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RESONANT INSTABILITY OF SEMI-INFINITE PLASMA WITH SHEARED ELECTRON STREAM

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We present a linear theory of surface wave instability due to electron stream shear near the plasma boundary. It is shown that in the case of a small shear the increment is exponentially small while for large shear it turns to be proportional to electron plasma frequency.

1. Introduction

Recently, resonant instability of bound plasma with inhomogeneous electron stream profile has been investigated by Vuković and Kyrie[1], where the streaming velocity was assumed fixed in direction parallel to the boundary $\mathbf{u} = u(z)\,\mathbf{e}_x$. It was shown that the surface wave, propagating with the wave vector $\mathbf{k}_{\parallel}$ along the plasma boundary, becomes unstable due to decay of the stream into the plasma wave and the surface wave

$$\mathbf{k}_{||}\,\mathbf{u}\,(z_r) = \omega_{pe} + \frac{\omega_{pe}}{\sqrt{2}}$$

in vicinity of the resonant point.

In the present paper we consider the monoenergetic electron stream propagating paralel to the plasma-vacuum interface (x, y plane) with the velocity

$$\mathbf{u}\,(z) = u\mathbf{e}\,(z); \quad |\mathbf{e}\,(z)| = 1. \tag{1}$$

Such streams frequently appear in experimental devices or in magnetospheric phenomena where inhomogeneous or shear magnetic fields are present. We shall, however, neglect the presence of external magnetic field assuming that electron gyro-frequency is much less than ω_{pe} and does not effect the considered high-frequency electrostatic modes.

Using the hydrodynamic approximation of cold collisionless plasma, we treat modes with frequency ω ($k_{||}\,c >> \omega$) assuming that all inhomogeneity characteristic scale lengths are much greater than electron Debye length. In this case electric field potential

$$\Phi\,(\mathbf{r}, t) = \Phi\,(z) \exp\,(-\mathrm{i}\omega t + \mathrm{i}\mathbf{k}_{||}\,\mathbf{r})$$

satisfies the following equation[2]:

$$\frac{\mathrm{d}}{\mathrm{d}z}\left|\varepsilon\,(z)\,\frac{\mathrm{d}\Phi}{\mathrm{d}z}\right| - k_{||}^2\,\varepsilon\,(z)\,\Phi = 0 \tag{2}$$

$$\varepsilon\,(z) = 1 - \frac{\omega_{pe}^2}{|\omega - k_{||}\,u\,(z)|^2}$$

where $\mathbf{u}\,(t)$ is defined by (1).

The absence of ion terms in (2) means, in fact, that we consider either the instability growth rates much greater than current-driven increments (Buneman 1959) or a current-free plasma.

The case of small velocity shear angles $\alpha\,(z)$ is particularly investigated

$$\cos\alpha\,(z) = \mathbf{e}\,(z = 0)\,\mathbf{e}\,(z) \tag{3}$$

where $\max_z \alpha\,(z) \ll 1$. The function $\alpha\,(z)$ monotonously changes from zero at $z = 0$ to α_0 at $z = \infty$. Moreover, it is supposed that:

$$\lim_{z\to 0} \frac{\alpha\,(z)}{z} \to 0; \quad \lim_{z\to\infty} z\,|\,\alpha\,(z) - \alpha_0\,| = 0.$$

2. *Streams with small shear*

At first we consider the case of small shear defined by condition:

$$\max_{z} \frac{\alpha(z)}{z} < \frac{\omega_{pe}}{u}. \tag{4}$$

The quantities

$$S_l = \frac{\alpha(z)}{z}; \quad S_d = \frac{\mathrm{d}\alpha}{\mathrm{d}z}$$

will be called the local and the differential shear, respectively.

