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SHELL STRUCTURE AND HIGH-SPIN STATES IN NUCLEI

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The interference of nuclear shell structure on rotational spectra of light nuclei is considered. $N = Z$ nuclei are chosen because the absence of pairing makes shell effects transparent even for relatively low-spin states. The rotational spectrum of ^{20}Ne is taken as an example and numerics evaluated.

1. Introduction

The nuclear structure and the dynamics of motion of light nuclei have received much attention. It is now an accepted fact that many of these nuclei are ellipsoidally deformed in their ground and (or) excited states (e. g. ^{12}C, ^{20}Ne or the 6.02 MeV excited state of ^{16}O). This has led to many investigations from the early Nilsson model[1)], an SU_3 classification scheme[2)] through deformed Hartree-Fock calculations[3)], the application of the generator co-ordinate method[4)], the α-cluster picture[5)], to the full-scale shell-model diagonalizations with realistic Kuo matrix elements[6)]. The complex shell-model calculations are able to reproduce experimental data, but the very complicated wave function does not allow a simple physical picture of the underlying dynamics. The other above mentioned approaches tacitly assume adiabaticity of the nuclear rotational motion. Because of the low-spin content of shells involved in the description of light nuclei, relatively low-spin states in these nuclei are to be considered as high-spin states. This implies that both the centrifugal and Coriolis forces affect the shape of the rotating nucleus.

Light nuclei may be considered in the framework of the simple model of rotated harmonic oscillator, because spin-orbit effects in these nuclei are negligible. Choosing $N = Z$ nuclei further simplifies the problem, for in these nuclei the pairing is absent. The cranked harmonic oscillator model can be exactly solved[8], whereas the solution given in Ref. 7 is only approximate.

The magnitude of shell effects[7] depends strongly on the frequencies characterizing the principal axis of an ellipsoid. The largest shell effects appear when the ratios of frequencies are ratios of small integers. Any departure from such ratios diminishes shell corrections.

2. *The model*

Assuming an independent particle motion, the total Hamiltonian H_0 can be written as a sum of single- particle Hamiltonians:

$$H_0 = \sum_{i=1}^{l} h(i). \tag{1}$$

For a harmonic oscillator

$$h = \frac{\vec{p}^{\,2}}{2m} + \frac{1}{2} m \left(\omega_1^2 x_1^2 + \omega_2^2 x_2^2 + \omega_3^2 x_3^2\right) \tag{2}$$

where ω_1, ω_2 and ω_3 define the shape. In the frame rotating with angular velocity ω around the intrinsic axis 1, the Hamiltonian obviously becomes

$$H_\omega = H_0 - \hbar\, \omega\, L_1 = \sum_{i=1}^{A} h_\omega(i) \tag{3}$$

where

$$h_\omega = h - \vec{\omega}\,(\vec{r} \times \vec{p})_1. \tag{4}$$

Expressed in the occupation-number representation with creation and annihilation operators $c^\dagger$ and c, satisfying the usual commutation relations

$$[c_i, c_j^\dagger] = \delta_{ij},$$

h_ω is

$$\begin{aligned} h_\omega = {} & \hbar\,\omega_1 \left(c_1^\dagger c_1 + \frac{1}{2}\right) + \hbar\,\omega_2 \left(c_2^\dagger c_2 + \frac{1}{2}\right) + \hbar\,\omega_3 \left(c_3^\dagger c_3 + \frac{1}{2}\right) \\ & - i\hbar \left[\omega \frac{\omega_2 + \omega_3}{2(\omega_2\,\omega_3)^{1/2}}\right] (c_2^\dagger c_3 - c_2 c_3^\dagger) \\ & - i\hbar \left[\omega \frac{\omega_3 - \omega_2}{2(\omega_2\,\omega_3)^{1/2}}\right] (c_2^\dagger c_3^\dagger - c_2 c_3). \end{aligned} \tag{5}$$

Equation (5) represents a system of coupled harmonic oscillators. The approximation made in Ref. 7 neglects the last term in expression (5), arguing its smallness due to the $N+2$ coupling (large energy denominators). Following Ref. 8, h_ω can be exactly diagonalized by means of the canonical transformation[9]

$$a_k = \sum_{m=2,3} (U^k_m c_m + V^k_m c^\dagger_m) \qquad (k = 2,3),$$

where the coefficients U and V are determined by

$$[a_k, h_\omega] = \hbar\, \Omega_k\, a_k,$$

yielding equations of the RPA type.

