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Modelling Differently Defined Dominant Stand Diameters of Monospecific Forest Plantations

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Abstract

Quadratic mean diameter is a widely used stand parameter present in the stand inventory summaries, while the top stand diameter is rarely reported in the literature, mainly in relation to dominant stand height. Since the dominant stand height is usually determined from the tree height-diameter curve of the stand, it is important how the top tree assemblage, used to estimate dominant diameter, is defined. The main objective of our study was to assess the bias between differently defined dominant diameter estimates for monospecific plantations of various species, to model the dominant diameter as a function of quadratic mean diameter and other relevant stand variables, and to estimate its goodness-of-fit in predicting dominant diameter and dominant height.

We used data records gathered in sample plots in monospecific plantations of four tree species: Scots pine, Black pine, black locust and hybrid black poplar. We calculated the quadratic and arithmetic mean diameters of the 20% thickest trees in the plots, and the quadratic and arithmetic mean diameters of the trees, whose number corresponded to the 100 thickest trees per hectare. For each dataset, we analyzed the range and the distribution of the relative deviations calculated for each pair of dominant diameter estimates. For the Black pine plantations, regression models were developed for the two dominant diameter definitions, whose values differed most. Their goodness-of-fit was assessed from model efficiency and error statistics. The same model derivation procedure, applied to the Scots pine data, was followed by substitution of the predicted dominant diameter into a height-diameter model to assess the goodness-of-fit of the dominant height predictions.

The differences between the arithmetic and quadratic means, estimated from the same subsample of trees, did not exceed 2% in all cases. However, dominant stand diameters calculated as averages of differently defined largest tree collectives differed by as much as 35%. Regardless of its definition, the dominant stand diameter was adequately predicted by a function of the quadratic mean diameter alone or considering stand basal area as a second predictor. The models showed very good accuracy of model efficiency above 0.92, average absolute error below 8%, with 90% of the relative errors less than 15%. The predicted dominant diameter value can be used in a height-diameter model to estimate with confidence the dominant stand height of a monospecific forest plantation, allowing the forecast of the stand attributes based on dominant trees when only average stand variables are known.

Keywords: Scots pine plantations, Black pine plantations, black locust plantations, hybrid black poplar plantations, quadratic mean diameter, height-diameter model, goodness-of-fit, regression model

1. Introduction

Curtis and Marshall (2000) described the quadratic mean diameter of the stand (*Dq*) as a broadly used stand statistic that is present in practically all yield tables, stand inventory descriptions and simulator outputs. It is preferred to the arithmetic mean diameter ($\bar{D}$) because it represents the average basal area tree and therefore is closely related to the mean tree volume, particularly in regular, even-aged stands (Curtis and Marshall 2000). At the same time, arithmetic and quadratic mean diameters are connected by the formula:

$$Dq^2 = \bar{D}^2 + var(D) \quad (1)$$

Where:

$var(D)$ is the variance of the tree diameters used to calculate the means.

From Eq. 1 follows that quadratic mean diameter is always bigger than the arithmetic mean, but in homogenous stands, where the individual tree diameter values fluctuate within a narrow range, the variance and, consequently, the difference between the two means will not be substantial (Curtis and Marshall 2000, Ducey and Kershaw 2023). Other representations of the average stand diameter are scarcely found in the literature. Van Laar and Akça (2007) mentioned the basal area central diameter as another used stand statistics, while the basal area-weighted mean diameter is popular in Finland (Pukkala and Miina 2005, Siipilehto and Mehtätalo 2013, Ruotsalainen et al. 2021).

Ducey and Kershaw (2023) generalized that another stand diameter measure – the top stand diameter, estimated as arithmetic or quadratic mean of the thickest 100 trees·ha^{-1} – is occasionally reported in the scientific literature, usually with little emphasis. Its use is mostly pronounced in height-diameter modelling (Tomé 1988, Pienaar et al. 1990, Cañadas et al. 1999, Cimini et al. 2011) and in studies on the response of dominant trees to, e. g. spacing (Gizachew et al. 2012), damage by extreme events (Albrecht et al. 2015), etc. The study by Ducey and Kershaw (2023) suggested that the top stand diameter could be the most substantial univariate predictor of many important stand parameters. The authors recommended that it should be considered a »standard variable« when characterizing the forest stand conditions and a valuable predictor when specifying more elaborate forest models.

Although the mean height of the 100 tallest trees per hectare is always bigger than the mean height of the 100 thickest trees (Van Laar and Akça 2007), the fraction of the largest trees that are considered top height trees is usually that at the right tail (large end) of the tree number-diameter distribution (Pretzsch 2009). According to Pretzsch (2009), in the standard investigations, the mean and top heights are always extracted from the tree height-diameter curve of the stand. Therefore, it is very important how the top tree collective used to estimate dominant diameter, is defined: the 100 thickest trees per hectare (the thickest tree in a 100 m^2 plot, but the 10 thickest trees in a 0.1 ha plot) or 20% of the thickest trees in the area (the 1000 thickest trees per hectare at density 5000 trees·ha^{-1}, but the 200 thickest trees·ha^{-1} at density 1000 trees·ha^{-1}). In addition, the average dominant diameter can also be assessed as an arithmetic or a quadratic mean. Sharma et al. (2002) compared the stand top height estimates calculated for loblolly pine plantations in 7 different ways. The authors used data from both thinned and unthinned stands that have been collected in permanent sample plots for 15 years and reported that all differently defined top heights differed significantly from each other, with a few exceptions registered.

