T}}$ denotes the $N_T \times N_T$ identity matrix and $\|\cdot\|$ stands for the Euclidean norm of a vector. The set of complex numbers is identified as $\mathbb{C}$.

II. Related Work

Several studies have proposed multiple approaches to mitigate the complexity of inverting high-dimensional matrices for massive MIMO applications [9]. The efficiency and scalability of iterative approaches have substantially enhanced signal detection approaches in massive MIMO systems. Moreover, the iterative method can reduce complexity by substituting matrix-vector multiplications for matrix-matrix operations. However, in contrast to direct matrix inversion, many iterations may increase the complexity [10]. Iterative techniques commonly utilized in massive MIMO detection include the JI [11, 12], Chebyshev technique (Cheby) [13], the CG technique (CG) [14], the Neumann-Series Approximation (NSA) [15], and many other iteration techniques [9]. The CG efficiently solves linear equations for the inverse by iteratively searching in conjugate directions. Modified detectors that use approximate matrix inversion techniques have proven vital for dealing with complexity and noise enhancement challenges. The ADMM method, which minimizes the optimization problem by breaking it down into smaller issues, is mostly used to resolve convex optimization issues [16]. One of its best features is that it can handle multiple computations differently. Iterative optimization methods can benefit greatly from using ADMM as an initial vector. Wherein this can lead to faster convergence and better solution quality. The results produced by ADMM are usually more in line with the optimum solution than those of random or zero vectors [17]. Thus, using the solution obtained from ADMM as the initial vector for iterative optimization methods can effectively seed the iterative process with a more informed starting point. The proposed technique's initial stage utilizes the ADMM to obtain the required detection performance with fewer iterations. These techniques only require matrix-vector multiplications, reducing the complexity from $\mathcal{O}({N_T}^3)$ to $\mathcal{O}({N_T}^2)$.

Various hybrid iterative algorithms have been suggested to enhance performance and convergence rate. The authors integrated the JI and GS techniques in [18]. The hybrid algorithms employed initialization based on the JI approach. The GS approach is then used for estimate. The suggested algorithms' computational complexity and bit error rate (BER) were compared with traditional 64QAM modulation schemes and 16x128 system configuration methods. The suggested algorithm achieves a compromise between complexity and performance. Joint steepest descent (SD) and non-stationary Richardson (NSR) iteration approaches were taken into consideration by the authors in [19]. To improve performance, the SD obtains an effective search direction for the ensuing NSR approach. To greatly accelerate convergence, they merged the system and iteration-dependent acceleration mechanism with the scaled-diagonal initialization. Compared to current iterative methods, the suggested joint detection strategy performs better and offers less computing complexity than the traditional MMSE detector. To enhance the effectiveness of the conventional iterative approaches, the authors in [20] suggested three hybrid algorithms: Newton–Schultz–Richardson (NS-RI), Newton–Schultz–Chebyshev (NS-Cheby), and Newton–Schultz–Gauss-Seidel (NS-GS). The likelihood ascent search (LAS) step significantly enhances the performance of the suggested detectors in terms of computational complexity and BER for 16QAM, 64QAM, and 256QAM modulation schemes, as well as for a 32x128 system configuration. The suggested algorithms' performance was examined and compared with that of other existing algorithms. The results showed that the complexity of the algorithms was decreased, and the performance was improved. In this work [21] a novel iterative method circumvents the matrix inversion computation problem by choosing CG and SOR techniques. The combined cascade structure of both iterative algorithms is the basis of this proposed uplink massive MIMO detection method. In the last iterations for terminal calculations, the SOR technique is used after the CG method has been used and adjusted for the initial answer.

Several approaches, including approximate inversions, have been suggested to balance performance and complexity. The objective is to identify the optimal combination of techniques customized to the massive MIMO structure. Recent findings show remarkable performance-complexity tradeoffs for practical implementation. Another issue with matrix inversion occurs when the channel matrix is almost singular, resulting in an ill-conditioned system. In these situations, matrix inversion may not equalize the incoming signal. We suggest using a novel hybrid iterative method called ADMM-CG to address these difficulties. It combines the advantages of CG and ADMM approaches with scaled-diagonal initialization to improve efficiency and reduce computational complexity. This method has made numerous important advancements, including:

• A blend of two iteration methods is utilized in this approach: ADMM is employed for the initial approximation, while CG is used for the final refinement. This approach takes advantage of the early iteration stability and efficiency of ADMM, along with the subsequent iteration effectiveness of CG.

• Low Computational Complexity: By merging ADMM with CG and using an initial algorithm based on trace tridiagonal, this algorithm greatly reduces computational complexity compared to traditional linear detectors or standalone iterative algorithms. It is important to note that we have many large-scale computations in massive MIMO systems.

• Enhanced Performance and Stability: The ADMM-CG algorithm is characterized by strong performance since it can provide near-optimal solutions within a few iterations, and it remains stable even at low Signal-to-Noise Ratios (SNR). As a result, it outperforms other iterative methods like JI, SOR, and standalone CG in terms of Bit Error Rate (BER).

