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ANALIZA PARAMETARSKE REZONANCIJE METODOM KONAČNIH ELEMNATA

ANALYSIS OF PARAMETRIC RESONANCE BY THE FINITE ELEMENT METHOD

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SAŽETAK

U radu je prikazana mogućnost dobivanja parametarske rezonancije metodom konačnih elemenata u programskom paketu COSMOS/M. Korištena je nelinearna metoda proračuna u modulu NSTAR s kontrolom prirasta sile po Newton-Raphson-ovoj metodi s konstantnom veličinom koraka. U radu su prikazana dva numerička modela. Prvi model sastojao se od jednostavnog slučaja zglobno pridržanog štapa od IPE 120 profila opterećenih uzdužnom promjenjivom silom na jednom kraju. Drugi složeniji, prostorni model betonskog okvira modeliran je s vertikalnom promjenjivom akceleracijom u osloncima stupova s namjerom da se parametarska rezonancija dobije kroz torzijski odziv okvira. Dobiveni rezultati su pokazali da je moguće dobiti parametarsku rezonanciju na konstrukcijama metodom konačnih elemenata.

Ključne riječi: *parametarska rezonancija, COSMOS/M, dinamička stabilnost, nelinearni model*

ABSTRACT

The paper explored the possibility of obtaining parametric resonance through the finite element method in the COSMOS/M software package. A nonlinear analysis method was employed in the NSTAR module, utilizing force increment control based on the Newton-Raphson method with a constant step size. The paper considered two finite element models. The first model consisted of a simple case of a hinge-supported IPE 120 profile column loaded with a longitudinal variable force at one end. The second, more complex, three-dimensional model of a concrete frame was analysed with a vertical dynamic acceleration at the columns support, aiming to achieve parametric resonance through the torsional response of the frame. The obtained results of the numerical models have shown that the parametric resonance in structures using the finite element method can be achieved.

Keywords: *parametric resonance, COSMOS/M, dynamic stability, nonlinear model*

1. UVOD

1. INTRODUCTION

The term stability is almost always related to the static stability of pressure-loaded constructions or individual elements. However, there is also the dynamic stability of structures, which is often overlooked. Dynamic stability, i.e., instability of structures occurs in the case of bringing the structure or some part of it into resonance with external excitation. Resonance occurs when the frequency of the structure and the external excitation are matched. Given that building structures in operation are often loaded with dynamic repetitive actions (presses, aggregates), their effect on the structure and the way the structure itself responds to these dynamic loads have to be taken into account [1], [2]. If the frequency of the external load on the structure differs greatly from the frequency of the structure, then the load is viewed as static with the use of dynamic factors to increase it, however, if the frequencies are similar, then it is necessary to analyse the structure for dynamic loads and observe its response. There are several ways in which the dynamic stability of the system can be violated, and we divide them according to the external cause:

- Harmonic resonance,
- Parametric resonance,
- Resonance caused by impulse forces,
- Resonance caused by aeroelastic forces,
- Resonance caused by circulation forces.

Of all the above-mentioned resonances, parametric resonance and the possibility of its modelling by the finite element method on a pressure-loaded rod and a spatial frame without additional damping within the system are specifically covered in the paper.

Parametric resonance of a structure or an element occurs under the action of an external periodic force that does not act in the direction of the observed oscillations. So, if the element is loaded with an oscillating longitudinal force of the appropriate frequency, it will cause transverse deformations of the element and it can be said that the load is parametric in relation to the caused deformations, and the work that the force brings to the top of the element is a small quantity of the second order in relation to the transverse displacements. The loss of stability due to parametric resonance occurs if the frequency of the longitudinal force is matched with the displacement of the top of the element, and in this way, energy is constantly introduced into the system by the operation of the variable part of the pressure force.

The problem of parametric resonance of a pressure-pressed element with a periodic force can be written mathematically [3]:

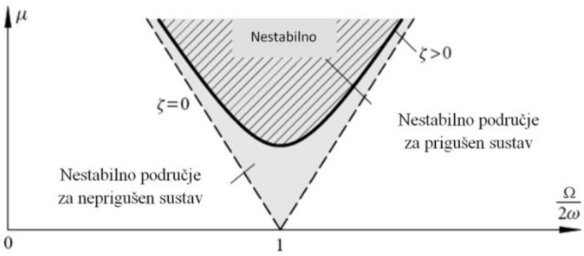
$$\ddot{f}(t)+\omega^2[1-2\mu cos\Omega t]f(t)=0 \quad (1)$$

where ω is the system frequency, Ω is the excitation force frequency, t is time, $f(t)$ is the system deformation function, while the coefficient μ can be expressed as follows:

$$2\mu=\frac{F_t}{F_{cr}-F_o}$$

From equation (1) it is evident that the trivial solution is f(t)=0, the equilibrium solution of the equation, that is, the trivial solution represents the stable motion of the element. The above equation is known as Mathieu's equation, and the very stability of the trivial solution of Mathieu's equation is well researched.