The Hamiltonian (5) becomes

$$h_\omega = \hbar\, \omega_1 \left(c^\dagger_1 c_1 + \frac{1}{2}\right) + \hbar\, \Omega_2 \left(a^\dagger_2 a_2 + \frac{1}{2}\right) + \hbar\, \Omega_3 \left(a^\dagger_3 a_3 + \frac{1}{2}\right),$$

with

$$\Omega^2_{2,3} = \frac{\omega_2^2 + \omega_3^2}{2} + \omega^2 \pm \frac{1}{2} \left[(\omega_2^2 - \omega_3^2)^2 + 8\omega^2 (\omega_2^2 + \omega_3^2)\right]^{1/2}. \qquad (6)$$

The eigenvalue of H_ω is

$$E_\omega = \hbar\, [\Sigma_1\, \omega_1 + \Sigma_2\, \Omega_2 + \Sigma_3\, \Omega_3],$$

where $\Sigma_i \equiv \sum_{k=1}^{A} \left(n_i + \frac{1}{2}\right)_k$ is the number of quanta of the filled oscillator orbits in the i direction.

For $\omega = 0$, $\Omega_{2,3}$ becomes $\omega_{2,3}$, as can be seen from expression (6).

All the required quantities can be immediately found provided the self-consistency condition is fulfilled. The usual Bohr-Mottelson[7] condition

$$\Sigma_1\, \omega_1 = \Sigma_2\, \omega_2 = \Sigma_3\, \omega_3$$

is equivalent to the requirement of isotropic velocity distribution, which in the rotated frame yields

$$\Sigma_1\, \omega_1 = \Sigma_2\, \Omega_2 = \Sigma_3\, \Omega_3. \qquad (7)$$

The conservation of volume requires that

$$\langle x_1^2 \rangle \langle x_2^2 \rangle \langle x_3^2 \rangle = \frac{\hbar^3}{m^3} \frac{\Sigma_1}{\omega_1} \frac{\Sigma_2}{\Omega_2} \frac{\Sigma_3}{\Omega_3} = \text{const.}$$

Clearly,

$$m\omega_i \langle x_i^2 \rangle = \frac{\partial E_\omega}{\partial \omega_i} = \left\langle \frac{\partial H_\omega}{\partial \omega_i} \right\rangle$$

and

$$-\hbar \langle L_1 \rangle = \frac{\partial E_\omega}{\partial \omega} = \left\langle \frac{\partial H_\omega}{\partial \omega} \right\rangle,$$

so that the acquired angular momentum is

$$\langle L_1 \rangle = \omega \left[\left(\frac{\Sigma_2}{\Omega_2} + \frac{\Sigma_3}{\Omega_3} \right) - \frac{2}{\Omega_2 \Omega_3} \left(\frac{\Sigma_2}{\Omega_3} + \frac{\Sigma_3}{\Omega_2} \right) \cdot \omega^2 \right] \tag{8}$$

and the energy of the system (i. e. the eigenvalue of $H = H_\omega + \hbar\omega L_1$) is

$$E = E_0 + \frac{\hbar}{2} \left(\frac{\Sigma_2}{\Omega_2} + \frac{\Sigma_3}{\Omega_3} \right) \omega^2 - \frac{3\hbar}{2} \left(\frac{\Sigma_2}{\Omega_3} + \frac{\Sigma_3}{\Omega_2} \right) \frac{\omega^4}{\Omega_2 \Omega_3}. \tag{9}$$

It is important to note that the self-consistency condition implies the constancy of Ω_2 and Ω_3, allowing the expressions for ω_2 and ω_3 as functions of ω. This means that the initial shape ($\omega = 0$) will change with acquired angular momentum (due to the action of the Coriolis force on the nucleonic motion), thus showing the non--adiabaticity of the problem. The maximum value of the acquired angular momentum for a given set of Σ_1, Σ_2 and Σ_3 is a consequence of the self-consistency requirement.

Shell structure represents non-uniformity in the distribution of stationary states. When Fourier-analyzed, the density of states can be represented to the lowest order[7)] as a cosine of energy with a period suitably chosen. Fig. 1 shows the degeneracy of the single-particle energies of the harmonic oscillator. It can be noted[7)] that whenever the ratio of frequencies (linked to the shape) is that of small integers, the shell structure becomes prominent. Any departure from the integer ratio (as illustrated in Fig. 1) diminishes the shell structure.