• Numerical Simulation Results: Numerous numerical simulations verify the proposed mixed algorithm's superiority to that of CG, which is one among its constituents, and ADMM, as well as other conventional detectors in various antenna configurations and high-order modulation. This means that the algorithm can be quite efficient and effective during massive MIMO scenarios.

These novel contributions make the ADMM-CG algorithm a notable step forward in massive MIMO signal detection by striking a balance between detection performance and computational complexity.

III. System Model

The BS in a huge uplink MIMO system has N_R antennas and simultaneously serves N_T users with a single antenna. where $N_R >> N_T$ [22]. Fig. 1 depicts the uplink of a massive MIMO system, including users with N_T and BS with N_R transmitting and receiving antennas, respectively.

The received signal vector $\tilde{\boldsymbol{y}} \in \mathbb{C}^{N_R \times 1}$ can be modeled as shown in

$$\tilde{\boldsymbol{y}} = \tilde{\boldsymbol{H}}\tilde{\boldsymbol{x}} + \tilde{\boldsymbol{n}}, \tag{1}$$

where $\tilde{\boldsymbol{x}} \in \mathbb{C}^{N_T \times 1}$ is the transmitted signal vector, $\tilde{\boldsymbol{n}} \in \mathbb{C}^{N_R \times 1}$ is additive white Gaussian noise, and $\tilde{\boldsymbol{H}} \in \mathbb{C}^{N_R \times N_T}$ is the channel matrix. Given knowledge of $\tilde{\boldsymbol{H}}$, estimating the sent signal vector $\tilde{\boldsymbol{x}}$ from the received signal is the goal of multiuser detection [23].

It is possible to transform the complicated equivalent model into the real equivalent signal model.

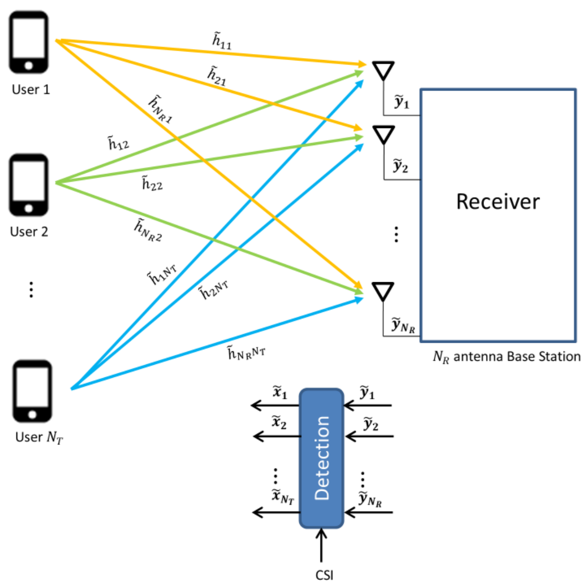


Fig. 1. An uplink massive MIMO system

Using Orthogonal Real-Valued Decomposition (ORVD) [20], in which the model of the received signal in real value can be expressed as:

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}, \tag{2}$$

the real equivalents of the received signal, transmitted vector, and noise vector, denoted as $\boldsymbol{y}$, $\boldsymbol{x}$, and $\boldsymbol{n}$, respectively, are expressed below:

$$\boldsymbol{y} = [\Re(\tilde{y}_1)\ \Im(\tilde{y}_1)\ \cdots\ \Re(\tilde{y}_{N_R})\ \Im(\tilde{y}_{N_R})]^{\boldsymbol{T}}. \tag{3}$$

$$\boldsymbol{x} = [\Re(\tilde{x}_1)\ \Im(\tilde{x}_1)\ \cdots\ \Re(\tilde{x}_{N_T})\ \Im(\tilde{x}_{N_T})]^{\boldsymbol{T}}. \tag{4}$$

$$\boldsymbol{n} = [\Re(\tilde{n}_1)\ \Im(\tilde{n}_1)\ \cdots\ \Re(\tilde{n}_{N_R})\ \Im(\tilde{n}_{N_R})]^{\boldsymbol{T}}. \tag{5}$$

In the model, the mean signal energy, E_x, is equal to $0.5\tilde{E}_x$ and the noise variance, σ^2, which is also equal to $0.5\tilde{\sigma}^2$. One way to represent the ORVD channel matrix $\boldsymbol{H}$ is as a reference.

