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Reliability Analysis and the Costs of Corrective Maintenance for a Component of a Fleet of Trams

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Abstract: This paper presents the results of reliability analysis investigations and the costs of corrective maintenance of the most frequently damaged and one of the most expensive to repair tram components. Having a database of failures times for individual components, it is easy to determine the average lifetime to failure the element and the standard deviation for this time, the problem arises when selecting the appropriate form of distribution. An incorrect determination of the type of failure distribution may cause a large error in the results of the reliability and durability assessment of the system. The subject of the investigation was the master controller in trams, which is a multiposition switch designed for transitioning electric motor windings or auxiliary elements and is responsible for the acceleration and braking of a tram. The research covered 5 years of operation of a fleet of 45 trams used in similar conditions in one city, with average mileage of 200 km per day for every tram. Furthermore, the study analysed the costs related to corrective maintenance related to this component. The results allow researchers and the fleet operator to predict possible failures of the components under review and calculate the associated financial costs. This comprehensive approach provides valuable information on the economic and operational aspects of tram systems in public transportation.

Keywords: corrective maintenance; optimal cost; reliability analysis

1 INTRODUCTION

In the reliability analysis of a technical system, various probability distributions are used to model failure data [1-3]. Common choices include normal, exponential, Weibull, and Gamma distributions. However, the research discussed in this article also considers other distributions that are not typically used but may offer better quality [4-6]. These include the log-normal distribution, the generalised gamma distribution, the logistic distribution, the log-logistic distribution, and the Gumbel distribution. The parameters of these distributions can be determined using analytical or numerical methods with specific computer programmes [7, 8]. Determining the parameters of a distribution involves data modelling to identify the most suitable distribution and calculate its parameters (shape, scale, position) using various estimation methods, both numerical and graphical [9, 10]. When evaluating different distributions, it is possible to find the one that best matches the empirical data, indicated by the smallest sum of squared deviations [11]. Common methods include the method of moments, the maximum likelihood method, the least squares method, the fitting in distribution grids, and the probability plot correlation coefficient (PPCC) method, among others [12].

The primary objective of the study discussed in the article was to create a way to evaluate the reliability and costs of corrective maintenance for the master controller. Understanding the most appropriate probability distributions for these components can improve transport safety, inform efficient maintenance schedules, and reduce maintenance costs [13-15]. We distinguish between scheduled maintenance, which includes planned inspections, and unscheduled maintenance, resulting from unexpected breakdowns, the latter often leading to more costly repairs [16-18]. This paper focusses primarily on unscheduled maintenance, supported by our reliability analysis. To identify the most accurately fitting distribution, the analysis used three measures. Akaike Information Criterion (*AIC*), Bayesian Information Criterion (*BIC*) and statistics derived from the log-likelihood function (LL) [20]. This approach also accounts for instances where an item remains operational at the end of the study, known as right-censored data [21, 22]. The research incorporated data censored to the right, referring to units that did not fail during the examination period. These units are considered 'still operational' with an expected future failure time. The technique to organise statistical data based on operational information was developed in previous research [23].

Before analysing how well different distributions fit, the empirical distribution's function or its reliability function is first determined using the Kaplan-Meier method. The parameters of the potential distributions are then estimated using the Maximum Likelihood Estimation (MLE) method [24, 25]. The likelihood function, especially when applied to suspended data, showcases the benefits of maximum likelihood estimation (MLE) over alternative parameter estimation methods. A primary advantage of MLE is its consideration of suspension times' actual values. Once these hypothetical distributions have been estimated, compliance statistics are calculated, which show how closely each hypothetical distribution matches the empirical distribution [26]. The distributions are then ranked on the basis of this analysis. Provided certain prerequisites are met, these rankings are performed independently according to the three criteria mentioned above [23]. A detailed description of this methodology for testing distribution compliance is provided in another part of the document. The remainder of the article is organised as follows. Section 2 discusses the design, usage, and reasons for damage to the master controller, along with the characteristics of the study group. Section 3 identifies the probability distribution that best fits the failure data. In Section 4, the main findings are presented, including interpretations of the data on the master controller.

2 SUBJECT OF THE STUDY AND RESEARCH METHODOLOGY

2.1 Subject of the Study

The subject of the study is the master controller located in the car seat of the tram driver. This is a modern thyristor-based traction control system that, compared to older mechanical-electrical designs, significantly improves passenger comfort and reduces energy consumption (see Fig. 1a, Fig. 1b.

(a) (b)

Figure 1 Master controller; a) main view, b) location in the driver's cabin

The device enables the tram driver to change the resistance value in the series circuit with the motor, and thereby change the voltage supplied to the motors. This allows for full control over the motor's rotational speed. In such designs, a manual drive selector was introduced for the first time. It combines a dead man's switch, brake, and throttle, which, together with a series of the latest electronic solutions, provides the driver with incomparably greater comfort in operating the vehicle and passengers with the convenience of travel. The contacts within the controller wore out, causing a lack of response at the set position of the lever. The cause was a lack of connections between the contacts that alter the position of the drive selector. Failed drive selectors were replaced with new ones; however, the cost of replacing the entire selector is quite high. In several cases, it was possible to implement preventive actions such as replacing only the contact connection system, which was prone to damage, during routine inspections, while the remaining electronic components were left unchanged. This allowed for the continued safe operation of vehicles and a significant reduction in repair costs [27, 41]. Therefore, the unreliability analysis focused on the components of the tram that are most susceptible to failure and have significant repair costs. The primary objective of the study discussed in the article was to develop a method for evaluating the reliability and costs of corrective maintenance for the master controller. The examination involved a group of 45 trams of the same model, operating under similar conditions in the same city. All trams were operated by employees of the same company, ensuring consistent usage conditions with an approximate daily mileage of 200 km.

The study was based on data collected during the first five years of service, which included two years under warranty and three years under a maintenance contract. The master controller was identified as one of the most frequently damaged and among the most costly parts to repair. Given its crucial role in ensuring passenger transport safety and its high failure rate, the reliability of this component was a key focus of our investigation.