Therefore, the shell-structure energy correction can be parametrized as

$$\delta E_{\text{shell}} = -\hbar\omega_{\text{shell}} \left| \cos 2\pi \frac{\omega_1}{\omega_2} \cos 2\pi \frac{\omega_2}{\omega_3} \cos 2\pi \frac{\omega_1}{\omega_3} \right|, \tag{10}$$

where (see Ref. 7)

$$\hbar\omega_{\text{shell}} = \frac{\hbar\omega_0}{(\langle x_1^2 \rangle \langle x_2^2 \rangle \langle x_3^2 \rangle)^{1/6}}$$

and denotes intershell spacing. Thus the total energy including the shell correction for the model considered is

$$E = E_0 + \frac{\hbar}{2} \left(\frac{\Sigma_2}{\Omega_2} + \frac{\Sigma_3}{\Omega_3} \right) \omega^2 - \frac{3\hbar}{2} \left(\frac{\Sigma_2}{\Omega_3} + \frac{\Sigma_3}{\Omega_2} \right) \frac{\omega^4}{\Omega_2 \Omega_3} + \delta E_{\text{shell}}. \tag{11}$$

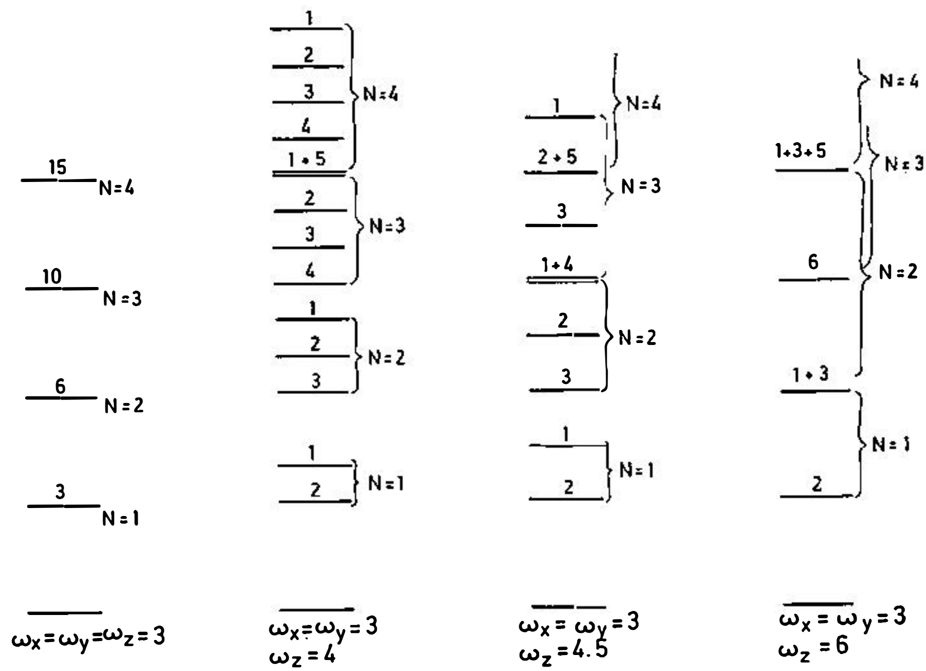

Fig. 1. The change in shell structure of a harmonic oscillator potential due to the change in frequency ω_z.

3. *Numerical results and conclusion*

The ^{20}Ne spectrum has been calculated to illustrate the change in shape (and consequently the difference in shell-structure corrections) as the nucleus is cranked with increasing angular velocity.

^{20}Ne is prolate in its ground state, so that the oscillator orbits (n_x, n_y, n_z) to be filled include (0, 0, 0), (0, 1, 0), (0, 0, 1), (1, 0, 0) and (0, 0, 2). It follows that

$$\Sigma_1 = \Sigma_2 = 14; \; \Sigma_3 = 22.$$

The mean oscillator frequency is chosen such as to fit the mean square radii of light nuclei:

$$\hbar\,(\omega_1\,\Omega_2\Omega_3)^{1/3} \equiv \hbar\omega_0 = 35.43\,A^{-1/3}\ \mathrm{MeV},$$

so that

$$\hbar\omega_1 = 15.18\ \mathrm{MeV} = \hbar\Omega_2$$

$$\hbar\Omega_3 = 9.66\ \mathrm{MeV}.$$

The band cut-off occurs at $\langle L_1 \rangle = \Sigma_3 - \Sigma_2 \doteq 8$ and the nucleus acquires an oblate shape, axially symmetric about the rotation axis 1.