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}_{1,1} & \boldsymbol{H}_{1,2} & \cdots & \boldsymbol{H}_{1,2N_T-1} & \boldsymbol{H}_{1,2N_T} \\ \boldsymbol{H}_{2,1} & \boldsymbol{H}_{2,2} & \cdots & \boldsymbol{H}_{1,2N_T-1} & \boldsymbol{H}_{1,2N_T} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \boldsymbol{H}_{2N_R,1} & \boldsymbol{H}_{2N_T,2} & \cdots & \boldsymbol{H}_{2N_R,2N_T-1} & \boldsymbol{H}_{2N_R,2N_T} \end{bmatrix}. \tag{6}$$

A submatrix in $\boldsymbol{H}$ with dimensions (i, j) is defined as

$$\boldsymbol{H_{i,j}} = \begin{bmatrix} \Re(h_{i,j}) & -\Im(h_{i,j}) \\ \Im(h_{i,j}) & \Re(h_{i,j}) \end{bmatrix}, \tag{7}$$

where the symbol $h_{i,j}$ represents the element $\boldsymbol{H}$, where i represents $1, 2, \ldots, 2N_R$($2N_R$ since extending to cover both in-phase and quadrature components), and j represents $1, 2, \ldots, 2N_T$ ($2N_T$ for similar reasons). $h_{i,j}$ represents the real model of the complex version $\tilde{h}_{n,m}$ which represents the element $\widetilde{\boldsymbol{H}}$, where n represents $1, 2, \ldots, N_R$, and m represents $1, 2, \ldots, N_T$.

Our work assumes a flat fading channel model so that a single coefficient can represent each signal path.

IV. Overview of MIMO Detection Methods

A. Minimum Mean-Squared Equalizer Detector

The MMSE detector is expressed as [20]

$$\widehat{\boldsymbol{x}}_{\boldsymbol{MMSE}} = (\boldsymbol{A})^{-\boldsymbol{1}} \hat{\mathbf{x}}_{\mathbf{MF}} \,, \tag{8}$$

$$\boldsymbol{A} = (\boldsymbol{H^H H} + \rho \boldsymbol{I}_{N_T}) \,, \tag{8-a}$$

$$\rho = N_0/E_s \,, \tag{8-b}$$

$$\hat{\mathbf{x}}_{\mathbf{MF}} = \boldsymbol{H^H y} \,, \tag{8-c}$$

where $\widehat{\boldsymbol{x}}_{\boldsymbol{MMSE}}$ is the estimated signal, $\mathbf{A}$ is the filtering matrix for MMSE, and $\hat{\mathbf{x}}_{\mathbf{MF}}$ is the matched filter output.

B. Conjugate Gradients Detection Method

The CG iteration detected signal is defined by [24]

$$\widehat{\boldsymbol{x}}_{\boldsymbol{k}} = \widehat{\boldsymbol{x}}_{\boldsymbol{k-1}} + \tau_{k-1} \widehat{\boldsymbol{p}}_{\boldsymbol{k-1}} \,, \tag{15}$$

where τ_{k-1} is a scalar parameter and $\mathbf{p}^{(\mathbf{k-1})}$ represents the conjugate direction concerning $\boldsymbol{A}$, i.e.,

$$(\widehat{\boldsymbol{p}}_{\boldsymbol{k-1}})^{\mathbf{H}} \mathbf{A} \widehat{\boldsymbol{p}}_{\boldsymbol{j-1}} = 0, \quad \text{for } k \neq j. \tag{16}$$

Our work assumes a flat fading channel model so that a single coefficient can represent each signal path.

V. The Proposed Hybrid Iterative Algorithm

Our focus here will be hybrid iteration algorithms, which enhance detection performance. This study examines three familiar iteration methods: JI, Cheby, and CG. The first iteration of the ADMM method is used in place of the first iteration of each technique.

The ADMM method minimizes the problem in Eq. (17) by breaking it down into smaller issues. One of its best features is that it can handle multiple computations in several different ways [25]

$$\widehat{\boldsymbol{x}} \triangleq \min_{x \in O^{N_T}} \|\boldsymbol{n}\|^{\boldsymbol{2}} = \textbf{arg} \min_{x \in O^{N_T}} \quad \|\boldsymbol{y} - \boldsymbol{Hx}\|^{\boldsymbol{2}}, \tag{17}$$

eq. (17) may be rewritten as

$$\widehat{\boldsymbol{x}} = \arg \min_{x,z \in O^{N_T}} \quad \mathbf{g}(\boldsymbol{z}) + \|\boldsymbol{y} - \boldsymbol{Hx}\|^{\boldsymbol{2}}, \tag{18}$$

where, function $\boldsymbol{g}(\boldsymbol{z})$ is a convex regularizer function. The augmented Lagrangian function $\boldsymbol{\mathcal{L}}$ for Eq. (18) is given as follows:

$$\boldsymbol{\mathcal{L}}(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{\lambda}) = \boldsymbol{g}(\boldsymbol{z}) + \tfrac{\mathbf{1}}{\mathbf{2}} \|\boldsymbol{y} - \boldsymbol{Hx}\|^2 + \tfrac{\beta}{2} \|\boldsymbol{z} - \boldsymbol{x} - \boldsymbol{\lambda}\|^2, \tag{19}$$