2.2 Data Acquisition

As an example, the study of long-life data over 5 years of operation for a fleet of homogenous trams $|J| = 45$, includes the exact failure time (in kilometres) and suspension time for the master controller, as shown in Tab. 1. Suspension time refers to right-censored data, which did not fail by the end of the test. During the time t_1 covered by the investigation, $n(t_1) = 25$ corrective maintenance actions were registered due to the failure of the master controller. Each failed item was replaced with a new one [28, 29]. It is important to note that due to the continuous operation of the vehicle fleet, the mileage l_i the i-th item may refer to a failed item that has been replaced or an item that is still functioning in the vehicle.

Table 1 Master controller failure time of master controller (1 - failure, 0 - censored)

Mileage / km	δ_i	Mileage / km	δ_i	Mileage / km	δ_i	Mileage / km	δ_i
4 799	0	122 934	1	239 666	0	326 816	0
20 106	0	123 494	0	244 106	1	326 904	0
24 143	1	128 339	0	245 495	0	327 707	0
26 807	1	138 921	0	258 866	1	328 106	0
35 855	0	142 315	1	262 359	1	331 688	0
39 927	0	143 058	1	262 694	0	333 298	0
40 274	0	148 551	0	265 598	1	334 596	1
41 638	0	148 909	0	282 281	1	344 645	0
47 422	0	152 554	1	283 162	1	347 089	0
48 858	1	154 706	0	295 784	0	348 917	0
71 490	1	156 048	1	299 457	0	350 587	0
77 576	1	192 863	1	304 031	0	359 661	0
81 652	0	192 935	1	305 563	0	359 877	0
83 077	0	216 144	1	306 514	0	361 756	0
87 310	0	219 815	1	312 813	0	368 581	0
94 706	0	227 441	1	314 151	0	374 672	0
95 854	0	235 029	1	323 755	0	-	-
108 027	1	235 268	1	326 742	0	-	-

At a fixed time t_1 during the fleet's operation, only operational items remain in the vehicles. For both operational and failed items, their mileage is determined based on the mileage of the vehicle in which they are or were installed. For the reliability tests conducted, data on the mileage of the master controller for all fleet vehicles up to the day t_1 were compiled in the form of pairs (l_i, δ_i), $i = 1, 2, ..., n(t_1) + |J|$, where l_i is expressed in kilometers of operational mileage of the i-th instance of the drive controller, and $n(t_1) + |J| = 70$ is the number of data records about the controller and:

$$\delta_i = \begin{cases} 1, \text{ if } l_i \text{ is the observed operational mileage,} \\ 0, \text{ if } l_i \text{ is the censored operational mileage.} \end{cases} \tag{1}$$

For the entire vehicle fleet, 70 pairs (l_i, δ_i) were obtained, and in 45 cases the drive controller was functional on the day t_1 of the interruption of observation.

2.3 Reliability Analysis Methodology

The computational software used offers directions to choose a distribution, taking into account statistical tests. These tests were executed separately for the investigated element. The research used Python programming language and Reliasoft software to guide the selection of a distribution based on statistical tests [30, 31]. In the first step, the Python programming language and the Reliability library were used to calculate the data censored by the right, that is, failures and suspensions. The software employs three methods for model selection in the context of regression to rank distributions, the first factor is *AIC* which was used to help choose the best model from a set of models [32, 33].

1. Based on information theory, *AIC* measures both the goodness of fit of the model to the data and the complexity of the model (the number of estimated parameters). The *AIC* value for a given model is calculated using the formula:

$$AIC = 2k - 2\ln(L) \tag{2}$$

Where k - the number of parameters in the model, L - the maximum value of the likelihood function for the model.

Lower *AIC* values indicate a model that fits well with the data while maintaining simplicity (fewer parameters). In practice, the model with the lowest *AIC* among a set of candidates is chosen.

However, *AIC* does not provide a direct hypothesis test (like a p-value) and should not be used to "prove" a model in a statistical sense. Instead, it is a tool for comparing different models and selecting the one that best balances complexity and fit to the data.

2. The second factor is the Bayesian Information Criterion (*BIC*), which is based on the likelihood function and is closely related to the *AIC*. However, *BIC* introduces a stronger penalty for the number of parameters in the model, making it more stringent against models with more parameters, especially as the sample size increases [34, 35]. The formula for *BIC* is the following.

$$BIC = \ln(n)k - 2\ln(L) \tag{3}$$

Where n - the number of data points (sample size), k - the number of parameters in the model, L - the maximum value of the likelihood function for the model.

Like *AIC*, with a lower *BIC* value indicating a better balance between model fit and complexity. The key difference from *AIC* is the inclusion of the sample size in the penalty term. This means that *BIC* tends to favour simpler models than *AIC* as the sample size grows. *BIC* is particularly useful in Bayesian model selection, where it approximates the posterior probability of a model being true under certain conditions. It is widely used in statistical inference for model selection purposes. However, similar to *AIC*, *BIC* does not provide a direct hypothesis test and is best used to compare the relative merits of different models.

3. The third factor is the logarithmic likelihood function value (*LL*) which calculates the value of the logarithmic likelihood function, given the parameters of the distribution [35]. The likelihood function, L, depends on continuous random variables $T_1, T_2, \dots, T_n, S_1, S_2, \dots, S_m$ and unknown parameters $\theta_1, \theta_2, \dots, \theta_k$ that need estimation. The logarithmic likelihood function, is often used, as it is mathematically more tractable. The maximum likelihood estimators of the parameters are derived by maximising L or Λ [36].

Because in this case, the likelihood function needs to be expanded to take into account the suspended master controller, the likelihood function is given by:

$$\begin{aligned} & L(\theta_1, \theta_2, ..., \theta_k | T_1, T_2, ..., T_n, S_1, S_2, ..., S_m) = \\ & \prod_{j=1}^{n} \left[1 - F(S_1; \theta_1, \theta_2, ..., \theta_k)\right] \end{aligned} \tag{4}$$

Where L - the likelihood function, n - observed failures at $T_1, T_2, \dots, T_n$, m - number of suspended data points at $S_1, S_2, \dots, S_m$, k - the number of estimated parameters, T_i - failure time of the i-th component, S_j - suspension of the j-th component, $\theta_1, \theta_2, \dots, \theta_k$, k - unknown parameters that need to be estimated, $f(T_i; \theta_1, \theta_2, \dots, \theta_k)$ probability density function *pdf*, $F(S_j; \theta_1, \theta_2, \dots, \theta_k)$ cumulative density function *cdf*.