While the mean stand height is required to estimate the stand volume in the forest inventories, the dominant stand height is regarded as a quantity that is more appropriate in site quality assessment, because it is less easily affected by thinnings (Van Laar and Akça 2007, Tarmu et al. 2020). Therefore, the estimation of the dominant stand height and consequently, dominant stand diameter are important. The inventory summary of each forest stand in Bulgaria contains information on quadratic mean diameter (cm) and mean stand height (m), but no data on stand-level attributes based on the fraction of the top trees are available. The establishment of a dominant-quadratic mean diameter relationship would be of practical importance for the estimation of both dominant stand height and diameter if sufficient accuracy is assured. The monospecific forest plantations have relatively homogenous spatial tree dispersion, of usually unimodal diameter distribution pattern suggesting that the high accuracy of such a relationship would be an achievable goal in this case. The main objectives of our study were:

- ⇒ to assess the presence of bias between differently defined dominant diameter values for monospecific plantations of various species and its magnitude
- ⇒ to model the dominant-quadratic mean diameter relationship, considering also multiple regression functional forms and to estimate the goodness-of-fit of the predictions
- ⇒ to test the accuracy of the dominant stand height predictions from height-diameter model, based on dominant diameter values predicted from a relationship to the quadratic mean diameter.

2. Materials and Methods

To achieve the study objectives, we used data records gathered in monospecific plantations of four tree species: Scots pine (*Pinus sylvestris* L.), Black pine (*Pinus nigra* L.), black locust (*Robinia pseudoacacia* L.) and hybrid black poplar (*Populus* x *euramericana* (Dode) Guinier). Data collection took place in 153 sample plots in Scots pine plantations and 143 plots in Black pine plantations (Table 1), which were of rectangular or circular form and of different sizes depending

Table 1 Characteristics of data sets used in the analyses

Variable *	Scots pine ($nSP = 153$)	Scots pine ($nSP = 100$)	Black pine ($nSP = 143$)	Hybrid black poplar ($nSP = 15$)	Black locust ($nSP = 25$)
PS, m^2	290.2 (60–1269)	265 (85–1042)	249.2 (54.9–1358.4)	1412 (1012–2079)	555.1 (165–720)
PN, trees	57 (27–165)	–	48 (20–239)	49 (37–61)	97 (40–136)
Age, years	37 (10–80)	–	45 (12–85)	3 (1–5)	16 (2–20)
G, m^2·ha^{-1}	42.18 (5.54–72.27)	44.29 (6.10 - 72.25)	48.72 (3.46–110.54)	3.47 (0.09–13.66)	20.65 (2.71–32.23)
N, trees·ha^{-1}	2983 (483–12200)	2854 (825–8210)	2800 (503–8700)	356 (244–543)	1856 (1299–3576)
Dq, cm	16.0 (3.6–35.3)	15.7 (2.5–32.8)	17.3 (3.5–35.3)	9.2 (2.0–17.9)	11.7 (3.1–15.7)
$n100$, trees	3 (1–13)	–	3 (1–14)	14 (10–21)	5 (2–7)
$n\%20$, trees	11 (5–33)	–	10 (4–48)	10 (7–12)	19 (8–27)
D_0100a, cm	22.7 (7.0–47.3)	–	24.4 (7.0–48.7)	11.0 (2.6–21.5)	18.7 (5.1–25.7)
D_020a, cm	20.6 (5.6–45.2)	–	22.1 (5.6–44.7)	11.3 (2.7–21.4)	16.5 (4.4–22.8)
D_0100q, cm	22.7 (7.0–47.6)	22.3 (7.0–42.3)	24.4 (7.0 – 48.9)	11.0 (2.6–21.5)	18.8 (5.1–25.7)
D_020q, cm	20.6 (5.7 – 45.6)		22.2 (5.7 – 45.0)	11.4 (2.7–21.4)	16.6 (4.4–22.9)
H_0100q, m	–	16.6 (4.0–27.2)	–	–	–

Abbreviations: nSP – number of sample plots, PS – plot size (m^2), PN – plot tree number (trees), Dq – quadratic mean diameter (cm), G – stand basal area (m^2·ha^{-1}), N – stand density (trees·ha^{-1}), $n\%20$ – 20% of the thickest trees in the plot (trees), $n100$ – number of trees in the plot corresponding to the 100 thickest trees per hectare (trees), D_0100a – dominant diameter, estimated as the arithmetic mean of the diameters of the n100 thickest trees in the plot (cm), D_020a – dominant diameter, estimated as the arithmetic mean of the diameters of the $n\%20$ thickest trees in the plot (cm), D_0100q – dominant diameter, estimated as the quadratic mean of the diameters of the n100 thickest trees in the plot (cm), D_020q – dominant diameter, estimated as the quadratic mean of the diameters of the $n\%20$ thickest trees in the plot (cm), H_0100q – dominant stand height (m) estimated as the Lorey's mean of the heights of the n100 thickest trees in the plot (m).