where $\boldsymbol{\lambda}$ is the Lagrange parameter with the constraint $\boldsymbol{z} = \boldsymbol{x}$ and $\beta > 0$ is a fixed penalty parameter. The ADMM method, which minimizes the modified Lagrangian, can be used to solve Eq. (18). ADMM is mostly used to resolve convex optimization issues, and one of its best features is that it can handle multiple computations in several different ways. To get the result, the modified Lagrangian's minimum is calculated over $\boldsymbol{x}$ and $\boldsymbol{z}$ in each step of ADMM. The steps in k ADMM iterations [23]:

$$\widehat{\boldsymbol{x}}_k = \boldsymbol{argmin}\{\tfrac{\mathbf{1}}{\mathbf{2}} \|\boldsymbol{y} - \boldsymbol{Hx}\|^2 + \rho * \|\boldsymbol{z} - \boldsymbol{x} - \boldsymbol{\lambda}\|^2\}, \tag{20-a}$$

$$\widehat{\boldsymbol{z}}_k = \textstyle\prod_{\boldsymbol{c}}(\widehat{\boldsymbol{x}}_{k-1} + \widehat{\boldsymbol{z}}_{k-1}) \,, \tag{20-b}$$

$$\widehat{\boldsymbol{\lambda}}_k = \widehat{\boldsymbol{\lambda}}_{k-1} + (\widehat{\boldsymbol{x}}_{k-1} - \widehat{\boldsymbol{z}}_{k-1}), \tag{20-c}$$

where, $\prod_{\boldsymbol{c}}$ is the Euclidean projection onto $\mathbb{C}$, and $\boldsymbol{\lambda}$ is the dual variable. The first step is to fix $\boldsymbol{z}$ and $\boldsymbol{\lambda}$ and minimize $\boldsymbol{x}$ using Eq. (20-a). Then, using Eq. (20-b) to fix $\boldsymbol{x}$ and $\boldsymbol{\lambda}$ and minimize the augmented Lagrangian over $\boldsymbol{z}$. Finally, the dual variable $\boldsymbol{\lambda}$ is updated by Eq. (20-c). $\boldsymbol{H}$ shows asymptotic orthogonality in the context of massive MIMO when $N_R \gg N_T$ [2, 26]; Consequently, we get

$$\frac{h_i^H h_j}{N_R} \to 0, \;\; i \neq j, \;\; i, j = 1, 2, \cdots N_T, \tag{21}$$

where h_i is the ith column vector of the $\boldsymbol{H}$. Thus, the MMSE filtering matrix $\boldsymbol{A}$ is diagonally dominant for massive MIMO uplink systems. Therefore, the diagonal matrix $\boldsymbol{D}$'s inverse approximates $\boldsymbol{A}$'s inverse. Moreover, the tridiagonal matrix, which is a special case of the diagonal matrix with partial pivoting, is defined as [27]

$$\boldsymbol{S} = stair(\boldsymbol{A}(N_T, N_T - 1); \boldsymbol{A}(N_T, N_T); \boldsymbol{A}(N_T, N_T + 1). \tag{22}$$

By observing that the number of receive antennas is much greater than the number of transmit antennas for large antenna arrays, the value of each element in the tridiagonal matrix may be estimated as follows:

$$s \approx \frac{tr(S)}{N_T}, \tag{23}$$

where $tr(S)$ is matrix $\boldsymbol{A}$'s tridiagonal element's trace. Thus, the initial vector $\widehat{\boldsymbol{z}}_{\boldsymbol{1}}$ is used as a first in our proposed work to accelerate its convergence rate, can be defined as:

$$\hat{\boldsymbol{z}}_{\mathbf{1}} = s\hat{\mathbf{x}}_{\mathbf{MF}}. \tag{24}$$

Observably, Eq. (22) reduced the required computations by substituting scalar–vector multiplications for matrix-vector multiplications. Then, Eq. (20-a) is modified with k=1 as follows.

$$\hat{\boldsymbol{x}}_1 = \boldsymbol{\Omega} * (\hat{\mathbf{x}}_{\mathbf{MF}} + \rho * (\hat{\boldsymbol{z}}_{\mathbf{1}})), \tag{25}$$

where $\boldsymbol{\Omega} = (\boldsymbol{L}^{\boldsymbol{H}})^{-\mathbf{1}}\boldsymbol{L}^{-\mathbf{1}}$, where $\boldsymbol{L}$ represents the Cholesky decomposition of $\boldsymbol{A}$, which prevents direct inversion.

Table I explains the ADMM-CG hybrid iterative algorithm in detail. The first iteration uses Eq. (25) in each scenario.

