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T' (10040) AS AN I = 1 VECTOR MESON RESONANCE

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We treat ϱ^0, ω, φ, ψ, T and T' as ground states of a six quark $Q\bar{Q}$ system and consider T' to be the neutral member of an isospin triplet. By determining ψ and T as appropriate mixtures of c, t and b quarks and assuming an extended vector meson dominance a good saturation of Weinberg's first spectral function sum rule is obtained with the implication $\Gamma(T \rightarrow e^+ e^-) = 4.5$ keV and $\Gamma(T' \rightarrow e^+ e^-) = 0.5$ keV for the electron pair decay widths.

Motivated by the observation that the $T' - T$ mass difference is somewhat too large[1] for a bottomonium interpretation of T', i.e. as a radial excitation of T, we consider T' (10040) as a member of a family of six vector meson resonances made up of quark-antiquark pairs and characterized by zero values of quantum numbers. This approach, occasionally mentioned as a possibility[2], postpones the need for the introduction of new quarkonium potentials[3], posibly different from the popular charmonium potential[4], until the eventual appearance of experimental data on separate radial excitations of T and T'. The predictive power of the model rests on the assumption that T' should be a heavy quark-antiquark $I=1$ state and thus make an analog to the ϱ^0 quark composition.

We start with six quarks: u, d, s, c, t and b. The most general unitary transformations, with the baryon number conservation included, introduce a $U(6)$ group. The masses of the quarks, however, are very broadly spread, from about 0.3 GeV to 4.9 GeV, and any assumed symmetry of the theory would necessarily be very badly broken. This does not prevent, however, that interaction currents transform like symmetry currents of the group, namely as linear combinations of the 36 components, of a tensor operator transforming by the regular representation of $U(6)$. As for the weak and electromagnetic interactions we assume that their currents are contained within the 16 components $j_\mu^1, \ldots, j_\mu^{15}$ and j_μ^0 of the $U(4)$ subgroup of $U(6)$, such that it contains quantum numbers B, I_3, Y and C. We maintain that with regard to this subgroup quarks transform like basis vectors of an irreducible, fundamental, six dimensional representation of $SU(4)$, whereby quarks obtain the following quantum numbers: $u(0,1/3,1/2)$, $d(0,1/3,-1/2)$, $s(0,-2/3,0)$, $c(1,2/3,0)$, $t(1,-1/3,1/2)$, $b(1,-1/3,-1/2)$ for C, Y, I_3 respectively, and from the point of $SU(3)_{I,Y}\ U(1)_C$ classification form an $SU(3)$ triplet (u,d,s) and an $SU(3)$ antitriplet (c,t,b). This coincides with Harari's view[5] on the six quark quantum numbers, but implies that the weak-electromagnetic interactions excite no new quantum numbers (»heaviness« H_1, H_2).

In order to confine the electromagnetic-weak currents to the 16 components of a $U(4)$ tensor operator, which in a $SU(2)\times U(1)$ unified theory amounts to the requirement for an embedding $SU(2)_W \times U(1) \subset U(4)$ we fix the phase ambiguity in the matrices of the $SU(4)$ generators $F_a^{(6)}$, $a = 1, \ldots, 15$ according to the convention of IUM-positivity[6] and take the electromagnetic current in the form[7]:

$$j_\mu^{em} = j_\mu^3 + \frac{1}{\sqrt{3}} j_\mu^8 + 2\sqrt{\frac{2}{3}}\,(j_\mu^{15} + j_\mu^0) \tag{1}$$

which corresponds to the quark charge assignment:

$$Q(u,d,s,c,t,b) = (2/3, -1/3, -1/3, 5/3, 5/3, 2/3). \tag{2}$$

Note that the assumption for T' forming an $I = 1$ state would not be possible had we ascribed as quark representation the $SU(6)$ generalization of the Gell--Mann λ-matrices because there the basis contains only one pair of $I = 1/2$ states and in order to obtain two $I = 1$ triplets (one for ϱ^0 and one for T') in the $Q\bar{Q}$ composition, we need two such states.

We assume that vector mesons ϱ^0, ω, φ, ψ, T and T' form a unitary mapping of quark-antiquark pairs: $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, $c\bar{c}$, $t\bar{t}$ and $b\bar{b}$. Also that ϱ^0, ω and φ contain only $SU(3)$ triplet quarks, u, d, and s and retain traditional structure with $\omega - \varphi$ ideal mixing. Finally, that ψ, T and T' contain only $SU(3)$ antitriplet quarks c, t and b and that T belongs to an isospin doublet. This determines the structure up to a constant a:

$$\begin{aligned} \sqrt{2}\,\varrho^0 &= u\bar{u} - d\bar{d}, \\ \sqrt{2}\,\omega &= u\bar{u} + d\bar{d}, \\ \varphi &= s\bar{s}, \end{aligned} \tag{3}$$