* Average variable value is shown with minimum – maximum in parentheses

on the density and homogeneity of the stands and the purpose of plot establishment. The plots were installed throughout the area of the distribution of these plantations, with the primary goal to encompass the range of growth stages, densities, and sites, specific for these monospecific stand types in Bulgaria. Data records from the broadleaf species were obtained in 15 plots installed in industrial plantations of juvenile-age hybrid black poplar and 25 plots in black locust industrial and progeny test plantations. Two measurements perpendicular to each other of the tree trunk diameter at breast height were taken with a caliper and used to calculate the breast-height diameter of each tree. The breast-height diameters of all trees in the plots were determined and their number (PN, trees) was counted. These data were used to additionally calculate other stand variables such as density (N, trees·ha^{-1}), basal area (G, m^2·ha^{-1}), quadratic mean diameter (Dq, cm), number ($n\%20$, trees), quadratic (D_020q, cm) and arithmetic (D_020a, cm) mean diameters of the 20% thickest trees in the plots, the number ($n100$, trees) of the trees in the plots, corresponding to the 100 thickest trees per hectare and their respective quadratic (D_0100q, cm) and arithmetic (D_0100a, cm) mean diameters (Table 1). Information on tree age (years) was obtained from the inventory descriptions of the stands. A validation dataset from 100 sample plots in Scots pine plantations was considered to address the third study objective (Table 1). It corresponds to the dataset denoted as »Validation Data Set 3« in the study by Stankova et al. (2022).

To address the first research objective, all four main datasets were used. For each of them, the relative deviations (Dif_i, %) of all 6 pairs of dominant diameter estimates were calculated and their range (minimum–maximum) and distribution (25th, 50th, 75th percentiles) were analyzed:

$$Dif_1 = 100(D_0100a - D_0100q) / D_0100a \tag{2}$$

$$Dif_2 = 100(D_0100a - D_020a) / D_0100a \tag{3}$$

$$Dif_3 = 100(D_0100a - D_020q) / D_0100a \tag{4}$$

$$Dif_4 = 100(D_0100q - D_020a) / D_0100q \tag{5}$$

$$Dif_5 = 100(D_0100q - D_020q) / D_0100q \tag{6}$$

$$Dif_6 = 100(D_020a - D_020q) / D_020a \tag{7}$$

In addition, F-test was performed to examine the hypothesis that the linear regression relating the values of the compared dominant diameter estimates has a slope equal to 1 and an intercept equal to zero.

The datasets from the coniferous plantations that were representative of the variety of these stands in

Bulgaria were used to achieve the second (with the Black pine dataset) and the third (with the Scots pine dataset) research objectives. To attain the second study objective, the two dominant diameter definitions were selected, whose values differed at most according to the results obtained for the Black pine plantations in Objective 1 and regression models were developed for their prediction. There is a clear correlation between the dominant and the quadratic mean diameter (*Dq*, cm); stand density (*N*, trees·ha^{-1}) and plot size (*PS*, m^2) affect the number of trees, used to estimate the dominant diameter according to the different definitions and, consequently, affect the dominant diameter magnitude. Stand basal area (*G*, m^2·ha^{-1}) is also viewed as a measure of the stocking rate, and diameter growth is age-related (Age, years). Therefore, all these variables were examined as predictors of the dominant diameter by stepwise multiple regression analysis and the condition number test statistics was used to control collinearity, with a reference value of a maximum of 30. The significant predictors were selected according to the Percent Relative Standard Error statistics (*PRSE*%=100·Standard Error(Parameter)/|Parameter|) that must attain values below 25%. The Breusch-Pagan analytical test and the plot of residuals against predicted values were used to check the assumption for homoscedastic residual distribution. When heteroscedasticity of errors was identified, the model was refitted by generalized linear least squares method. To check the assumption of normality of errors, both analytical (Anderson-Darling test of normality, test for kurtosis of Anscombe-Glynn (1983) and test for skewness of D'Agostino (1970)) and graphical (Quantile-Quantile plot) tests were used. The presence of bias was assessed according to a *t*-test for zero mean error and by *F*-test to examine if the observed and predicted values are related by a linear regression of slope equal to 1 and a zero intercept. When the model derived was a linear regression of the variables quadratic mean diameter and density, an attempt to express density as a function of *Dq* was carried out, and its predicted value was substituted in the final regression equation, as suggested by Van Laar and Akça (2007). The regression statistics adjusted coefficient of determination (*Adj. R*^{2}) and residual standard error (*RMSE*, cm) were computed for the models that proved adequate and their predictability was further assessed according to the parameters model efficiency (*ME*), the quartiles of the relative errors (*RE*%) distribution and the mean of the absolute values of relative errors (*MARE*%) that characterize the spread and the size of prediction errors as compared to the observed dominant diameters:

$$ME = 1 - \frac{\sum (y_{\mathrm{i}} - \hat{y}_{\mathrm{i}})^2}{\sum (y_{\mathrm{i}} - \bar{y})^2} \tag{8}$$

$$RE\% = \frac{y_{\mathrm{i}} - \hat{y}_{\mathrm{i}}}{y_{\mathrm{i}}} 100 \tag{9}$$

$$ARE\% = \frac{\left| y_{\mathrm{i}} - \hat{y}_{\mathrm{i}} \right|}{y_{\mathrm{i}}} 100 \tag{10}$$