In simpler terms, to make calculations and analysis easier, we often use a mathematical trick where we take the logarithm of the function [37, 38]. This process transforms it into what is called a log-likelihood function, symbolised as Λ. For data that are right censored, meaning we know when observations begin but not when they end, this log-likelihood function has a special format that is more convenient to work with [39]:

$$\begin{aligned} & L\Lambda = \ln L = \sum_{i=1}^{n} \ln f(T_i; \theta_1, \theta_2, ..., \theta_k) + \\ & + \sum_{j=1}^{m} ln\left[1 - F(S_1; \theta_1, \theta_2, ..., \theta_k)\right] \end{aligned} \tag{5}$$

In the mentioned equation, there are two parts: the first part adds up information from complete data, and the second part does the same for right-censored data, which are data where we don't know the ending point. To find the best values for the unknown parameters $\theta_1, \theta_2, \dots, \theta_k$ we aim to find the highest possible value of either L or Λ. To figure out these best values, known as Maximum

Likelihood Estimators (MLE), we use a process involving partial derivatives. This means we slightly tweak the log-likelihood function Λ by changing each parameter θ_e, $e = 1, 2, \ldots, k$, one at a time, while keeping the others constant. It was done for each parameter and we found the point where this derivative equals zero:

$$\frac{\partial \Lambda}{\partial \theta_e} = 0,\ e = 1,\ 2,\ ...,\ k \tag{6}$$

This helps define the exact values of the parameters that best fit our data.

As mentioned above, we used the Maximum Likelihood Estimation (MLE) method to determine the parameters for the chosen lifetime distributions [31, 40]. The most challenging aspect of MLE is the optimisation process, which we will briefly discuss in the next section. Here, we will explore how to perform optimisation in Python using the scipy.optimize.minimize library [41]. This library requires four key inputs:

a) an objective function that should be minimised,
b) an initial guess,
c) the optimisation routine to use,
d) bounds on the solution.

For this study, the truncated Newton Algorithm (TNC) optimiser was employed. The TNC method uses a truncated Newton algorithm, referenced in sources [42, 43], designed to minimise a function with bounded variables. Newton's method aims to find the minimum (or maximum) of a function by iteratively moving towards the zero of its derivative (gradient). The method assumes that the function can be locally approximated as a quadratic near the optimum, which is why the second derivative (Hessian) is included in the formulation. This algorithm utilizes gradient information and is also known as the truncated Newton conjugate gradient. TNC is particularly useful for multidimensional functions, where finding direct minima is challenging or inefficient with traditional methods.

It is crucial to understand that in maximum likelihood estimation (MLE), the goal of the optimiser is to adjust the model parameters to maximise the log-likelihood. However, there are instances where the optimizer might not succeed or might yield suboptimal results [48]. In such scenarios, it is advisable to either switch to a different optimizer or instruct the reliability package to test various optimizers by setting optimizer='best'.

The operating principles of the TNC algorithm, also known as Hessian-free optimisation, are a family of optimisation algorithms designed for optimising nonlinear functions with large numbers of independent variables [45]. The process is as follows: A starting point x_0 is selected, along with initial parameters, such as the maximum number of iterations. In each iteration, the gradient of the objective function $\nabla f(x)$ at the current point is calculated, indicating the direction of the fastest increase of the function. To minimise the objective function, the algorithm moves in the opposite direction of the gradient, utilising information from the second derivative (Hessian). In TNC, Hessian approximations are used instead of the full Hessian to reduce the computational load, making the algorithm more efficient, especially for large-scale problems. Points are updated using the calculated direction and an appropriately chosen step size to move closer to the minimum, with steps adjusted to accommodate any existing constraints. The algorithm continues iterations until a termination condition is met, such as a minimal change in the objective function value, a minimal change in variable values, reaching the maximum number of iterations, or achieving a sufficiently small norm of the gradient. The method is used in the following context: $x = (x_1, \ldots, x_n)$ is a vector of unknown variables, d is a vector considered to be well known or where uncertainty is negligible, and $y = h(x, d)$ is the scalar variable of interest [46]. The objective here is to determine the extreme values (minimum and maximum) of y when x varies. The TNC algorithm addresses the issue by solving the following:

$$\min_{x \in [a,\, b],\, b \in \mathbb{R}^n} f(x) \tag{7}$$

Where x - represents a vector of unknown variables in an optimisation problem, h - represents the objective function that depends on the vector of variables x and another vector d, y - scalar variable that is the output of a function $h(x, d)$, d - the vector represents known or fixed parameters within the function h, a, b - bounds within which the vector x is constrained, $\mathbb{R}^n$ - the n - dimensional real vector space.

And continues as per the appropriate regularity conditions of the objective function f:

$$\begin{cases} \nabla f(x) = 0 \\ \nabla^2 f(x) \text{is definite positive} \end{cases} \tag{8}$$

Where $\nabla f(x)$ - gradient of the function f with respect to x, $\nabla^2 f(x)$ - the Hessian matrix of the function f.

The Hessian matrix at x_k, represents the second derivatives of the function. Provides information on the curvature of the function's graph [43]. The Taylor development of the second order of f around x_k leads to the determination of the iterate x_{k+1} such as:

$$\begin{cases} \Delta_k = x_{k+1} - x_k \\ \nabla^2 f(x_k)\Delta_k = -\nabla f(x_k) \end{cases} \tag{9}$$

Where Δk - the update step in the k-th iteration, x_k - the approximation of the solution at the k-th iteration, $-\nabla f(x_k)$ - the negative gradient at x_k, Δ_k represents the step change in the variable x at iteration k. This step is calculated to move the current estimate x_k closer to the function's minimum [44]. The goal of each step is to find a point where the gradient (first derivative) of the function is zero, because this condition is necessary for a local minimum (or maximum). The equation $\nabla^2 f(x_k)\Delta_k = -\nabla f(x_k)$ is used to calculate the step Δ_k that is expected to lead directly towards a local minimum by linearly approximating the behaviour of the function based on its current gradient and curvature [47]. The negative gradient $-\nabla f\left(x_k\right)$ at x_k, indicating the direction in which the function decreases most rapidly.