$$\sqrt{2}\,\psi = \sqrt{2}\,\alpha c\bar{c} + \beta\,(t\bar{t} + b\bar{b}),$$
$$\sqrt{2}\,T = -\sqrt{2}\,\beta c\bar{c} + \alpha\,(t\bar{t} + b\bar{b}),$$
$$\sqrt{2}\,T' = t\bar{t} - b\bar{b},$$

with,

$$\alpha^2 + \beta^2 = 1. \tag{4}$$

To specify the undetermined constant α we consider implications of $U(4)$ symmetry on ($V \to e^+e^-$) decay widths, where, in the zero width approximation, one has

$$\Gamma(V \to e^+e^-) = \frac{4\pi\alpha^2\, m_V}{3f_V^2} + 0\left(\left(\frac{m_e}{m_V}\right)^4\right), \tag{5}$$

with f_V defined by

$$< 0\,|\,j_\mu^{em}(0)\,|\,V;\lambda, p > \; = \varepsilon_\mu^{(\lambda)}(p)\cdot\frac{m_V^2}{f_V(p^2)}. \tag{6}$$

To compare the matrix elements $< 0\,|\,j_\mu^{em}\,|\,V >$ for different vector mesons we need to express ϱ^0, ω, φ, ψ, T and T' as linear combinations of the six states, with zero quantum numbers, that appear in the reduction: $6 \times 6^* \to 1 + 15 + 20$.

Using notation ($SU(4)$, $SU(3)$, $SU(2)$, C, Y, I_3) these states are: (1, 1, 1), (15, 1, 1), (15, 8, 1), (15, 8, 3), (20, 8, 1) and (20, 8, 3), where $C = Y = I_3 = 0$ is understood. The first four of these basis vectors belong to the $U(4)$ regular representation associated, respectively, with the generators F_0, F_{15}, F_8 and F_e, and it is convenient to write $|0>$, $|15>$, $|8>$, $|3>$, $|20, 8, 1>$, $|20, 8, 3>$, respectively.

The SU (4) reduction coefficients for $6 \times 6^*$ give:

TABLE 1.

	$\|0>$	$\|15>$	$\|8>$	$\|3>$	$\|20,8,1>$	$\|20,8,3>$
$u\bar{u}$	$\frac{1}{\sqrt{6}}$	$-\frac{1}{\sqrt{6}}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{2}$
$d\bar{d}$	$\frac{1}{\sqrt{6}}$	$-\frac{1}{\sqrt{6}}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{2}$
$s\bar{s}$	$\frac{1}{\sqrt{6}}$	$-\frac{1}{\sqrt{6}}$	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	0
$c\bar{c}$	$\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	0
$t\bar{t}$	$\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{2}$
$b\bar{b}$	$\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{2}$

and consequently

$$| V > = \sum_{a} C_a^V | a >, \quad a \in \{0, 3,8,15, (20,8,1), (20,8,3)\} \tag{7}$$

with C_a^V given by:

TABLE 2.

	$\|0>$	$\|3>$	$\|8>$	$\|15>$	$\|20,8,1>$	$\|20,8,3>$
ϱ^0	0	$\frac{1}{\sqrt{2}}$	0	0	0	$-\frac{1}{\sqrt{2}}$
ω	$\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{6}}$	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{6}}$	0
φ	$\frac{1}{\sqrt{6}}$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{3}}$	0
ψ	$\frac{1}{\sqrt{6}}(\alpha + \sqrt{2}\beta)$	0	$\frac{1}{\sqrt{6}}(\sqrt{2}\alpha - \beta)$	$\frac{1}{\sqrt{6}}(\alpha + \sqrt{2}\beta)$	$\frac{1}{\sqrt{6}}(\sqrt{2}\alpha - \beta)$	0
T	$\frac{1}{\sqrt{6}}(\sqrt{2}\alpha - \beta)$	0	$-\frac{1}{\sqrt{6}}(\alpha+\sqrt{2}\beta)$	$\frac{1}{\sqrt{6}}(\sqrt{2}\alpha - \beta)$	$-\frac{1}{\sqrt{6}}(\alpha + \sqrt{2}\beta)$	0
T'	0	$\frac{1}{\sqrt{2}}$	0	0	0	$\frac{1}{\sqrt{2}}$

For the $U(4)$ regular representation basis vectors $| \text{reg}, \beta>$ one has

$$< 0 | j_\mu^a | \text{reg}, \beta; \lambda, \text{p} > = \delta_{a\beta}\, \varepsilon_\mu^{(\lambda)} (\text{p})\, \text{g} (\text{p}^2). \tag{8}$$

Also, since the reduction of 15 × 20 does not contain trivial representation, one has

$$< 0 | j_\mu^a | 20 > = 0. \tag{9}$$

This implies

$$< 0 | \varepsilon \cdot j^{em} | V > = C_V\, \text{g} (\text{p}^2), \tag{10}$$

with

$$C_V = C_3^V + \frac{1}{\sqrt{3}} C_8^V + 2 \sqrt{\frac{2}{3}} (C_{15}^V + C_0^V). \tag{11}$$