Where:

y_{i} measured dominant diameter value of the *i*-th sample plot, cm

$\hat{y}_{\mathrm{i}}$ predicted dominant diameter value of the *i*-th sample plot, cm

$\bar{y}$ mean observed dominant diameter value, cm

ARE% absolute value of the relative error, %

To achieve the third study objective, the described model derivation procedure was applied to the variable D_0100q of the Scots pine plantations dataset in this study. The selected adequate models were then used to predict the dominant diameter values of the validation data set (Table 1), which were substituted afterwards as predictors of the dominant height according to the established relationship (Stankova et al. 2022):

$$H_0100q = 1.3 + (0.968H_{\mathrm{m}} - 0.748) \cdot e^{0.32(1 - D_q / D_0\widehat{100}q) + (1/D_q - 1/D_0\widehat{100}q)} \tag{11}$$

Where:

H_0100q dominant stand height, m

H_{m} mean stand height, m

$D_0\widehat{100}q$ dominant diameter value, predicted from the derived regression models, cm

The accuracy of the dominant stand height predictions was assessed according to the test statistics used for verification of the predicted dominant diameters of Black pine plantations in Objective 2 (Eqs. 8–10). To examine for bias, we used simultaneous *F*-test for slope equal to 1 and zero intercept of the linear regression that relates the observed and predicted height values.

The data management, graphical representation and statistical analyses were performed using the packages of R software environment: *tidyverse* (Wickham et al. 2019), *dplyr* (Wickham et al. 2023b), *readxl* (Wickham and Bryan 2023), *plyr* (Wickham 2011), *purrr* (Wickham and Henry 2023), *tidyr* (Wickham et al. 2023a), *psych* (Revelle 2023), *janitor* (Firke 2023), *car* (Fox and Weisberg 2019), *ggplot2* (Wickham 2016), *ggprubr* (Kassambara 2023), *ggplotify* (Yu 2021), *patchwork* (Pedersen 2022), *nlme* (Pinheiro and Bates 2000, 2023), *olsrr* (Hebbali 2020), *relaimpo* (Grömping 2006), *heplots*

(Friendly et al. 2022), *lattice* (Sarkar 2008), *corrplot* (Wei and Simko 2021), *effectsize* (Ben-Shachar et al. 2020), *moments* (Komsta and Novomestky 2022), *nortest* (Gross and Ligges 2015), *lmtest* (Zeileis and Hothorn 2002).

3. Results

3.1 Objective 1: Estimation of Bias and its Magnitude

The differences between arithmetic and quadratic means, estimated from the same subsamples of trees (Dif_1 and Dif_6) did not exceed 2% in all cases (Table 2), the quadratic mean surpassing the arithmetic as suggested by Eq. 1. The largest deviations, up to nearly 35%, for the coniferous plantations were recorded between the arithmetic means based on differing subsamples of trees ($n100$ vs $n\%20$) – Dif_2. For the smaller datasets from the broadleaves, the largest deviations, those between D_0100a and D_020q (Dif_3), were less than 20% (Table 2). The deviations of bigger magnitude (Dif_2 to Dif_5) decreased with the diameter increase for the coniferous plantations, but remained constant for the juvenile broadleaf plantations (Figs. 1b, 2b, 3b, 4b). Statistically significant bias, as indicated by the simultaneous *F*-test for slope equal to 1 and zero intercept of the linear regression relating the compared values, was found in practically all cases (Figs. 1a, 2a, 3a, 4a). Significant deviations from the reference values of both the slope and the intercept, when examined separately, were unconditionally proven for the two larger datasets (data not shown).