Unlike "ordinary" resonance, which occurs for some range of the observed frequency that is close to or equal to its own frequency, parametric resonance occurs for several different ranges of values of the observed parameter. Theoretically, there is an unlimited number of such regions of instability, however the surfaces of these regions of instability are reduced proportionally by reducing the ratio $\Omega/2\omega$, and the primary region is at the location $\Omega/2\omega=1$ and is of the greatest importance due to its size and the possibility of resonance even with system damping (Figure 1).



Slika 1 *Nestabilno područje Ω/2ω=1*

Figure 1 *Unstable range Ω/2ω=1*

If we now add the system damping ζ, we get:

$$\ddot{f}(t)+2\zeta\omega\dot{f}(t)+\omega^2[1-2\mu cos\Omega t]f(t)=0, \quad (2)$$

that is, with the addition of non-linear influences:

$$\ddot{f}(t)+2\zeta\omega\dot{f}(t)+\omega^2[1-2\mu cos\Omega t+\gamma f(t)^2]f(t)=0, \quad (3)$$

where γ is the non-linearity coefficient.

One of the characteristics of the oscillations of a damped system loaded with a parametric force in the stable region (the frequency of the external force far from the system frequency) is an exponential decrease in amplitude independent of the initial conditions (eccentricity). Therefore, systems exposed to parametric external excitation will be by nature responsive, exponential in nature.

2. DINAMIČKA ANALIZA U PROGRAMSKOM PAKETU COSMOS/M

2. DYNAMIC ANALYSIS IN SOFTWARE PACKAGE COSMOS/M

The dynamic nonlinear analysis of the system in the COSMOS/M software package is performed by the NSTAR module [4], [5], with control, iteration and force limitation. Force control is performed using a time curve that is determined by a series of points. The curve added to the force determines its intensity over the time in which the analysis is carried out. The iterative procedure is carried out within fixed time steps using the Newton-Raphson method so that the equilibrium condition is satisfied at the end of each time interval.

The time to which the analyses in this paper are limited was derived from the amount of data provided by this type of analyses. For the observed case of a straight stick, the analysis was carried out in 14,000 iterations, which corresponds to a time period of 14 min. The numerical model of the frame has slightly more nodes, but the number of iterations is reduced, resulting in an analysis time of 70 min.

The discretized equations of the dynamical system, as solved by COSMOS, can be written in the following form

$$[M]^{t+t\Delta}\{\ddot{U}\}^{(i)} + [C]^{t+t\Delta}\{\dot{U}\}^{(i)} + {}^{t+t\Delta}[K]^{(i)}\ {}^{t+t\Delta}\{\Delta U\}^{(i)} = {}^{t+t\Delta}\{\mathrm{R}\} - {}^{t+t\Delta}\{\mathrm{F}\}^{(i-1)} \quad (4)$$

where:

$[M]$ – is the mass matrix,

$[C]$ – is the system damping matrix,

${}^{t+t\Delta}[K]^{(i)}$ – is the system stiffness matrix,

${}^{t+t\Delta}\{\mathrm{F}\}^{(i-1)}$ - the vector of internal generated forces at the nodes in the step (i)

${}^{t+t\Delta}\{\mathrm{R}\}$ - vector of external forces in nodes

${}^{t+t\Delta}\{\Delta U\}^{(i)}$ - vector of incremental displacements of the nodes in the step (i)

${}^{t+t\Delta}\{\ddot{U}\}^{(i)}$ - vector of absolute displacements of the nodes in the step (i)

${}^{t+t\Delta}\{\mathring{U}\}^{(i)}$ - vector of absolute velocities in a step (i)

${}^{t+t\Delta}\{\ddot{U}\}^{(i)}$ - vector of absolute accelerations in a step (i)

By using implicit integration methods such as Newmark-Beta and Wilson-Theta and by using Newton's iterative method, expression (4) can be written shorter

$$^{t+t\Delta}[\bar{K}]^{(i)}\,\{\Delta U\}^{(i)} = {}^{t+t\Delta}[\bar{R}]^{(i)}\ , \quad (5)$$

more extensively in [4].