Eq. (1) is truncated: The Eq. (1) is truncated: the iterative research of Δ_k is stopped as soon as Δ_k verifies:

$$\left\|\nabla^2 f\left(x_k\right)\Delta_k+\nabla f\left(x_k\right)\leq\|\eta\|\nabla f\left(x_k\right)\right\| \tag{10}$$

Where η - a constant or variable that controls the precision of the stopping condition for the iterative process.

Finally, the iteration is defined by the following:

$$x_{k+1}-x_k+\delta_k\Delta_k \tag{11}$$

Where δ_k the step size in the k-th iteration.

In the next step, the software first evaluates and arranges the selected distributions based on the degree to which they match the input data. Then it organises these distributions in order of their fitting with the input data. To complete this classification process, it combines three different tests, assigning a specific importance (or weights) to each. In Tab. 2, the second column shows the results of the Akaike Information Criterion (*AIC*) method [30]. The next column shows results from the Bayesian Information Criterion (*BIC*). The likelihood values (*LL*) are presented in the fourth column. The fifth column contains the given values. These weighted values are then combined to form a single *WDV* (weighted decision variable) value, as described in equation:

$$\begin{aligned}WDV&=\left(LL\ \text{Rank}\times40\%\right)+\left(AIC\ \text{Rank}\times30\%\right)+\\&+\left(BIC\ \text{Rank}\times30\%\right)\end{aligned} \tag{12}$$

The weights assigned to each statistical method are derived from engineering practices, which consider the strengths and weaknesses of each method.

In the study, it was demonstrated that the two-parameter Weibull distribution provided the optimal fit for the data across both subsystems. Consequently, the subsequent discussions and equations presented in the paper were based on this distribution. The two-parameter Weibull distribution is characterised by a density function [48]:

$$f\left(t;\beta,\alpha\right)=\frac{\beta}{\alpha}\cdot\left(\frac{t}{\alpha}\right)^{\beta-1}\cdot e^{-\left(\frac{t}{\alpha}\right)^{\beta}},\ t\geq0,\beta>0,\ \alpha>0 \tag{13}$$

Where β - shape parameter, α - scale parameter.

The subsequent parameter determined in the analysis is the Mean Residual Life *MRL* (Mean Residual Life), which applies for any t where $R(t)>0$ $R(t)>0$. This can be straightforwardly expressed using the reliability function as follows: [49]:

$$MRL\left(t\right)=\frac{1}{R\left(t\right)}\int_0^{\infty}R\left(t+l\right)\mathrm{d}l=\frac{1}{R\left(t\right)}\int_0^{\infty}R\left(l\right)\mathrm{d}lt\geq0 \tag{14}$$

Where $MRL(t)$ - the mean residual life of the object at time t, $R(t)$ - the reliability function, l - is the additional lifetime of the object beyond time t, $\int_0^{\infty}R\left(l\right)\mathrm{d}l\geq0$ t, - the integral from t to infinity of the reliability function $R(l)$ with respect to l. When $R(0)=1$ and $t=0$, the *MRL* is equivalent to the mean lifetime, specifically, that is $MRL(0)=MTTF$ (Mean Time to Failure). If a component has survived up to time t, then the mean residual life at time t, denoted as $MRL(t)$, indicates the expected remaining lifetime until failure for that component. Regarding the two parameters of the Weibull distribution, it holds that [9, 28]:

$$MRL\left(t\right)=\alpha\cdot e^{\tau}\cdot\Gamma\left(1+\frac{1}{\beta}\right)\cdot\left(1-\frac{\Gamma_{\tau}\left(\frac{1}{\beta}\right)}{\Gamma\left(\frac{1}{\beta}\right)}\right),\ \tau=\left(\frac{t}{\alpha}\right)^{\beta} \tag{15}$$

Where α - the scale parameter, β - the shape parameter, e - the base of the natural logarithm, t – the time or distance for which we compute *MRL*, Γ - the gamma function, $\Gamma_\tau(\mathrm{r})$ - the incomplete gamma function defined as $\Gamma_{\tau}\left(r\right)=\int_0^{\tau}t^{r-1}e^{-t}\mathrm{d}t$.

The Mean Time to Failures (*MTTF*), in our case the Mean Distance Between Failures (*MDBF*), is a statistical measure that indicates the average time or distance in which units in a group are likely to function without failing. Essentially, it quantifies the average period or distance of dependable operation until a failure occurs [39].

The Mean Time (Distatnce) to Failure (*MDTF*) describes the expected time to failure for a nonrepairable system and for the 2P-Weibull distribution can be calculated as follows:

$$MDTF=\int_0^{\infty}R\left(t\right)\mathrm{d}t=\int_0^{\infty}e^{-\left(\frac{t}{\alpha}\right)\mathrm{d}t} \tag{16}$$

Where $R(t)$ - the reliability function, α - the scale parameter, β - the shape parameter, e - the base of the natural logarithm. After applying appropriate variable substitutions and algebraic transformations, we obtain the following form:

$$MDTF=\alpha\cdot\Gamma\left(1+\frac{1}{\beta}\right) \tag{17}$$

Where Γ - the gamma function. A reliable life, often referred to as warranty time, is the projected period until reliability reaches a predetermined target. The reliable life of a unit, denoted as (T_R) for a specified reliability R, starting from age zero, can be determined using the following formula [49]:

$$T_R=\alpha\left[-\ln\left(R\right)\right]^{\frac{1}{\beta}} \tag{18}$$

Where T_R is the reliable life or warranty time, α is a scale parameter, β is a shape parameter, R is the specified reliability goal, $-\ln\left(R\right)$ represents the natural logarithm of the specified reliability. This formula is commonly applied in reliability analysis to calculate the point at which the reliability of a unit will meet a designated level (R), beginning from age zero.