TABLE I
PROPOSED ADMM-CG ALGORITHM PSEUDO CODE

Input: H, y, N_0, E_s, N_T, N_R, s, **L**
Output: $\hat{\boldsymbol{x}}_k$
Initialization:
$\rho = N_0/E_s, \hat{\mathbf{x}}_{\mathbf{MF}} = \boldsymbol{H^H y}, \boldsymbol{A} = \boldsymbol{H^H H} + \rho \boldsymbol{I}_{\boldsymbol{N_T}}$
$\boldsymbol{\Omega} = (\boldsymbol{L^H})^{-1}\boldsymbol{L}^{-1}, \hat{\boldsymbol{z}}_{\mathbf{1}} = \hat{\mathbf{x}}_{\mathbf{MF}}/s$
First iteration:
$\hat{\boldsymbol{x}}_{\mathbf{1}} = \boldsymbol{\Omega}(\hat{\mathbf{x}}_{\mathbf{MF}} + \rho\hat{\boldsymbol{z}}_{\mathbf{1}})$
$\hat{\boldsymbol{r}}_{\mathbf{1}} = \hat{\mathbf{x}}_{\mathbf{MF}} - \boldsymbol{A}\hat{\boldsymbol{x}}_{\mathbf{1}}, \hat{\boldsymbol{p}}_{\mathbf{1}} = \hat{\boldsymbol{r}}_{\mathbf{1}}$
Other iteration:
for k=2 to K **do**
$\hat{\boldsymbol{p}}_k = \boldsymbol{A}\,\hat{\boldsymbol{p}}_{\boldsymbol{k-1}}, \tau_{\boldsymbol{k-1}} = \Vert \hat{\boldsymbol{r}}_{\boldsymbol{k-1}}\Vert^2/(\hat{\boldsymbol{p}}_{\boldsymbol{k-1}})'\,\hat{\boldsymbol{p}}_{k-1},$
$\hat{\boldsymbol{x}}_{\boldsymbol{k}} = \hat{\boldsymbol{x}}_{\boldsymbol{k-1}} + \hat{\tau}_{k-1}\hat{\boldsymbol{p}}_{\boldsymbol{k-1}},$
$\hat{\boldsymbol{r}}_k = \hat{\boldsymbol{r}}_{\boldsymbol{k-1}} - \tau_{\boldsymbol{k-1}}\hat{\boldsymbol{p}}_{\boldsymbol{k-1}},$
$\delta = \Vert \hat{\boldsymbol{r}}_k\Vert^2/\Vert\hat{\boldsymbol{r}}_{\boldsymbol{k-1}}\Vert^2, \hat{\boldsymbol{p}}_k = \hat{\boldsymbol{r}}_k + \delta\hat{\boldsymbol{p}}_{\boldsymbol{k-1}},$
end for

VI. COMPLEXITY ANALYSIS

Our next step is to assess the current detectors and proposed hybrid approaches for their level of complexity. Since multiplications provide the majority of complexity, we have considered their number while estimating detector complexity.

Initialization complexity is almost identical since the proposed hybrid techniques use Eq. (25) in the first iteration.

(1) Compute $\hat{\boldsymbol{x}}_{\boldsymbol{MF}}$: $\hat{\boldsymbol{x}}_{\boldsymbol{MF}}$ requires $4N_T N_R$ multiplications.

(2) Compute $\boldsymbol{\Omega}$: the computation of $\boldsymbol{\Omega}$ requires $4N_T^2 + 4N_T$ multiplications.

(3) Compute $\hat{\boldsymbol{z}}_{\mathbf{1}}$: $\hat{\boldsymbol{z}}_{\mathbf{1}}$ come from $s * \hat{\boldsymbol{x}}_{\boldsymbol{MF}}$, where s is a scalar quantity, and the multiplication between s and $\hat{\boldsymbol{x}}_{\boldsymbol{MF}}$ requires $2N_T$ multiplications. Therefore, the total number of multiplications required for $\hat{\boldsymbol{z}}_{\mathbf{1}}$ is $4N_R N_T + 2N_T$.

(4) The multiplication between ρ and $\boldsymbol{z}_{\mathbf{1}}$ is also $2N_T$.

(5) Compute the multiplications between $\boldsymbol{\Omega}$ and $(\hat{\mathbf{x}}_{\mathbf{MF}} + \rho * \boldsymbol{z}_{\mathbf{1}})$): result in $4N_T^2$.

While the initial ADMM iteration is essential for improving performance, it also involves many multiplications. It takes $(8N_R N_T + 8N_T + 8N_T^2)$ multiplications to compute Eq. (25). The rest of the iterations for ADMM-CG are equivalent to the CG method. For $N_R/N_T = 4$ and three iterations.

Fig. 2 illustrates the number of multiplications used along with the number of users. The figure demonstrates that the suggested hybrid approach needs more operations than the conventional algorithms. However, it requires much less multiplication than MMSE.

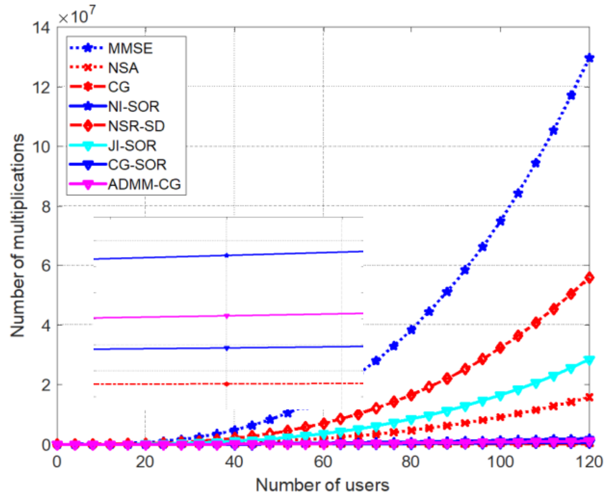


Fig. 2. Real complexity vs users' number for the conventional and suggested hybrid approaches.