Table 2 Relative deviations (%) between different estimates of dominant diameter

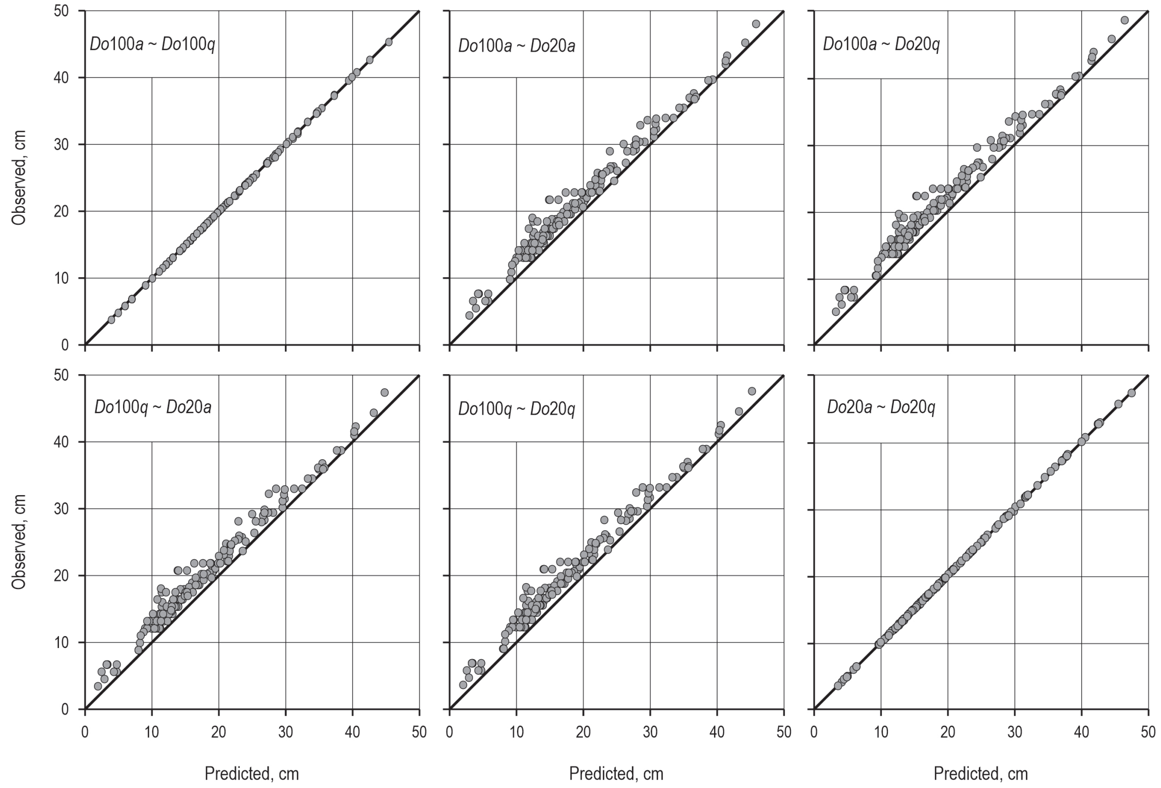
Species	Percentile	Dif_1	Dif_2	Dif_3	Dif_4	Dif_5	Dif_6
Scots pine	0	–1.31	0	–0.39	0.39	0	–1.79
	25	0	6.47	6.19	6.47	6.25	–0.63
	50	0	9.33	8.95	9.43	9.19	–0.40
	75	0	13.33	13.30	13.33	13.33	0
	100	0	32.22	31.11	32.22	31.11	0
Black pine	0	–0.98	0	–0.47	0	0	–1.79
	25	0	5.30	5.10	5.48	5.28	–0.60
	50	0	10.00	10.00	10.00	10.00	–0.31
	75	0	14.32	13.96	14.40	14.00	0
	100	0	34.74	33.68	34.74	33.68	0
Hybrid black poplar	0	–1.56	–10.16	–10.94	–9.37	–9.43	–1.75
	25	0	–4.12	–4.12	–4.12	–4.12	0
	50	0	–3.03	–3.33	–3.03	–3.33	0
	75	0	–1.34	–1.34	–1.15	–1.15	0
	100	0	0.47	0.47	0.47	0.47	0
Black locust	0	–1.61	6.77	6.25	7.25	6.74	–1.01
	25	–0.43	10.00	9.68	10.48	10.00	–0.55
	50	0	12.33	11.74	12.61	11.93	–0.50
	75	0	12.99	12.60	12.99	12.61	–0.45
	100	0	19.44	19.44	19.44	19.44	0

3.2 Objective 2: Dominant – Quadratic Mean Diameter Relationships for Black Pine Plantations – Goodness-of-fit in Dominant Diameter Predictions

There were two maximum differences of the same magnitude: D_020a–D_0100a and D_020a–D_0100q estimated for the Black pine plantations, and we chose to model the dominant diameter as a arithmetic (D_020a) and a quadratic (D_0100q) mean. Plantation age and plot size were not kept as significant predictors of either of the dominant diameters, while basal area was the second significant independent variable, beside the quadratic mean diameter, selected through the stepwise regression analysis (Table 3). Stand density (trees·ha^{-1}) could have been included as a predictor after expressing stand basal area as the product of density and quadratic mean diameter. An exponential and a power relationships of stand density to quadratic mean diameter was tested and the exponential model showed higher accuracy and predictability. It was approximated in a log-transformed functional form and a ratio correction coefficient (Snowdon 1991) was applied to the back-transformed dependent variable values to correct for bias (Table 3). By including the predicted density value as an independent variable, a second regression model was derived for each of the dominant diameter definitions (Table 3). To cope with the manifested residual heteroscedasticity, all four adequate models developed in fulfillment of Objective 2 were fitted through generalized least squares method employing variance functions of different forms. Three of the regression equations derived passed through the origin, all parameters assessed showed substantial stability (*PRSE*%<15%) and all fitted models showed relatively small residual mean squared error (*RMSE* (D_020a)<1.2cm, *RMSE* (D_0100q)<2.2cm) (Table 3).