2.4 Optimum Replacement of Preventive Maintenance

Reliability is crucial in engineering, deeply influencing decisions based on the total cost of the life cycle, which includes significant expenses for operation, maintenance, and repairs. Costs related to the purchase and storage of spare parts, labour hours, and financial losses due to system failures. In some situations, it may be more economically advantageous to replace a component proactively (i.e. before it fails) rather than reactively (i.e. after it fails). For any specific component, the initial step is to decide whether a preventive replacement is suitable; if so, the subsequent step is to determine the optimal timing for the replacement [30]. Preventive replacement of a component is appropriate only if the vulnerability of the component to failure increases with time and if its preventive (planned) replacement cost (C_P) is significantly less than its corrective (unplanned) replacement cost (C_f). When considering the timing of preventive maintenance in the vehicle fleet, it is crucial to weigh the economic benefits of replacing the component before failure versus after. Preventive replacement can be economically more viable than corrective replacement under specific conditions. There are two key factors to consider for it to be beneficial [50]:

1. The overall cost of replacement after a failure must exceed the cost prior to the failure.
2. The failure rate of the master controller should be on an upward trend.

To determine the optimum time for preventive actions, it is necessary to develop a mathematical model that describes the associated costs and risks. The model assumes that if the unit fails before a certain time t, corrective maintenance will occur. In other words, if the unit does not fail at time t, preventive maintenance will be performed. Essentially, the unit is replaced either upon failure or after operating for a designated period, whichever comes first. Therefore, quantitative analysis and model construction are necessary to determine the most appropriate time for preventive maintenance to avoid potential issues [51]. This approach involves calculating costs and risks associated with both scenarios: immediate repair after failure or preventive replacement after a certain period of use [50, 51].

It was also assumed that replacing the mechanical part of the master controller with a new one will result in the item being "as good as new" after the repair. This is also called a perfect repair. By analysing the cost over time, it becomes possible to identify the most cost-effective replacement time to prevent significant issues and achieve cost savings [53]. This assumption was made because no damage to the electronic parts of the master controller was recorded during the period studied. Therefore, the optimal replacement time can be determined by minimising the cost per unit time, denoted as $C(t_p)$ [30, 31,52]:

$$C(t_t)=\frac{\text{Total Expected Replacement Cost per Cycle}}{\text{Expected Cycle Length}}=\\ =\frac{C_p\cdot R(t_t)+C_u\left[1-R(t_p)\right]}{\int_0^{t_p}R(s)\,\mathrm{d}s} \tag{19}$$

Where $C(t_p)$ - is the total cost per unit of time, C_p - is the cost of a planned (preventive) replacement, C_f - is the cost of an unplanned (corrective) replacement, $R(t_p)$ - is the reliability of the master controller at time t_p, t_p - optimal time interval for preventive replacement once the item has reached a specific age. The optimum replacement time interval, t_p, is the time that minimizes $C(t_p)$. Hence, the optimum replacement time can be obtained by solving for t_p:

$$\frac{\partial\left[C(t_p)\right]}{\partial t_p}=\frac{\partial\left[\frac{C_p\cdot R(t_t)+C_u\left[1-R(t_p)\right]}{\int_0^{t_p}R(s)\,\mathrm{d}s}\right]}{\partial t_p}=0 \tag{20}$$

The total cost $C(t_p)$ is minimized by means of the maintenance interval t_p. This formula cannot be solved analytically. To find the value of t_p that minimizes $C(t_p)$. numerical optimization technique using the TNC method was employed, which is suitable for problems where derivatives are not readily available. The optimisation process was initiated with a reasonable guess for (e.g. 50000 kilometres) and iteratively adjusted to minimise the cost function. The other mathematical details are beyond the scope of this article. This approach helps to effectively manage maintenance costs while mitigating risks associated with potential failures.

3 RESULTS
3.1 Parameters Estimation and Distribution Fitting

Based on Eq. (6), a ranking of the best-fitting distributions was established, in which the smallest WDV value means the best fit of the empirical data to the model (see Tab. 3). In Fig. 2, all the distributions analysed are presented. The probability plot illustrates the trend in failure probability over time, capturing trends in master controller failure behaviour for each distribution within two-sided 95% confidence bounds.

From this plot, we can infer that six distributions (i.e., Weibull 2P and 3P, log-logistic, normal, and Gamma 2P and 3P) are reasonably well fitted. However, the last three distributions (i.e., Gumbel, Exponential 1P, and lognormal) are not well fitted, with a significant number of failure points lagging considerably. Based on this analysis, we can determine that the best-fit distribution in this case is Weibull 2P. In Tab. 3, the distribution rankings with statistical test values are presented. In this study, the best fitted distribution is Weibull 2P.

In the next step, in addition to estimating the two basic scale and shape parameters, $\hat{\alpha}=433110$ and $\hat{\beta}=1.6342$ for all studied distribution parameters studied, the mean time/distance between failures was calculated using formula 11. When the data were substituted, the following results were obtained: first, the *MDTF* parameter was calculated.

Table 2 Estimated parameters for all distributions

Distribution	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\lambda}$
Weibull_2P	433110	1.6342	-	-	-	-
Gamma_2P	221409	1.9065	-	-	-	-
Loglogistic_2P	355089	1.8663	-	-		-
Lognormal_2P	-		-	12.831	1.03	-
Exponential_1P	-		-	-	-	1.693e–06
Weibull_3P	432881	1.5687	4799	-	-	-
Gamma_3P	237512	1.7858	4799	-	-	-
Normal_2P	-	-	-	331148	160556	-
Gumbel_2P	-	-	-	376202	114 94	-

$$MDTF = 433110 \cdot \Gamma\left(1+\frac{1}{1.6342}\right) = 387595.9\ \text{km} \tag{21}$$

An estimation was also made of the reliable life parameter T_R for the required reliability of 0.9, which corresponds to a B10 life indicator, i.e., the time, by which 10% of the objects will fail. Therefore, upon substituting the data the following was obtained:

$$T_R = 433110 \cdot \left(-\ln(0.9)\right)^{\frac{1}{1.6342}} = 116915\ \text{km} \tag{22}$$

Table 3 Rank of distributions for master controller according to 6 equations

Distribution	*LL*	*BIC*	*AIC*	*WDV*	Ranking
Weibull_2P	–353.9	712.0	716.3	21561.1	**1**
Gamma_2P	–354.0	712.1	716.5	21565.9	2
Loglogistic_2P	–354.0	712.2	716.5	21568.2	3
Lognormal_2P	–354.8	713.7	718.0	21613.6	4
Exponential_1P	–357.2	716.5	718.7	21633.0	5
Weibull_3P	–353.8	714.0	720.4	21685.1	6
Gamma_3P	–354.0	714.3	720.7	21692.5	7
Normal_2P	–356.4	717.0	721.4	21713.5	8
Gumbel_2P	–358.9	721.9	726.2	21860.0	9