VII. SIMULATION RESULTS AND DISCUSSION

This section compares the proposed method's detection performance with traditional algorithms to current state-of-the-art detectors such as NSA [15], CG [9], JI-GS [18], NSR-SD [19], NI-SOR [20], CG-SOR [21]. These detection algorithms also used a massive MIMO uplink system. The value of the SOR relaxation parameter used in NI-SOR is 1.05, while in CG-SOR, it is 1.2. The matched filter output is the initial vector for CG. Fig. 3 demonstrates the results of comparing the suggested hybrid approach and the traditional algorithm for a $N_R \times N_T = 128 \times 32$ employing a 64-QAM antenna scenario. The BER performances of the iterative detectors are evaluated relative to the MMSE detection. The suggested hybrid detection technique provides near-optimum performance for just two iterations.

The proposed method outperforms the performance of the other algorithms, like JI-GS, NSR-SD, NI-SOR, and CG-SOR, for three iterations, as shown in Fig. 4.

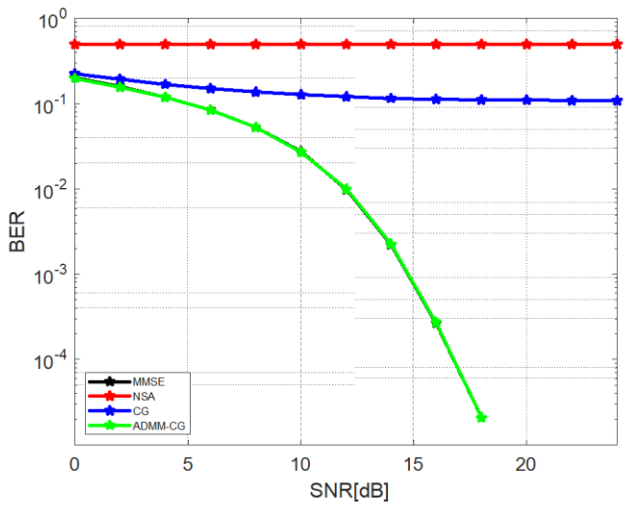
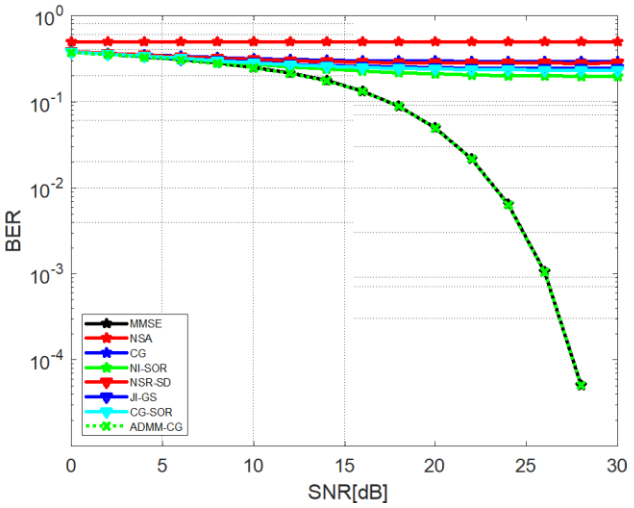
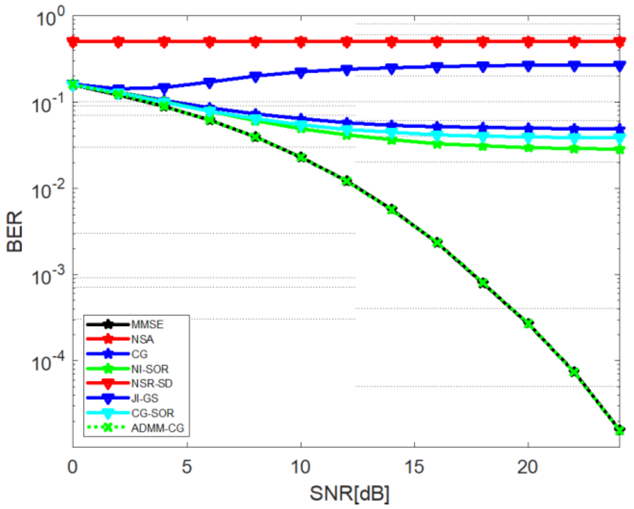