In real constructions, one of the main causes of non-linearity in addition to material non-linearity is the occurrence of large displacements, i.e., geometric non-linearity. The analysis of the system with large deformations included can return significantly different characteristics of the system response than those obtained by linear analysis.

These changes can be explained on the steel cable, where the increase in deformation and longitudinal force increases the transverse initial stiffness, resulting in a non-linear response (geometrical non-linearity).

Geometrical nonlinearity is taken into analysis by selecting the options "Finite Strain Analysis" or "Large Deflection Analysis". The difference between these two ways of solving geometric nonlinearity is in the simplifications and assumptions that are introduced for faster problem solving.

"Finite Strain Analysis" is used in analyses where there are large changes in the form of finite elements. The application of this type of analysis is necessary for membrane constructions where there are large differences in the shape of the element, initial (modelled) and deformed state.

"Large Deflection Analysis" is used in cases where there are large changes of the local coordinate system in relation to the global coordinate system (large rotations).

The observed case of rod oscillation requires a numerical analysis with the "Large Deflection Analysis" option enabled, because large rotations of the finite elements of the rod are expected during the analysis, which has a great influence on the change of the stiffness matrix.

Another significant phenomenon in real constructions is the non-linear dependence between stress and deformation. Material nonlinearity, as we call this phenomenon, can be of great importance in the following cases: with elements loaded with a time-varying force in the area of plasticity, long-term loading that can cause material creep, and temperature in the case of thermo-plastic materials. The program package supports analysis with mechanical characteristics of materials: linear elastic, non-linear elastic, plastic, highly elastic. Numerical analyses attempted to repeat the research of oscillations of a straight rod carried out for the linear region, and for this reason the material was modelled as linearly elastic. Specifying a linearly elastic material in the numerical rod model means that at each step there will be a linear relationship between stress and strain. Also, regardless of the deformation of the rod, there will be no residual deformation in the rod after unloading.

To perform the analysis with all nonlinear problems (geometrical and material nonlinearity and nonlinearity of boundary conditions), an interactional approach to the problem is needed [4], [6]. Non-linear analysis is carried out during the time period linked to an external load. In each observed time step, the equilibrium condition must be satisfied, and in order to satisfy this condition, the following calculation methods are used: force increment control, displacement control, and arc length control. For the mentioned numerical calculation methods, two methods are available with which the software package solves nonlinear equations: the Newton-Raphson method and the Modified Newton-Raphson method.

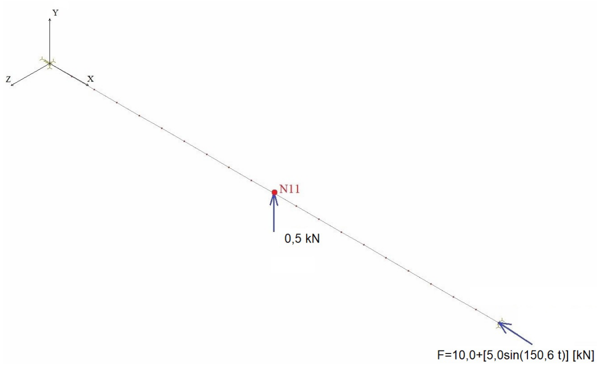
3. ANALIZA PARAMETARSKE REZONANCIJE U PROGRAMSKOM PAKETU COSMOS/M

3. PARAMETRIC RESONANCE ANALYSIS IN SOFTWARE PACKAGE COSMOS/M

A numerical model (Figure 2) was created with a 3.0 m long rod with a IPE120 cross-section rod supported by joints. The longitudinal force at the top of the rod was adopted according to the expression:

$$F=10.0+[5.0\cdot\sin(150.6\cdot t)]\ \text{kN} \tag{6}$$

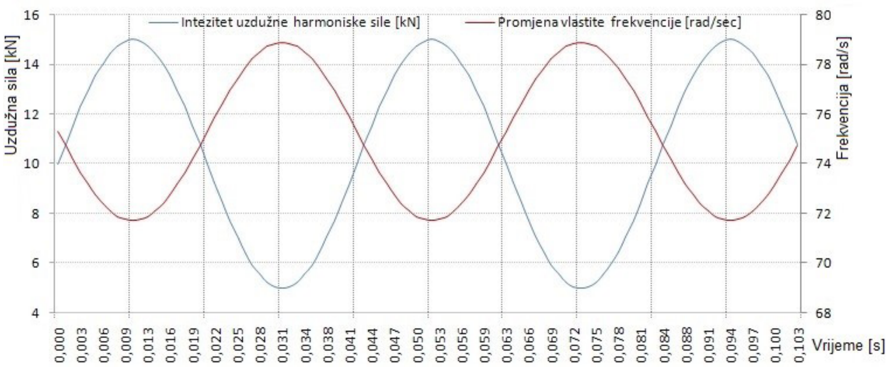
which renders it evident that the constant part of the force is 10.0 kN, while the variable part is 5.0 kN. Given that the condition for selecting the frequency is $\Omega/2\omega\approx1$, the force frequency of 150.6 rad/sec was adopted because the frequency of the rod loaded with a constant part of the longitudinal force is 75.3 rad/sec. The graphic representation of the sinusoidal longitudinal force according to expression (6) is shown in Figure 3 (blue line). The given longitudinal force gives the coefficient $\mu=0.0465$.



Slika 2 *Numerički model okvirnog sustava*

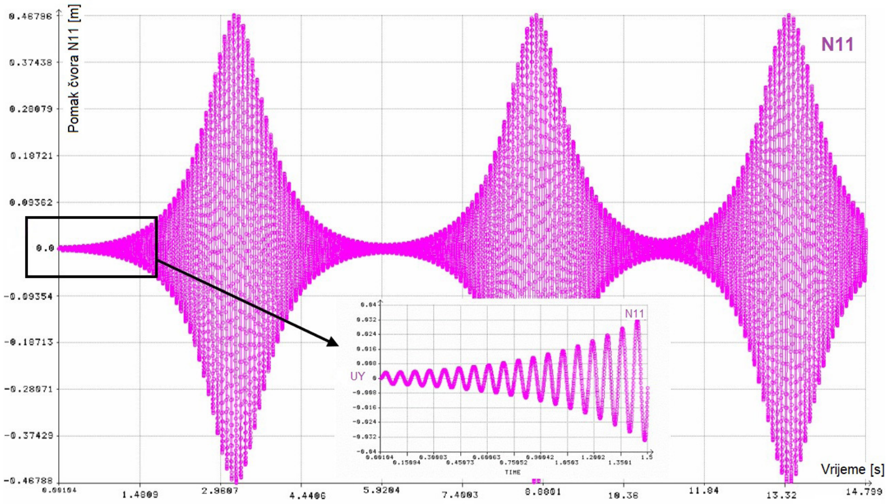
Figure 2 *Numerical model of the frame system*

The initial eccentricity of the rod is given by a transverse force of 0.5 kN in the direction of the local axis Y of the IPE cross-section, which results in the initial deformation of the rod of 0.464 mm, i.e., 0.015% of the total length of the rod, and it is turned off when the first full value of the longitudinal force is reached. The non-linearity of the material was not taken into account in these studies because an attempt was made to repeat the results of analytical studies [7], therefore the material was taken as linearly elastic. The analysis of the numerical model was carried out with control of the force increase according to the Newton-Raphson method with a constant step size, which was determined according to the amplitude of the variable part of the excitation force. In order to obtain parametric instability results, it is a necessary prerequisite that the software package enables analysis with geometric nonlinearity. With the NSTAR module, it is possible to perform this type of analysis, with the fact that the analysis is done with the assumption of large displacements (Large Deflection Analysis).



Slika 3 *Promjena frekvencije sustava u ovisnosti o uzdužnom harmonijskom opterećenju*

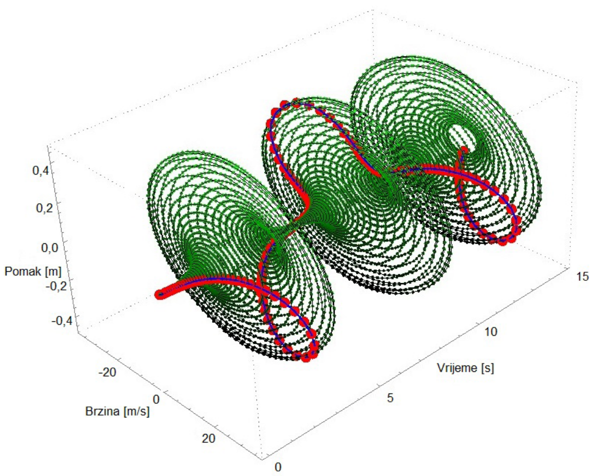
Figure 3 *System frequency change accordingly to longitudinal harmonic load*