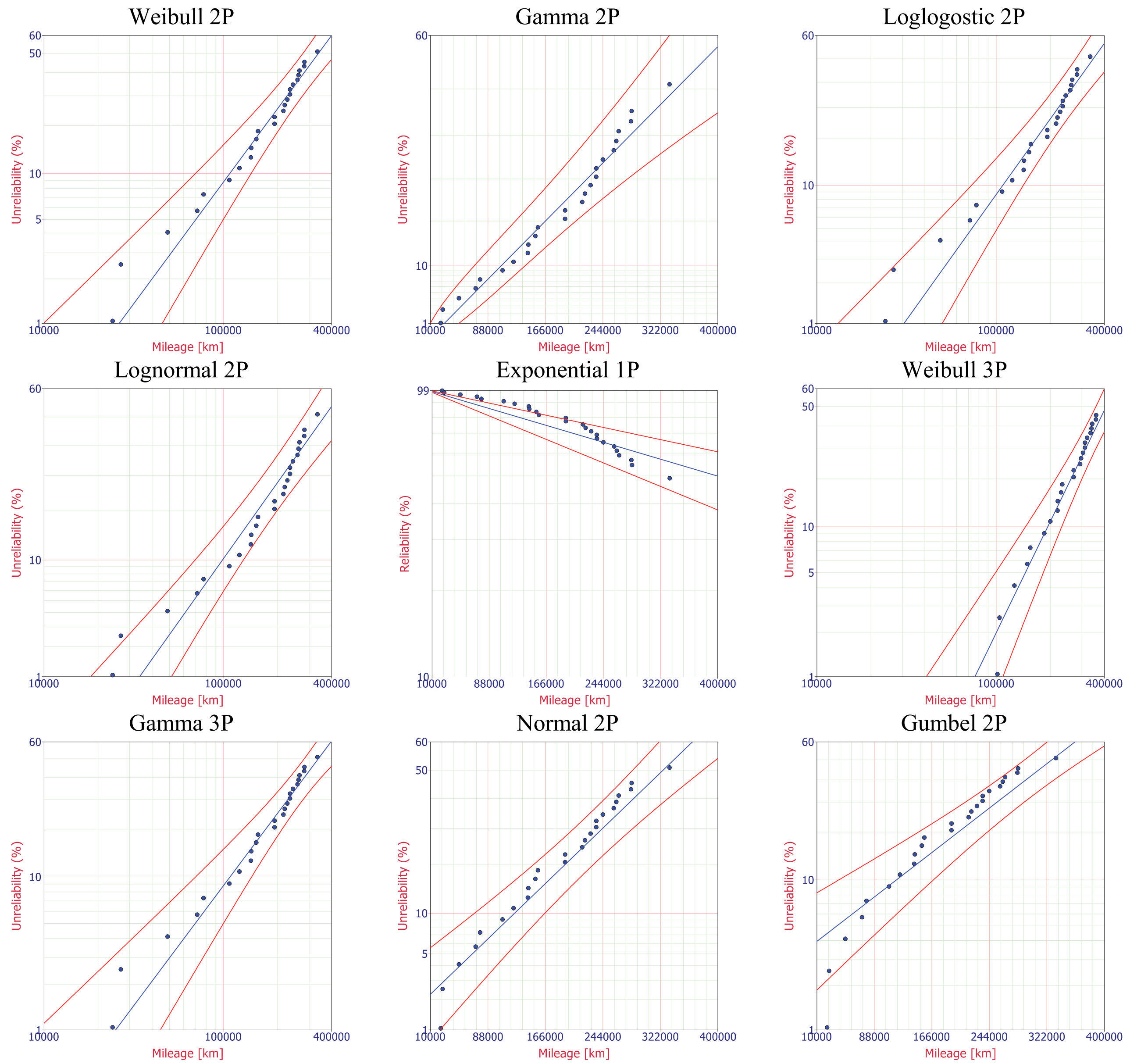

Figure 2 Probability plots of the fitted distribution were estimated using the Python Reliability library

In the following steps, we also calculated the selected parameters for age (t) at the end of the study, that is, after five years of operation, and for the master controller it was t = 368581 km. We estimate the values of the following parameters: the probability of failure Q, the failure rate h and the mean residual life MRL were calculated. From the distribution function formula, the probability of failure was calculated.

$$Q(368581) = 1 - e^{-\left(\frac{368581}{433110}\right)^{1.6342}} = 0.5462 \tag{23}$$

In the next part, the authors calculated the failure rate and the assumed time:

$$h(368581) = \left(\frac{1.6342}{433110}\right) \cdot \left(\frac{368581}{433110}\right)^{1.6342-1} = \tag{24}$$
$$= 0.000003377 \text{ km}$$

Then, from Eq. (10), the mean residual time was calculated after substituting the data for the superstructure:

$$MRL(368581) = 433110 \cdot e^{\left(\frac{368581}{433110}\right)^{1.6342}} \cdot \Gamma\left(1 + \frac{1}{1.6342}\right) \cdot$$
$$\cdot \left(1 - \frac{\Gamma_\tau\left(\frac{1}{1.6342}\right)}{\Gamma\left(\frac{1}{1.6342}\right)}\right) = 212523.5 \text{ km}$$
(25)

Table 4 Estimated model parameters for the master controller

Parameter	Value
Shape parameter	$\hat{\beta} = 1.6342$
Scale parameter	$\hat{\alpha} = 433110$ km
Mean time/distance to failures	$MDTF$ = 387595.9 km
Reliable life for t(R=0.9)	T_R = 116915 km
For age (t) at end of the study	t = 368581 km
Probability of failure	Q = 0.5462
Failure rate	h = 0.000003377 km
Mean residual life	MRL = 212523.5 km