Considering the outcome of Objective 1, which showed neglectfully small differences between arithmetic and quadratic means based on the same tree subsample, the goodness-of-fit in predictions of all four differently defined dominant diameters were examined (Table 4). The regression models derived

a. Comparison of differently defined dominant diameter values



b. Relative deviations *Difi*

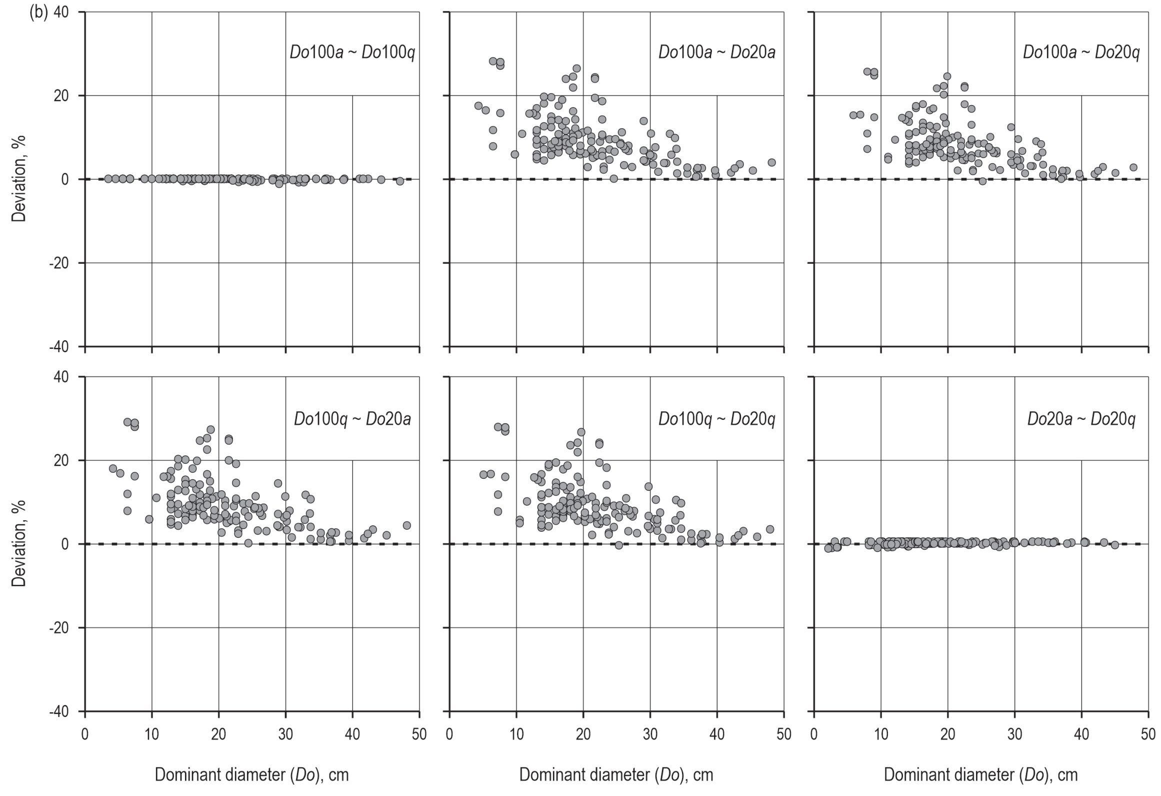
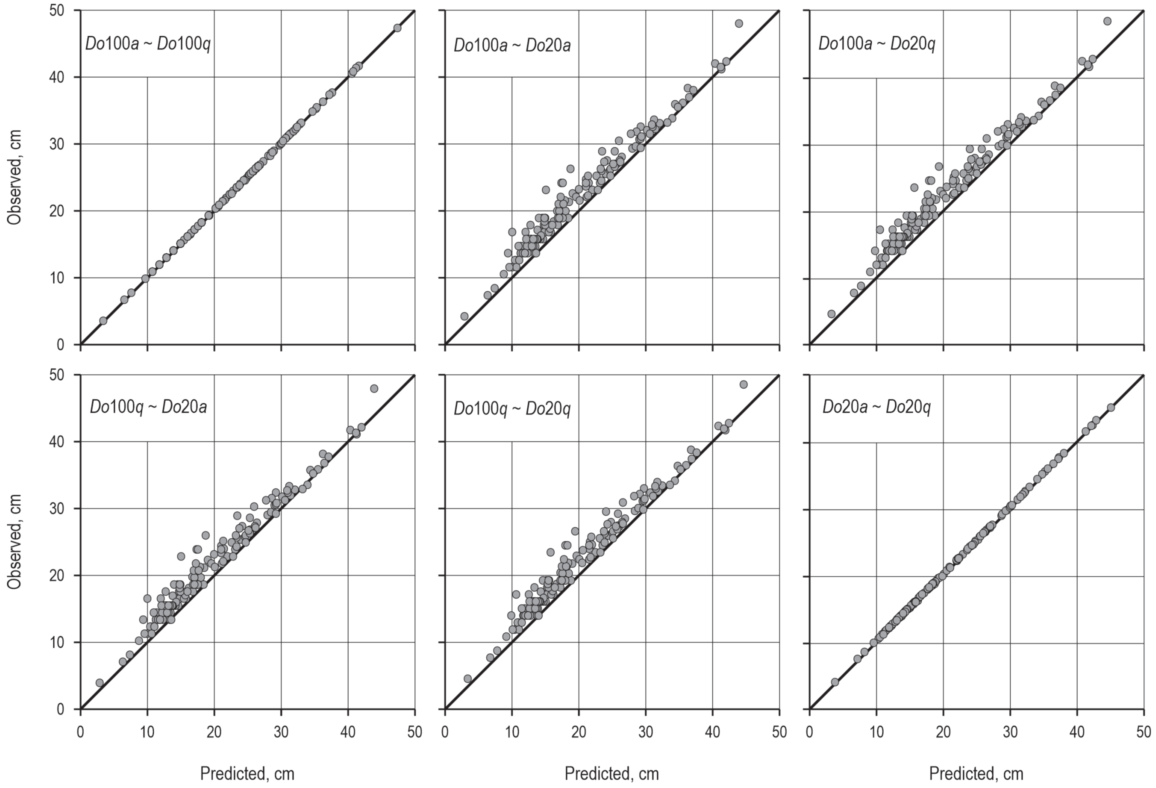