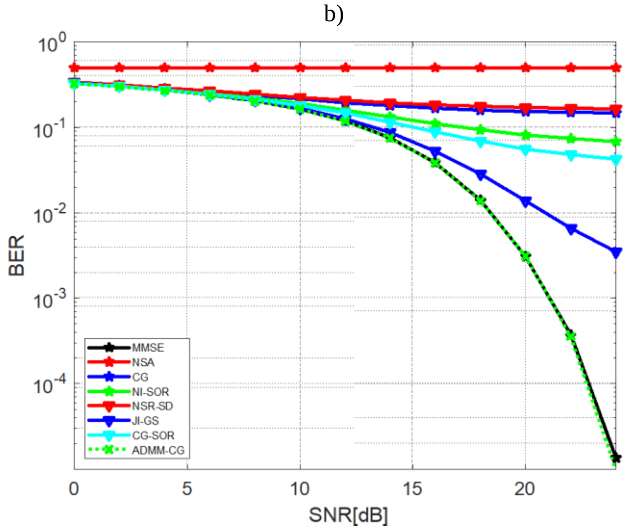
Fig. 3. The BER performance comparison of the proposed algorithm and other algorithms for two iterations with 128×32 and 64-QAM



a)



b)



c)

Fig. 4. The BER performance comparison of the proposed algorithm and other algorithms for three iterations with the following system dimensions: a. 128×64 and 256-QAM, b. 128×128 and QPSK, c. 256×64 and 256-QAM

Fig. 4-a shows that the proposed method outperformed the other methods in the high-order modulation 256-QAM. Fig. 4-b shows that it maintained its performance and outperformed the rest of the methods when the number of users equaled the number of antennas in the BS. Fig. 4-c shows that it outperformed the other methods when the modulation order was high, and the number of antennas in the BS was 256. We conclude from Fig. 4 that the existing methods are sensitive at high modulation order and when the number of users approaches the number of antennas in the BS. The proposed algorithm showed performance close to optimum in all cases.

Fig. 5 demonstrates how transmit antenna modifications affect BER performance for $N_R = 256$ at $SNR = 20$ and 256-QAM with $k = 3$. The proposed method showed performance near to optimum in all user values and at low N_T, the rest of the methods approached optimum, but the performance gap became clearer as the number of users increased. Compared with the mentioned hybrid algorithms, the proposed algorithm performs near-optimal with growing system dimensions such as BS antennas, users, or modulation orders.

Table II illustrates the required computation for each algorithm's initial and first iteration vectors. It can be observed that our proposed method showed performance close to optimum with reasonable complexity compared to other conventional and hybrid algorithms

Fig. 6 demonstrates the complexity comparison regarding the number of multiplications performed with every iteration.The CG method has the least number of multiplications among all algorithms, followed by CG-SOR and then the proposed ADMM-CG method. However, they require more iterations than the proposed ADMM-CG with high SNR to obtain the best BER performance, which is still lower than the proposed method performance

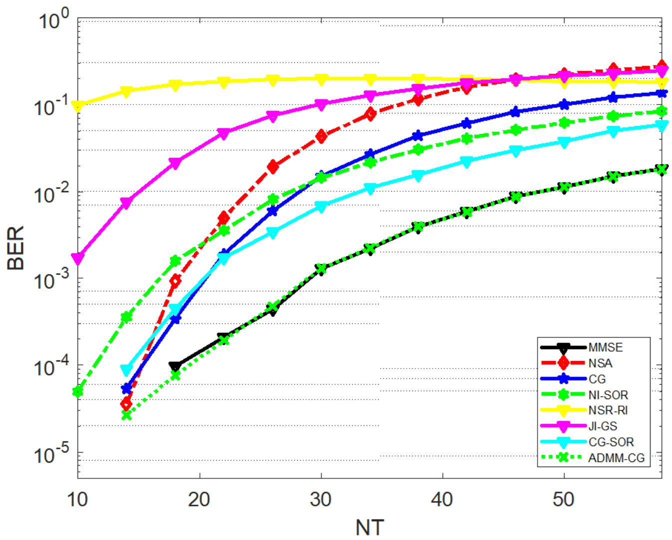


Fig. 5. The BER performance against the user number comparison of the proposed algorithm and other algorithms for three iterations when the BS is equipped with 256 antennas using 256-QAM, at SNR=20dB