Using the method described by Vuković and Kyrie[1] we solve Eq. (2) for small differential shear and obtain the following expressions for the frequency ω and the increment γ:

$$\omega = \frac{\omega_{pe}}{\sqrt{2}} + 2k_{||}^2 u \int\limits_0^\infty \mathrm{d}z \cos|\Theta - \alpha(z)|\, \mathrm{e}^{-2k_{||} z} \tag{5}$$

$$\gamma = \frac{\omega_{pe}}{\sqrt{2}} \mathrm{e}^{-2k_{||} z_0} \operatorname{sign}\left\{2k_{||} u \sin\left|\Theta - \frac{\alpha(z_0)}{2}\right| \sin\frac{\alpha(z_0)}{z} - \frac{\omega_{pe}}{\sqrt{2}}\right\}. \tag{6}$$

Here Θ is the angle between $\mathbf{k}_{||}$ and $\mathbf{e}(z = 0)$ and the point z_0 is defined by:

$$2k_{||} u \sin\left|\Theta - \frac{\alpha(z_0)}{2}\right| \sin\frac{\alpha(z_0)}{z} - \frac{\omega_{pe}}{\sqrt{2}} = \pm\, \omega_{pe}. \tag{7}$$

This relation coincides with the resonant condition up to small corrections connected with spatial dispersion of surface wave

$$\mathbf{k}_{||}\, \mathbf{u}(z_r) = \omega \pm \omega_{pe}. \tag{8}$$

It is evident from Eq. (6) that the instability occurs only for positive sign on the right side of Eqs. (7) and (8).

The physical nature of this instability arises from the anomalous Doppler effect (see e. g. Ref. 3). In the considered plasma configuration, surface wave carries negative energy and becomes more unstable while exciting the local plasma oscillations. Negative sign in Eqs. (7) and (8) corresponds to the normal Doppler effect, and instead of instability, damping occurs.

From Eqs. (6) and (7), using the condition $\alpha(z) \ll 1$ in the instability region $\gamma > 0$, it follows:

$$\gamma = \frac{\omega_{pe}}{2\sqrt{2}} \mathrm{e}^{-2k_{||} z_0}, \tag{9}$$

$$k_{\|} \left| a(z_0) \sin \Theta - \frac{a^2(z)}{2} \cos \Theta \right| = \varkappa, \tag{10}$$

and

$$\varkappa \equiv \frac{1}{u} \left(\omega_{pe} + \frac{\omega_{pe}}{\sqrt{2}} \right). \tag{11}$$

It can be seen from Eqs. (9) and (10), that the largest increment, for given $k_{\|}$, is obtained for $\sin \Theta = \sin a(z_0)$ i. e. when the surface wave propagation is perpendicular the velocity direction at the boundary **e** (0). The increment (9) is determined by local shear S_l because

$$k_{\|} z_0 = \frac{\varkappa}{S_l(z_0)}. \tag{12}$$

Also, from Eq. (10), it is not difficult to see that the largest increment corresponds to the largest value of the local shear, i. e. for $z_0 = z_m$, where the local shear is equal to the differential one:

$$\begin{aligned} S_l(z_m) &= S_d(z_m) = S, \\ \frac{a(z_m)}{z_m} &= \left(\frac{\mathrm{d}a}{\mathrm{d}z}\right)_{z = z_m}. \end{aligned} \tag{13}$$

The corresponding value of the wave number $k_{\|, m}$, is determined as:

$$k_{\|, m} = \frac{\varkappa}{a(z_m)}. \tag{14}$$

In the vicinity of the increment maximum:

$$\frac{z - z_m}{z_m} = - \frac{k_{\|} - k_{\|, m}}{k_{\|, m}}$$

from where it follows:

$$\gamma \approx \frac{\omega_{pe}}{\sqrt{2}} \exp \left\{ - \frac{\varkappa}{S \sin \Theta} \left| 1 - \frac{z_m^2}{2a(z_m)} \left(\frac{\mathrm{d}^2 a}{\mathrm{d}z^2}\right)_{z=z_m} \left(\frac{k_{\|} - k_{\|, m}}{k_{\|, m}}\right)^2 \right| \right\}. \tag{15}$$

It is clear that $\left(\frac{\mathrm{d}^2 a}{\mathrm{d}z^2}\right)_{z=z_m} < 0$ because γ is largest for $k_{\|} = k_{\|, m}$. This means that the point z_m is further away from the boundary than the saddle-point z_n of the function $a(z)$. It can be seen from (15) that the condition $\varkappa > S$ coincides with $|\omega - ku\cos\Theta| \gg \gamma$ which was used in calculations of (5) and (6). Note, that electrostatic condition $k_{||} c \gg \omega$ corresponds to:

$$a(z_m)\, u \ll c$$

which is always fullfiled for $a \ll 1$.