From (7), we find:

$$\sqrt{2}\, C_{\varrho^0} = \sqrt{2}\, C_{T'} = 1,$$

$$3\sqrt{2}\, C_\omega = - 3\sqrt{2}\, C_\varphi = 1,$$

$$\sqrt{18}\, C_{\psi} = 5\sqrt{2}\alpha + 7\beta,$$
$$\sqrt{18}\, C_{T} = 7\alpha - 5\sqrt{2}\beta. \tag{12}$$

In the CM system, $p^2 = m_V^2$, we have

$$\Gamma(V \to e^+e^-) = \frac{4\pi a^2}{3m_V} C_V^2 \frac{g(m_V^2)}{m_V^2}. \tag{13}$$

In order to relate $g(m_V^2)$ for different vector mesons we use $\varrho^0, \omega, \varphi, \psi, T, T'$ to saturate Weinberg's first spectral function sum rule[8] and this results in the independence of the ratio $g^2(m_V^2)/m_V^2$ from the vector meson mass. Formula (13) is then used to obtain an extended Das-Mathur-Okubo sum rule[9]:

$$\Gamma_{\varrho} m_{\varrho} = 3(\Gamma_{\omega} m_{\omega} + \Gamma_{\varphi} m_{\varphi}) = \frac{1}{11}(\Gamma_{\psi} m_{\psi} + \Gamma_T m_T) = \Gamma_{T'} m_{T'}, \tag{14}$$

and also the ratio

$$\frac{m_{\psi}\Gamma_{\psi}}{m_T \Gamma_T} = \left(\frac{5\sqrt{2}\alpha + 7\beta}{7\alpha - 5\sqrt{2}\beta}\right)^2. \tag{15}$$

Inserting experimental data for $m_{\varrho}, m_{\psi}, m_T, m_{T'}, \Gamma_{\varrho}$ and Γ_{ψ} into (14) one finds: $\Gamma(T \to e^+e^-) = 4.5$ keV and $\Gamma(T' \to e^+e^-) = 0.5$ keV. This makes the ratio of $m_{\psi}\Gamma_{\psi}$ to $m_T \Gamma_T$ very close to 1/3 and subsequent determination of α produces the following quark composition for ψ and T:

$$3\sqrt{3}\,\psi = 5\,c\bar{c} - (t\bar{t} + b\bar{b})$$
$$3\sqrt{6}\,T = 2\,c\bar{c} + 5\,(t\bar{t} + b\bar{b}) \tag{16}$$

giving the mix of $c\bar{c}$, $t\bar{t}$ and $b\bar{b}$ pairs in the ratio 5 : 1 : 1, in ψ, and the ratio 2:5:1, in T. The appearance in ψ, in this model, of a $t\bar{t} + b\bar{b}$ ingredient in addition to $c\bar{c}$, should not be too puzzling since t,b quarks each being heavier than the ψ particle, cannot make appearance in ψ decays. Also note that the conventional $c\bar{c}$ structure of ψ cannot saturate Weinberg's first sum rule, not even in the orthodox charm theory with four $SU(4)$ quarks and four vector mesons.

To check the consistency of (3) and (16) with the experimental values for vector meson masses we apply the naive quark model and obtain:

$$100\, m_C^2 + 4\, m_t^2 + 4\, m_b^2 = 27\, m_{\psi}^2,$$
$$8\, m_C^2 + 50\, m_t^2 + 50\, m_b^2 = 27\, m_T^2, \tag{17}$$
$$2\, m_t^2 + 2\, m_b^2 = m_{T'}^2.$$

System (17) is a dependent set of equations and produces the constraint:

$$2\, m_{T'}^2 - 25\, m_T^2 + 23\, m_{T''}^2 = 0. \tag{18}$$

This requires $m_{T''} \cong 9.9$ GeV and gives good consistency check of the proposed vector meson quark structure.

Introduction of experimental data for the vector meson masses gives the estimate $m_C = 0.8$ GeV, and the assumption $m_t \cong m_b$, based on the existence of isospin doublet containing t and b quarks, gives $m_t \cong m_b \cong 4.8$ GeV.

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T' (10040) KAO VEKTORSKI MEZON SA I = 1

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Originalni znanstveni rad

U radu se T' (10040) tretira kao izovektor sastavljen od teških kvark-antikvark parova, suprotno uobičajenom kvarkoniumskom tumačenju ove rezonance kao pobuđenog stanja sistema $b\bar{b}$. Razmatranje se zasniva na modelu šest kvarkova, klasificiranih po $SU(4)$ — 6 reprezentaciji, sa nabojima: 2/3, −1/3, −1/3, 5/3, 5/3, 2/3. Primjena proširene vektor-mezonske dominantnosti, uz eksperimentalne mase i širine raspada u leptonski par, dobro zasićuje prvo Weinbergovo sumaciono pravilo.