Fig. 1 Scots pine plantations

a. Comparison of differently defined dominant diameter values



b. Relative deviations *Difi*

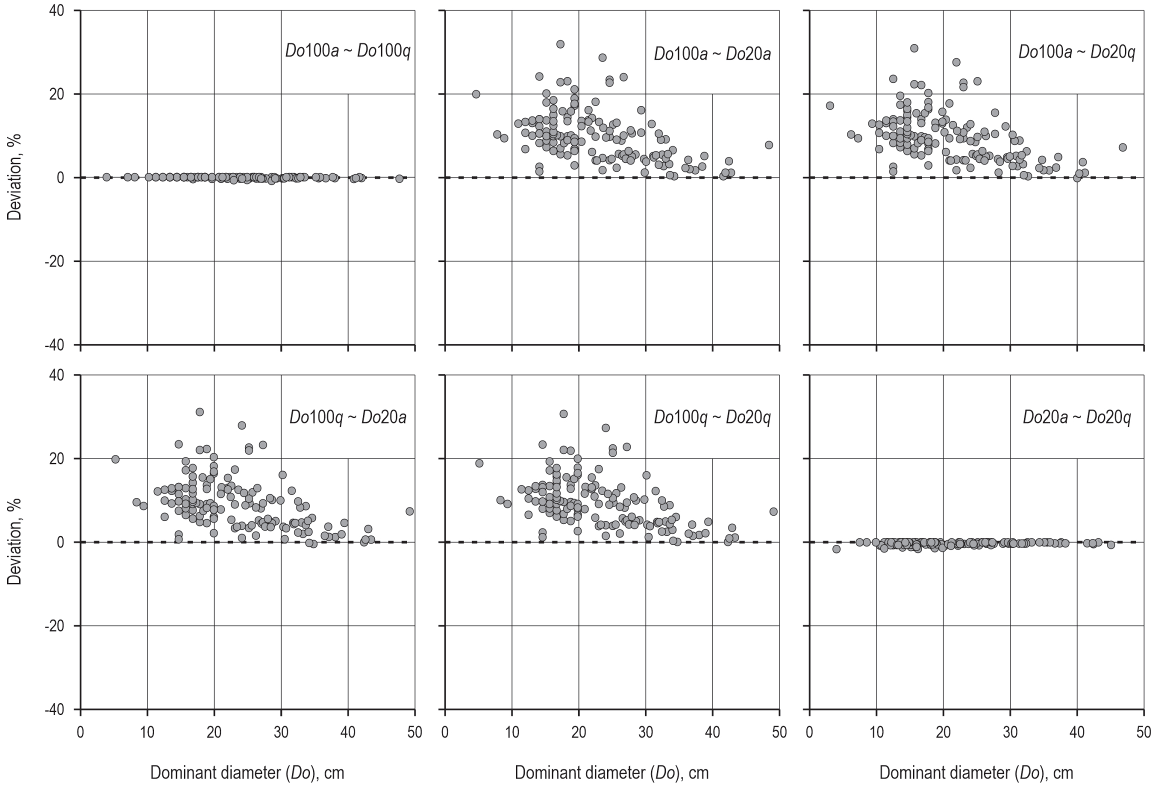
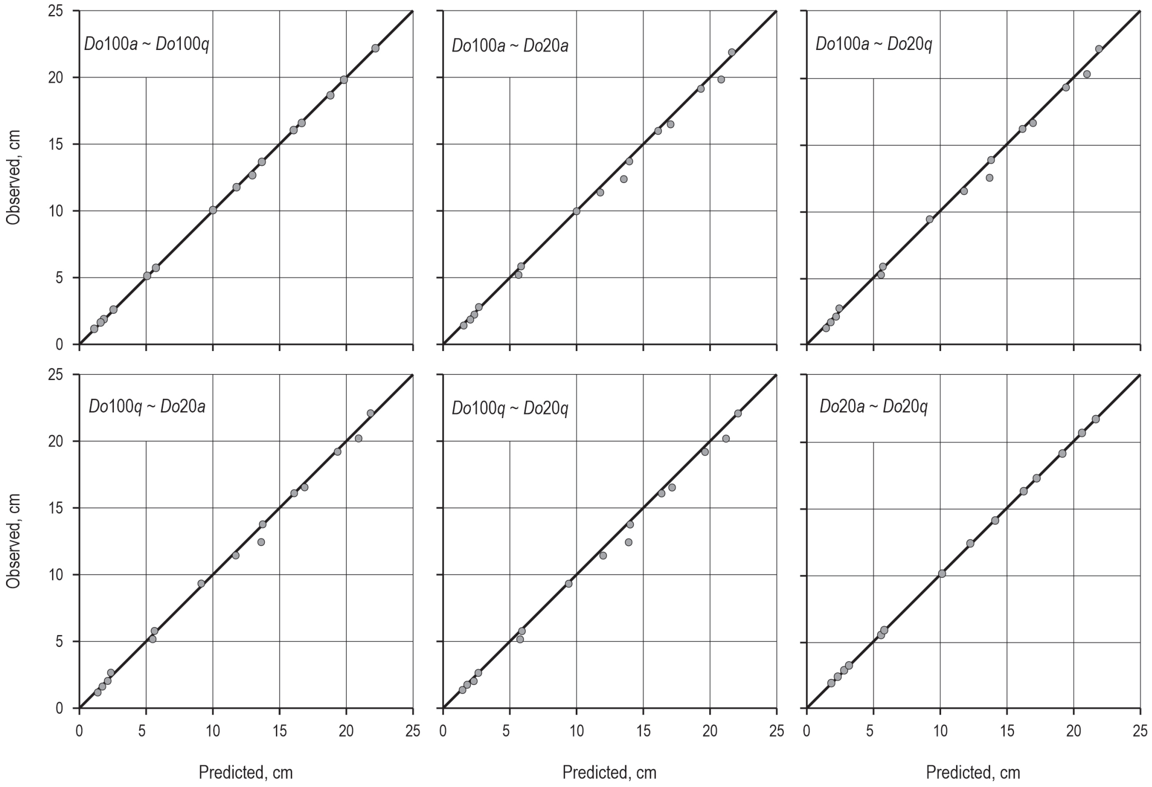