Slika 4 *Poprečni pomaci sredine štapa*

Figure 4 *Transverse displacements in the middle of the element*

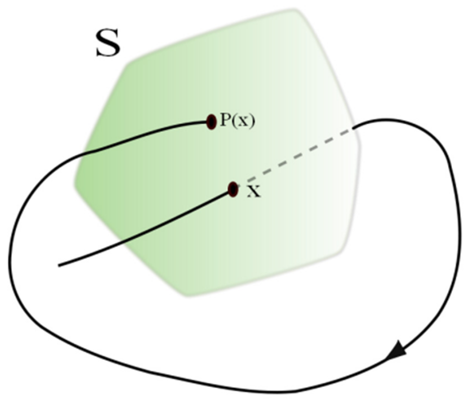
The numerical model analysis with the force control method and a fixed time step of 0.001 gave results in the time period from 0.0 s to 14.8 s. The displacement of the node in the middle of the rod (at node 11) in the direction of the Y axis is shown in Figure 4.



Slika 5 *Parametarski prikaz rezultata: brzina, pomak, vrijeme*

Figure 5 *Parametric display of results: speed, displacement, time*

In order to more easily analyse and understand the phenomenon of reduction of deformations that periodically appears after a deformation of 0.48m is reached, the results are also shown as a function of speed and thus parametrically shown in the spatial speed-displacement-time diagram in Figure 5. the Poincare map method was used (Figure 7).



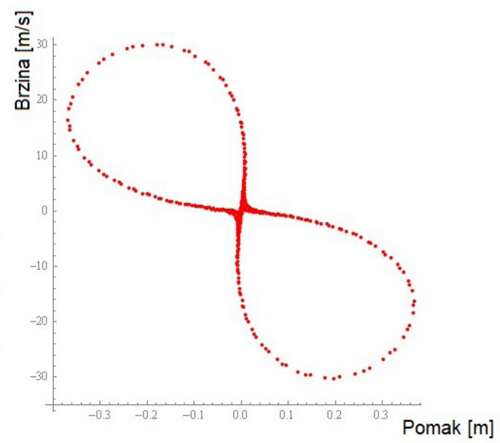
Slika 6 *Prikaz Poincare-ove mapa*

Figure 6 *View of the Poincare map*

Poincare maps [8], [9] show periodic intersections of the orbit of the results of the dynamic system through a surface placed perpendicular to them (Figure 6). They can be interpreted as a dynamic system with time discretization, that is, a system that has one dimension less than the real dynamic system. By time discretization, spatial dynamic systems can be presented as planar, that is, results can be presented in a two-variable coordinate system.

The places where the spatial curve of the result (Figure 5) intersect the plane set in periods equal to the oscillating nodal force are marked with red dots. The indicated red points are then reduced to a two-dimensional diagram (Figure 7).

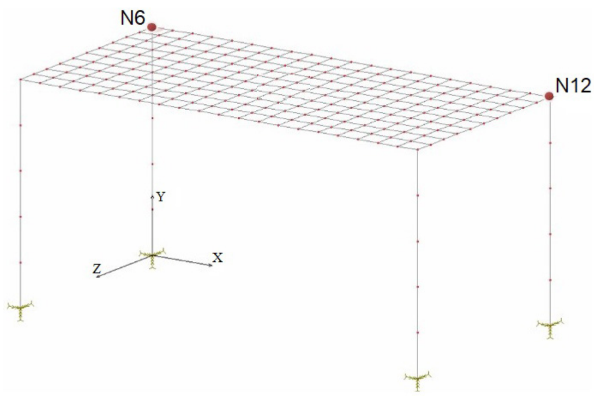
From the processed results via the Poincare map, a gradual increase in the phase deviation between the rod and the oscillating excitation force is visible. At the moment when the phase shift between the rod and the force reaches the value π/2, the rod calms down, which continues until the phase shift π, when the system begins to oscillate again with an exponential increase in transverse deformations.



Slika 7 *Prikaz rezultata za štap preko Poincare-ove mape*

Figure 7 *Display of the results for the elements via the Poincare map*

Another example, which is processed in this paper, is a spatial model of a reinforced concrete frame. The frame is modelled with BEAM3D (columns) and SHELL4 (slab) finite elements. The mechanical characteristics of the frame are adopted according to the analytical example ([7] p. 121). The dimensions of the AB columns are 40.0cm×23.00cm, which results in a frame stiffness ratio of β=3.0. The ratio of the width and length of the frame is γ=2.0, according to which the floor plan dimensions of the frame are 5.0 m × 10.0 m, and the height is 5.0 m (Figure 8). The AB plate was adopted with a thickness of 90 cm in order to obtain sufficient stiffness and the size of the constant part of the excitation force.