Fig. 2 shows the energy of the rotated oscillator including shell corrections (11) (full circles) as a function of acquired angular momentum, and the energy of the cranked oscillator model without shell corrections (9) (open circles).

δE_{shell} (full line in Fig. 2) shows a strong oscillatory behaviour. It is seen that the largest shell-structure effects are present in the state with $\langle L_1 \rangle = 4\hbar$, for which $\omega_1 : \omega_2 \simeq 1 : 1$ and $\omega_1 : \omega_3 \simeq \omega_2 : \omega_3 \simeq 3 : 2$.

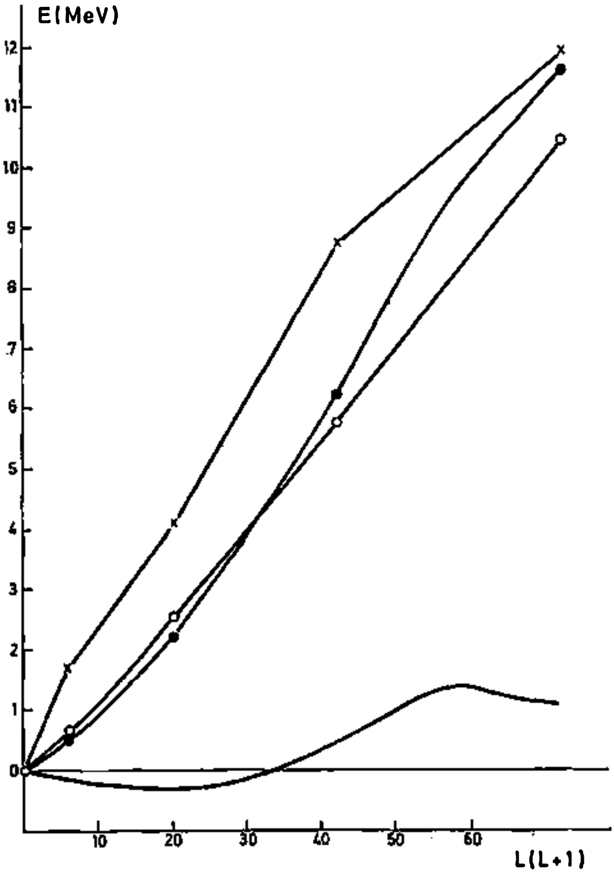

Fig. 2. The ground-state rotational spectrum of ^{20}Ne. The experimental spectrum (taken from Ref. 10) is marked with crosses for L = 2, 4, 6, $8\hbar$. The energy of the rotated oscillator model (see Eq. (9)) (open circles) and the energy including shell corrections (Eq. (11)) (full circles) is drawn. The full line represents the calculated δE_{shell}.

The experimental spectrum (taken from Ref. 10, marked with crosses in Fig. 2 of this paper), also shows oscillations around the straight $L(L+1)$ line and these closely follow the δE_{shell} pattern.

The discrepancy between the experimental and calculated spectrum is due to the simplicity of the model (e. g. as a consequence of the isotropic distribution of velocities (7), the moment of inertia is that of the rigid body).

Moreover, it should be pointed out that given the occupation of oscillator orbits and the adjustment of the mean oscillator frequency to the r. m. s. radius there are no parameters in the model. In conclusion, the importance of the shell correction in understanding the rotational motion with high frequencies is clearly demonstrated. In heavier nuclei, however, such a simple model is not expected to be adequate; namely, the shape of the potential is of the Woods-Saxon type rather than of the harmonic oscillator type and the *l-s* coupling plays important role. Even so effects of shell structure should be clearly seen only at very high angular velocities because of the presence of proton and neutron pairing in lower-spin states. This means that when the rotating nucleus becomes normal through the disappearance of superconductivity (the nuclear Meissner effect), it is still at sufficiently low temperature for a given angular momentum to exhibit shell-structure effects in the rotational motion.

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LJUSKASTA STRUKTURA I STANJA VISOKIH KUTNIH KOLIČINA GIBANJA JEZGRI

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Razmatra se utjecaj nuklearne ljuskaste strukture na rotacione spektre lakih jezgara. U jezgrama sa $N = Z$ efekti ljusaka su jasno uočljivi i za stanja relativno niskih spinova, zbog odsutnosti sparivanja. Za ilustraciju je izračunat rotacijski spektar ^{20}Ne.