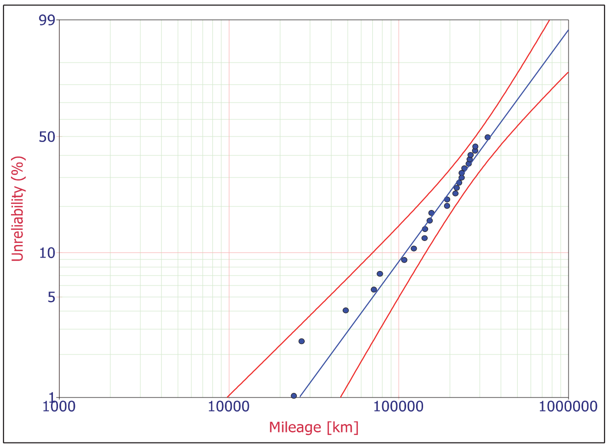

Figure 3 2P-Weibull probability graphic representation

The estimated parameter values of the $\hat{\beta}$, $\hat{\alpha}$ distribution and all calculated characteristics for each subsystem are shown in Tab. 7. The Weibull 2P shape parameter indicates whether the failure rate is increasing, constant, or decreasing. The estimated values of the shape parameter are greater than 1, which means that the failure rate is increasing (Fig. 3). The scale parameter is a measure that represents the time at which approximately 63.2% of the systems or components being analysed are expected to fail. It is closely related to the mean time to failure. Tab. 4 contains all the estimated parameters.

The following figures present distributions to the probability grids, the curves of the probability distribution function and the probability distribution, density function, histogram, MRL, and contour plot for the master controller.

The blue line represents the Weibull probability and the red line are two sided 95% confidence bounds (Fig. 4). For these analysed master controllers, most of the failures occurred between 120000 and 300000 km. The reliability vs. time plot shows the reliability values over time, capturing trends in the product failure behaviour.

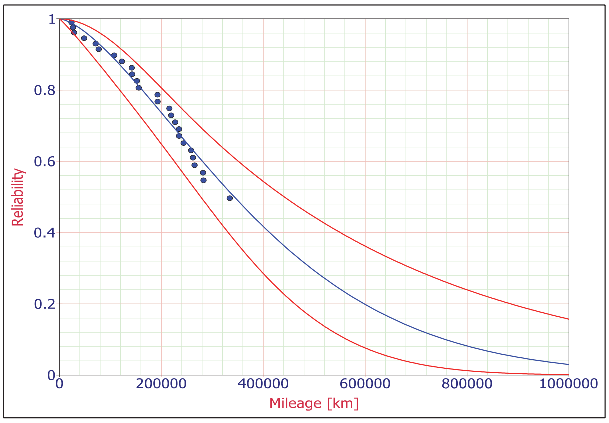

Figure 4 Weibull reliability graphic representation

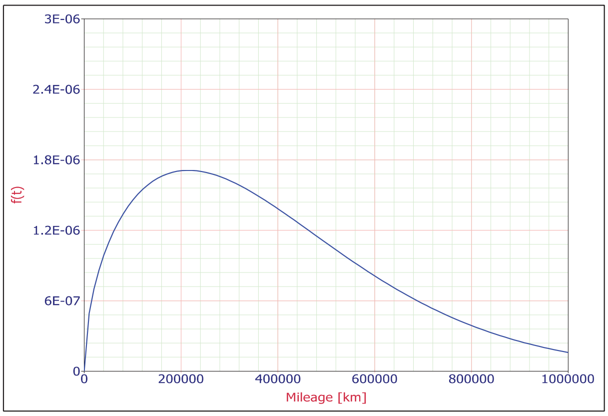

Figure 5 Probability density plot

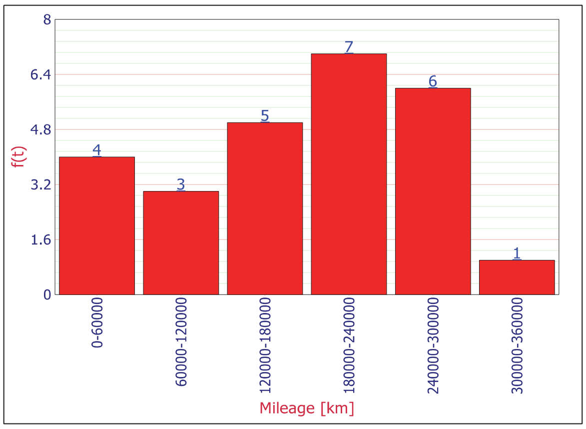

Figure 6 Histogram

The pdf plot shows the probability density function of the data over time, allowing you to visualise the distribution of the data set (see Fig. 5). In Fig. 6. the histogram is presented for the data with a range size of 60000 km. From this histogram, it can be characterised where the data set fails with respect to its values. This graph illustrates that a relatively high proportion of the data falls between the values of 60000 km and 300000 km.

3.2 Optimal Preventive Maintenance Replacement Time

Determining the optimal time for preventive replacement of appliance parts before failure occurs is similar to proactive financial planning (Tab. 5). It is considered that replacing a component before its failure can be economically advantageous under certain conditions, as the cost of corrective repairs may exceed the cost of preventive replacement. Moreover, the shape parameter is greater than one, indicating that the failure rate increases with time, which means a wear-out failures process. This insight reinforces the need to take action for corrective maintenance; otherwise, performing preventive maintenance is not economical. During the time covered by the investigation, in addition to the 25 failures and associated corrective maintenance actions previously described, 5 preventive maintenance actions were taken.

The total costs of the unplanned maintenance of the master controller amounted to approximately 273327.6 EUR. For example, the cost of planned (preventive) maintenance is listed at 527.60 EUR, whereas corrective maintenance, after a failure, costs significantly higher at 2521.25 EUR. This higher cost for corrective maintenance is determined by data from several rolling stock repair workshops, comprising both the price of the new master controller (2432.43 EUR) and additional replacement costs 89.12 EUR.

Table 5 Optimum replacement cost target computation example

Calculation start time from:		20000	
Increment by:		10000	
Cost for planned replacement:		528 EUR	
Cost for unplanned replacement:		2463 EUR	
Time units	Cost/unit time	Time units	Cost/unit time
200 000	0.00564	360 000	0.005656
210 000	0.005604	370 000	0.005675
220 000	0.005577	380 000	0.005696
230 000	0.005557	390 000	0.005717
240 000	0.005544	400 000	0.005738
250 000	0.005537	410 000	0.00576
260 000	**0.005534**	420 000	0.005781
270 000	0.005535	430 000	0.005803
280 000	0.00554	440 000	0.005825
290 000	0.005548	450 000	0.005847
300 000	0.005558	460 000	0.005868
310 000	0.005571	470 000	0.00589
320 000	0.005585	480 000	0.005911
330 000	0.005601	490 000	0.005932
340 000	0.005618	500 000	0.005952
350 000	0.005636	**Minimum at:**	**0.005534**

Specifically, the average cost of replacing the part with a new one amounted to a substantial 5260 EUR. Furthermore, the average time taken for repairs was 4.5 hours. Taking into account the hourly labour rate of 50 EUR, the labour cost for part replacement comes to 225 EUR. This implies that the total expense of replacing a single master controller represented approximately 1.64% of the total repair expenses associated with the corrective maintenance fleet of trams.