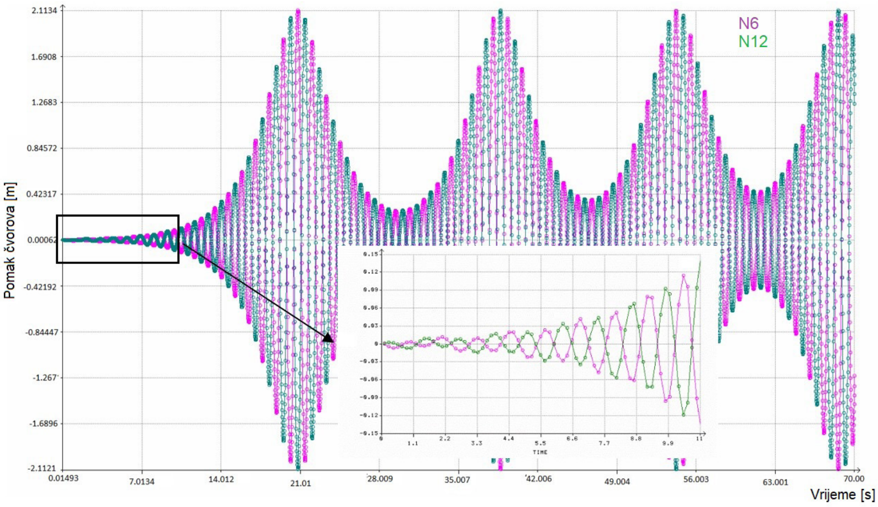
Slika 8 *Numerički model okvirnog sustava*

Figure 8 *Numerical model of the frame system*

The harmonic load on the numerical model is given over the time curve of the vertical gravitational acceleration. The constant part of the force is given by the gravitational acceleration g, i.e., the system's own weight. The variable part of the force is given by the oscillating gravitational acceleration of amplitude 0.3g connected to the supports (Figure 9). The natural frequency of the system is 7.94 rad/sec (torsion), according to

Slika 9 *Krivulja kontrole vertikalne akceleracije*

Figure 9 *Control curve for vertical acceleration*



***Slika 10** Horizontalni pomaci čvorova 6 i 12*

***Figure 10** Horizontal displacements of nodes 6 and 12*

which a frequency of 15.87 rad/sec was selected and adopted for excitation acceleration of a harmonic character.

In order to obtain the initial eccentricity, the excitation force at node 6 was set to 100 kN.

The reaction of the system to the vertical acceleration and the excitation horizontal force is shown through the horizontal displacements of points 6 and 12 (Figure 10). The graph shows displacements in the y direction: node 6 in red and node 12 in green. The graph shows the correct behaviour of the system, i.e., the parametric resonance of the system in terms of torsion around the centre of the plate with a pronounced harmonic oscillation of the displacement amplitude.

4. ZAKLJUČAK
4. CONCLUSION

Hitherto research on parametric resonances was primarily carried out by means of analytical analyses on a hinged stick or some similar simpler system.

The goal of the work was to achieve parametric resonance with a software package for numerical modelling, which at first glance is unusual since numerical models have been used for 30 years to solve engineering problems. However, to obtain parametric resonance, the software package must support analysis with geometric nonlinearity included.

Both numerical models gave the results of decreasing and increasing the vibration amplitude. This phenomenon of decreasing and increasing amplitude is the result of the influence of the variable part of the longitudinal force on the rod frequency and geometric nonlinearity. All results show the same, the period of oscillation of the rod gradually increases until the moment when the phase shift between the oscillation of the system and the force is $\pi/2$. At the moment of the force-system phase shift $\pi/2$, the system calms down (the force takes away energy from the rod).

After the resynchronization of the rod oscillations and the excitation force, but with a phase shift π, the system starts to oscillate again, which can be seen from the Poincare map (Figure 7).

The numerical model of the space frame obtained the results of the parametric resonance, and the characteristics of the parametric resonance are identical to those described for the rod. Namely, a periodic decrease and increase in the displacement amplitude and a gradual increase in the period of the system were again observed.

5. REFERENCE
5. REFERENCES

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