If $a(z)$ is a sufficiently smooth function, we see from (15) that γ is exponentially small compared to γ_{max} already for $|(k_{||} - k_{||,m})/k_{||,m}| < 1$. For $k_{||} = k_{||,min}$ defined by

$$k_{||,min} = \frac{\varkappa}{a_\Theta}$$

γ vanishes. The resonant condition (10) cannot be fullfiled for $k_{||} < k_{||,min}$, while there are no restrictions when $k_{||} > k_{||,m}$.

If surface wave is damped by the usual collision mechanism and/or the linear wave transformation (due to inhomogeneity of plasma density near the boundary), the instability growth rate can be written in the following form:

$$\gamma = \frac{\omega_{pe}}{\sqrt{2}} e^{-2k_{||} z_0} - \tilde{\gamma} \tag{16}$$

where

$$\tilde{\gamma}(k_{||}) = \frac{\nu_e}{2} + \frac{\omega_{pe}}{4\sqrt{2}} \pi k_{||} \int_0^a dz\, \delta \left[\varepsilon\left(z, \frac{\omega_{pe}}{\sqrt{2}}\right)\right],$$

where a — is the width of the inhomogeneous boundary layer. From (16) we obtain the shear threshold for the instability

$$S_{th} = \frac{2\varkappa}{\ln\left(\frac{\omega_{pe}}{2\sqrt{2}\,\tilde{\gamma}(k_{||})}\right)}.$$

3. *Streams with large shear*

We turn now to the case of large shear:

$$S > \varkappa \tag{17}$$

and consider surface waves with the wavelengths much larger than the transition layer width a ($k_{||} a \ll 1$). The equation (2) near the boundary is now solved by successive approximations using the small parameter $k_{||} a$. This solution can be matched with $\Phi(z)$ outside plasma, as well as, with the asymptotic solution in the region

$$\left|\frac{d\, ln\, \varepsilon}{d\, k_{||} z}\right| \ll 1.$$

This procedure gives the following expressions for the frequency and the increment of surface wave:

$$\omega = \frac{\omega_{pe}}{\sqrt{2}} \left| 1 + \frac{k_{||}}{4} \int_0^\infty \mathrm{d}z \frac{1 - \varepsilon^2 (z, \omega)}{\varepsilon (z, \omega)} \right| - ku \cos \Theta \tag{18}$$

$$\gamma = \frac{\pi \sqrt{2}}{8 \left(1 + \sqrt{2}\right)} \; \frac{\varkappa}{|\sin \Theta| \; |S_d (z_0)|} . \tag{19}$$

As in § 2 the point z_0 is determined by (6), (7) keeping the »+« sign. The condition $\gamma \ll | \omega\text{-}ku \cos \Theta |$ was taken into account in calculation of (19). Here Θ is the angle between $k_{||}$ and $\mathbf{e}(\infty)$. The conditions $k_{||} z_0 \ll 1$ and $\gamma \ll | \omega\text{-}ku \cos \Theta |$ can be written in the form:

$$|\sin \Theta | \gg \frac{\varkappa}{S_t (z_0)}$$

$$|\sin \Theta | \gg \frac{\varkappa}{S_d (z_0)} .$$

At the point of largest local shear z_m defined by (12) these inequalities coincide i. e.

$$|\sin \Theta | \gg \frac{\varkappa}{S} .$$

Therefore, in the limit of the applicability of the theory the increment differs from ω_{pe} only by numerical coefficient and much exceeds the corresponding value for current instability which is proportional to $(\omega_{pi}^2 \; \omega_{pe})^{1/3}$.

4. *Conclusion*

The resonant instability investigated in the present paper can excite surface waves with large growth rates for sufficiently large stream shear. Development of such instability brings about the electron energy transformation into the surface waves and shear decreasing in vicinity of the point z_m. However, the complete image about the considered phenomena could be obtained only by non-linear study of excited wave interaction with inhomogeneous plasma stream shear. Finally, we would like to mention that throughout this paper we have not considered the hydrodynamic Cherenkov instability, recently explained by Andreev et al.[4], which can also contribute to the considered growth rates.

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REZONANTNA NESTABILNOST POLUOGRANIČENE PLAZME SA ŠIROM ELEKTRONSKOG TOKA

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