TABLE II
COMPLEXITY OF THE INITIAL VECTOR FOR VARIOUS ALGORITHMS

Algorithms	The initial and first stage	Complexity of the initial vector
NI-SOR	$\hat{\boldsymbol{x}}_{\mathbf{0}} = (\alpha + \phi\rho)\hat{\mathbf{x}}_{\mathbf{MF}} + \phi \boldsymbol{H}^{\boldsymbol{H}}\boldsymbol{H}\hat{\mathbf{x}}_{\mathbf{MF}}$ $\overline{\boldsymbol{x}}_{\mathbf{0}} = \boldsymbol{H}^{\boldsymbol{H}}\boldsymbol{H}\hat{\boldsymbol{x}}_{\mathbf{0}}$ $\hat{\boldsymbol{x}}_{\mathbf{1}} = (2 - \alpha\rho + \phi\rho^2)\hat{\boldsymbol{x}}_{\mathbf{0}} \cdots$ $+(2\phi\rho - \alpha)\overline{\boldsymbol{x}}_{\mathbf{0}} + \phi\boldsymbol{H}^{\boldsymbol{H}}\boldsymbol{H}\overline{\boldsymbol{x}}_{\mathbf{0}}$ Where α, ϕ are scalars	$16N_RN_T + 8N_T$
NSR-SD	$\lambda_{min} = N_R\left(1 - \sqrt{\frac{N_T}{N_R}}\right)^2,$ $\lambda_{max} = N_R\left(1 + \sqrt{\frac{N_T}{N_R}}\right)^2$ $c = \frac{N_T}{N_R} + \frac{1}{\sqrt{\lambda_{min}}}\hat{\boldsymbol{x}}_{\mathbf{0}} = (c\boldsymbol{D})^{-1}\hat{\mathbf{x}}_{\mathbf{MF}},$ $\hat{\boldsymbol{r}}_{\mathbf{0}} = \hat{\mathbf{x}}_{\mathbf{MF}} - \boldsymbol{A}\hat{\boldsymbol{x}}_{\mathbf{0}},$ $a = \Vert \hat{\boldsymbol{r}}_{\mathbf{0}}\Vert^2/(\mathbf{A}\hat{\boldsymbol{r}}_{\mathbf{0}})^H \cdot \hat{\boldsymbol{r}}_{\mathbf{0}}$ $\hat{\boldsymbol{x}}_{\mathbf{1}} = \hat{\boldsymbol{x}}_{\mathbf{0}} + \boldsymbol{a}\boldsymbol{r}_{\mathbf{0}}$	$3{N_T}^2N_R + 4N_RN_T + 2N_T$
JI-GS	$\hat{\boldsymbol{x}}_{\mathbf{1}}$ $= \boldsymbol{D}^{-\mathbf{1}}(\hat{\mathbf{x}}_{\mathbf{MF}} + (\mathbf{D} - \mathbf{A})(\boldsymbol{D}^{-\mathbf{1}}\hat{\mathbf{x}}_{\mathbf{MF}}))$	$2{N_T}^2N_R + 4N_RN_T + 8N_T^2$
CG-SOR	$\hat{\boldsymbol{x}}_{\mathbf{1}} = (\Vert \hat{\mathbf{x}}_{\mathbf{MF}}\Vert^2/\hat{\mathbf{x}}_{\mathbf{MF}}{}' \cdot \boldsymbol{A}\hat{\mathbf{x}}_{\mathbf{MF}})\hat{\mathbf{x}}_{\mathbf{MF}}$	$4N_TN_R + 4{N_T}^2 + 10N_T$
Proposed ADMM-CG	$\hat{\boldsymbol{x}}_{\mathbf{1}} = (\boldsymbol{L}^{\boldsymbol{H}}\mathbf{L})^{-1}(\hat{\mathbf{x}}_{\mathbf{MF}} + \rho\hat{\boldsymbol{z}}_{\mathbf{1}})$	$8N_RN_T + 8N_T + 8N_T^2$

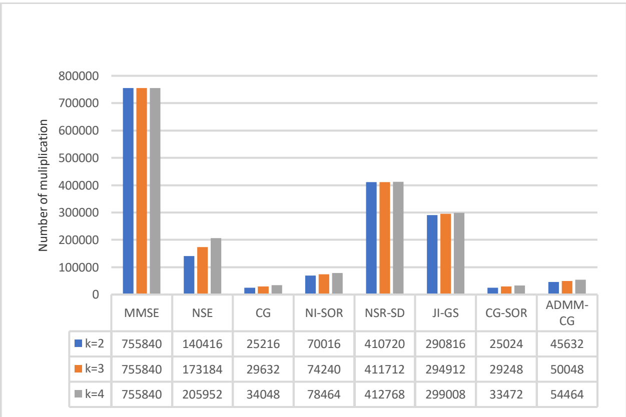


Fig. 6. Complexity comparison of the proposed hybrid and the conventional approaches for a 128×32 scenario

VIII. CONCLUSION

This paper proposes a hybrid iterative algorithm-based uplink massive MIMO system detection to meet the next-generation demand in increasing the number of users and high data rate requirements. The proposed algorithm incorporates the ADMM technique with the CG algorithm and integrates with the trace tridiagonal initial method to achieve a complexity-performance tradeoff for uplink massive MIMO. Its initial iteration utilizes the ADMM to improve the convergence of the CG technique. The Monte Carlo simulation results show significant performance improvement compared to the current techniques. The proposed algorithm performs near-optimal with growing system dimensions such as BS antennas, users, or modulation orders. Moreover, the suggested method performs near optimum for only two iterations. Despite its increased complexity compared to conventional iterative algorithms, the ADMM-CG algorithm offers a compelling solution by reducing the overall computational burden compared to MMSE detection.

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