The determination of the optimal replacement time, denoted as t_p, involves minimising the total cost per unit time $C(t_p)$, which is formulated in Eq. (19) and Eq. (20). The optimal replacement time can be determined by minimising the cost per unit of time. The computational steps in Python were as follows:

1. Define the cost function $C(t_p)$.
2. Use the scipy.optimize.minimize function with the TNC method to find the t_p that minimizes $C(t_p)$. This method is used to find the zero-crossing point of the derivative, indicating where $C(t_p)$ is minimized. In the Tab. 4 detailed calculations have been presented.

$$C(t_t)=\frac{\text{Total Expected Replacement Cost per Cycle}}{\text{Expected Cycle Length}}= \\ =266621.17\text{ km} \quad (26)$$

Therefore, the optimal replacement time can be determined by calculating the value of t (Tab. 5):

$$\frac{\partial\left[C\left(t_p\right)\right]}{\partial t_p}=0 \text{ thus } t_p=0.0055339\text{ EUR/km} \quad (27)$$

The resulting optimal replacement time, t_p was approximately 266621.17 kilometres, indicating this is the most cost-effective point to perform maintenance assuming that the component behaves "as good as new" after replacement. Some details of computation are presented in the Tab. 5. A plot of the derivative across a range of t_p values helps to visually verify where the derivative approaches zero (Fig. 7).

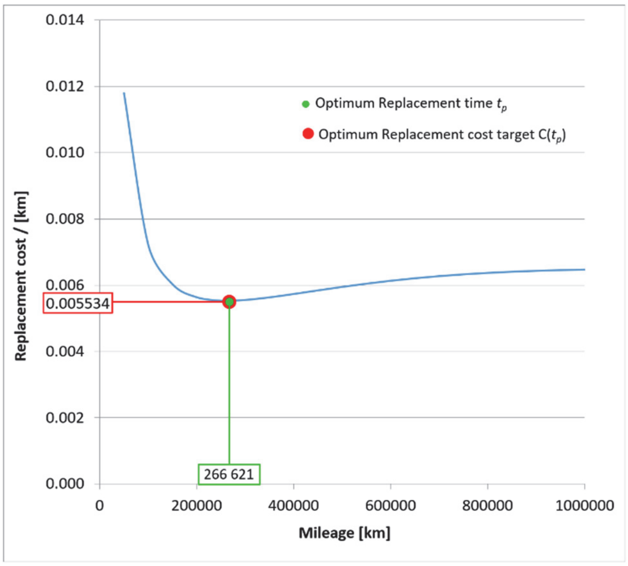

Figure 7 Cost per unit time vs. replacement time

Graphically, the optimal preventive maintenance time and the corresponding costs per unit of time for the master controller are presented in Fig. 7. This figure illustrates the best-scheduled replacement intervals to minimise costs over time. According to the results, the optimal replacement interval is approximately 266621 km, with the lowest cost being around 0.005534 EUR per kilometre, taking into account that vehicles from the fleet are considered as one surveyed system.

4 CONCLUSIONS

A thorough analysis of the reliability and cost implications of corrective maintenance for the master controller in a fleet of trams has been presented by this research. Rigorous statistical methods were applied to a substantial dataset obtained from the operation of these trams, allowing several significant conclusions to be drawn. A statistical distribution was fitted to life data from a representative sample of units, and predictions about the life of all items in the research area were attempted. The Weibull statistical distribution, among other techniques, is beneficial in gauging the reliability parameters of vehicles. Critical product life parameters, including dependability or likelihood of failure at a particular mileage, mean life, and failure rate, were then estimated using the parameterized data distribution. Thanks to performed life data analysis, many management choices concerning life-cycle costs and maintenance may now be made more confidently using reliability predictions. The study clearly indicates that preventive maintenance, when executed at the optimal time, is more cost-effective than corrective maintenance. The findings reveal that the optimal replacement interval for the master controller is approximately 266 621 km, which not only minimizes downtime but also significantly reduces maintenance costs per unit time to about 0.005534 EUR per kilometres. By transitioning to a preventive maintenance schedule based on the results of this analysis, tram operators can expect a reduction in both the frequency and severity of unscheduled repairs, which are notably more expensive and disruptive. This shift is supported by the predictive capabilities of the chosen statistical model, which can inform maintenance schedules with high accuracy. The employment of advanced statistical tools like the Maximum Likelihood Estimation method and reliability software proved essential in analysing the censored data effectively. These tools facilitated a deeper understanding of the failure mechanisms involved and allowed for the precise estimation of maintenance intervals.

It is recommended that tram fleet operators adopt this analytical approach to determine the maintenance schedules for other critical components of their vehicles. This strategy will likely yield improvements in operational reliability and cost efficiency across their fleets. Further research could explore the integration of real-time data acquisition systems in tram operations to continuously update and refine the predictive models. Additionally, studying the impact of different operational environments on the reliability of tram components could provide more tailored maintenance strategies for diverse urban settings. By leveraging the insights gained from this study, tram operators can enhance their maintenance strategies, thereby improving the reliability of public transport systems and optimizing the lifecycle costs associated with these assets.

Proper data quality ensures that the conclusions drawn from the analysis are valid and can be used confidently in decision-making processes [54] related to maintenance [55], warranty, safety, and customer satisfaction.

Future research should explore the acquisition of real-time data from tram operations to improve failure prediction models. The analysis of operational scenarios and the application of machine learning algorithms could create adaptive tram maintenance strategies for various operating conditions. This can improve tram operators' work strategies, optimize costs, and ensure adequate data quality for decisions related to maintenance, warranty, safety, and customer satisfaction.

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