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LETTER TO THE EDITOR

SIX-DIMENSIONAL SPECIAL RELATIVITY?

J. STRNAD

Department of Physics and Institute »J. Stefan«,

University of Ljubljana, Ljubljana

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Six-dimensional space-time with symmetrical spatial and temporal parts allows a one-to-one mapping of the interior of the light cone unto its exterior and vice versa. It was, however, not possible to relate three temporal components to one experimentally accesible time component. The letter shows that this is true also for a proposal based on a matrix relative velocity and a corresponding averaging procedure over three temporal components.

Attempts to extend the Lorenzt transformation to superluminal reference frames lead to difficulties, since in a four-dimensional space-time with one temporal and three spatial dimensions the interior of the light cone cannot be transformed in a consistent way into its exterior and vice versa. Owing to this, recently six-dimensional space-time with three temporal and three spatial dimensions attracted considerable interest[1–5]. While in a symmetrical six-dimensional theory there are no general arguments against an extension of the Lorentz transformation, however, the question arises how the measurable ordinary time should be constructed with three time components. The proposition to take the scalar product of the time part of the world six-vector $\vec{t} \cdot \vec{t} = t_1^2 + t_2^2 + t_3^2$ directly as this quantity did not give satisfactory results[3-5]. Recently, a more sophisticated scheme has been introduced in which by means of a relative velocity matrix with elements $\partial x_i/\partial t_j$, $(i, j = 1, 2, 3)$ subluminal and superluminal transformations can be considered on an equal footing[6]. Though this six-dimensional generalization is formally appealing one should not forget that it must meet a severe condition: in the subluminal case it must reproduce the ordinary four-dimensional relativity together with experimentally established facts.

The intention of this letter is to show that the proposed scheme[6] in this respect apparently suffers from a serious deficiency. This can be realized by considering the subluminal part only. The new transformation from an inertial reference frame S to another such frame S′ in the usual orientation reads

$$\begin{aligned} \vec{t}' &= \vec{t} - \gamma x_1 \vec{v} + \gamma^2 \vec{v} (\vec{v} \cdot \vec{t})/(\gamma + 1) \\ x_1' &= \gamma x_1 - \gamma (\vec{v} \cdot \vec{t}), \; x_2' = x_2, \; x_3' = x_3. \end{aligned} \tag{1}$$

The light velocity is put equal to 1. The remainder of the velocity matrix $\vec{v} = (\partial x_1/\partial t, \partial x_1/\partial t_2, \partial x_1/\partial t_3)$ pertains to the motion of the origin of S′ in S and can be considered as an abstract parameter, whereas $\gamma = (1 - \vec{v} \cdot \vec{v})^{-1/2}$. The transformation (1) leaves invariant the scalar product $-\vec{t} \cdot \vec{t} + \vec{x} \cdot \vec{x}$ and has the usual properties: the inverse transformation is got by replacing $\vec{t}$ by $\vec{t}'$ and $\vec{x}$ by $\vec{x}'$ and vice versa and $\vec{v}$ by $-\vec{v}$. The relation

$$\vec{t}' \cdot \vec{t}' = \vec{t} \cdot \vec{t} + \gamma^2 x_1^2 \vec{v} \cdot \vec{v} + \gamma^2 (\vec{v} \cdot \vec{t})^2 - 2\gamma^2 x_1 (\vec{v} \cdot \vec{t})$$

gives for a pure time vector $\vec{t}'$ $(\vec{x} = 0)$ averaged over all orientations

$$t_m'^2 = t_m^2 \left(1 + \frac{1}{2} \gamma^2 \vec{v} \cdot \vec{v}\right). \tag{2}$$

To get the well-known time dilatation for the measured coordinate (t_m') and proper (t_m) time $t_m'^2 = t_m^2/(1 - V^2)$ the ordinary velocity V of the origin of S′ is to be determined by the parameter $\vec{v}$ as $V^2 = \vec{v} \cdot \vec{v}/(3 - 2\vec{v} \cdot \vec{v})$. If x_1 is a proper length on the x_1-axis in S, the contracted length measured in S′,

$$x_1' = x_1/\gamma = (1 - V^2)^{1/2} x_1 (1 + 2V^2)^{1/2} \tag{3}$$

contains an additional factor $(1 + 2V^2)^{-1/2}$. Among all these results[6] the last (3) makes the scheme suspect. Though length contraction cannot be measured directly it is difficult to believe that other results of special relativity would be preserved.

As a test whether this is true or not dynamical quantities, e.g. the energy-momentum vector, could be used for which experimental verification exist[7]. But the introduction of these quantities into the six-dimensional scheme appears not to be unique. There is, however, a more appropriate test: the Doppler effect. One has only to introduce a six-vector $(\vec{\omega}, \vec{k})$ lying on the light cone: $-\vec{\omega}\cdot\vec{\omega}+\vec{k}\cdot\vec{k}=0$. If the phase of electromagnetic waves is invariant under transformation (1), $-\vec{\omega}\cdot\vec{t}+\vec{k}\cdot\vec{x}=-\vec{\omega}'\cdot\vec{t}'+\vec{k}'\cdot\vec{x}'$, the vector $(\vec{\omega}, \vec{k})$ transforms like $(\vec{t}. \vec{x})$:

$$\vec{\omega}'=\vec{\omega}+\gamma^2\vec{v}(\vec{\omega}\cdot\vec{v})/(\gamma+1)-\vec{v}\,k_1\,\gamma$$

$$k_1'=\gamma\,k_1-\gamma\,(\vec{\omega}\cdot\vec{v}),\ k_2'=k_2,\ k_3'=k_3.$$

Following the above procedure of averaging over all orientations of the temporal part of the vector the equation $\vec{\omega}'\cdot\vec{\omega}'=\vec{\omega}\cdot\vec{\omega}+\gamma^2 k_1^2\vec{v}\cdot\vec{v}+\gamma^2(\vec{\omega}\cdot\vec{v})^2-2\gamma^2 k_1(\vec{\omega}\cdot\vec{v})$ gives

$$\omega'^2_m=\omega^2_m(1+\gamma^2\vec{v}\cdot\vec{v}+\frac{1}{3}\gamma^2\vec{v}\cdot\vec{v})=\omega^2_m(1+\frac{4}{3}\gamma^2\vec{v}\cdot\vec{v})$$

if the wave propagates in the x_1-direction ($k_2=k_3=0$). In this case $\vec{\omega}\cdot\vec{\omega}=k_1^2$ since the vector lies on the light cone.

The equation

$$\omega'^2_m=\omega^2_m(1+3V^2)/(1-V^2)\approx\omega^2_m(1+4V^2+\ldots)$$

does not agree with the well-known Doppler formula

$$\omega'^2_m=\omega^2_m(1+V)/(1-V)\approx\omega^2_m(1+2V+2V^2+\ldots)$$

which was confirmed indirectly up to terms of the second order[8]. One has to be careful in averaging since $\vec{\omega}'$ contains $-k_1\,\gamma\,\vec{v}$ and is not a pure time vector. Nevertheless, one cannot see how to get averaging $\vec{\omega}'\cdot\vec{\omega}'$ a term, linear in $\vec{v}$ or V, that is essential for the Doppler effect.

Such reasoning forces us to meet, as before, sixdimensional space-time, and the proposed averaging procedure above, all, with restraint.

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ŠESTRAZSEŽNA POSEBNA TEORIJA RELATIVNOSTI

J. STRNAD

Institut »J. Stefan«, Univerza v Ljubljani, Ljubljana

UDK 630.12

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