Fig. 2 Black pine plantations


a. Comparison of differently defined dominant diameter values

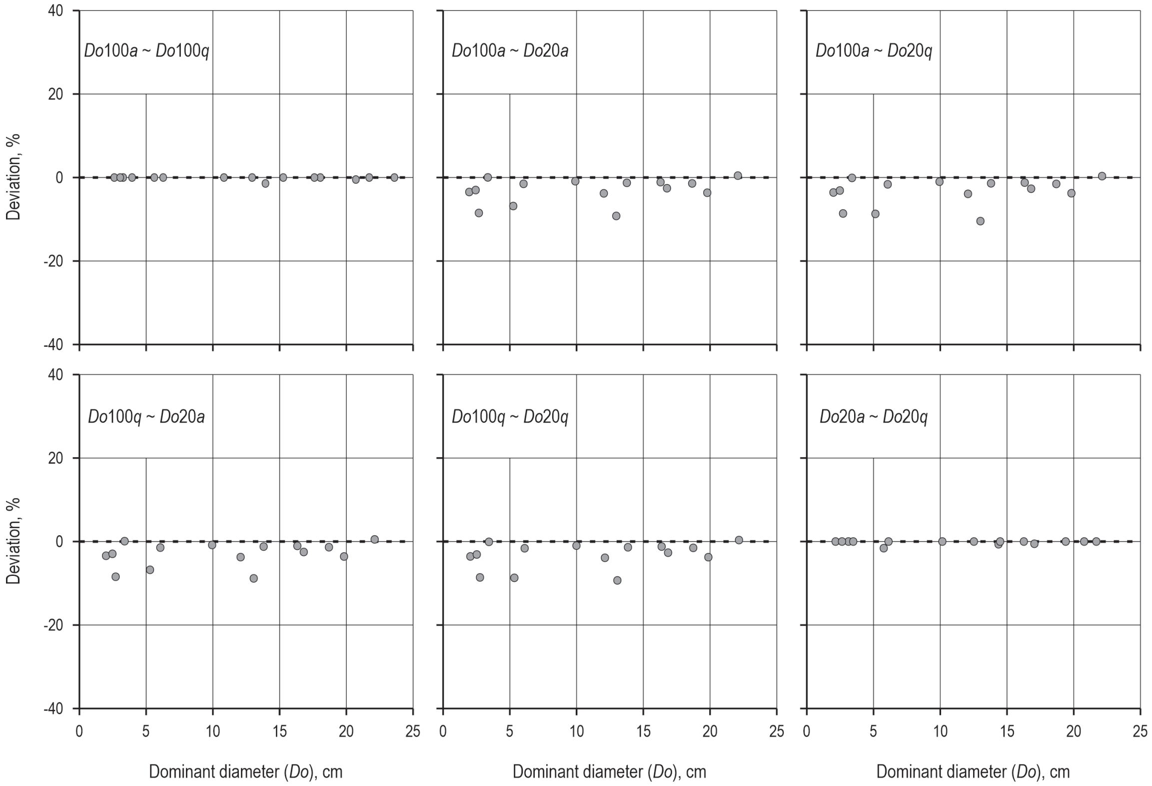
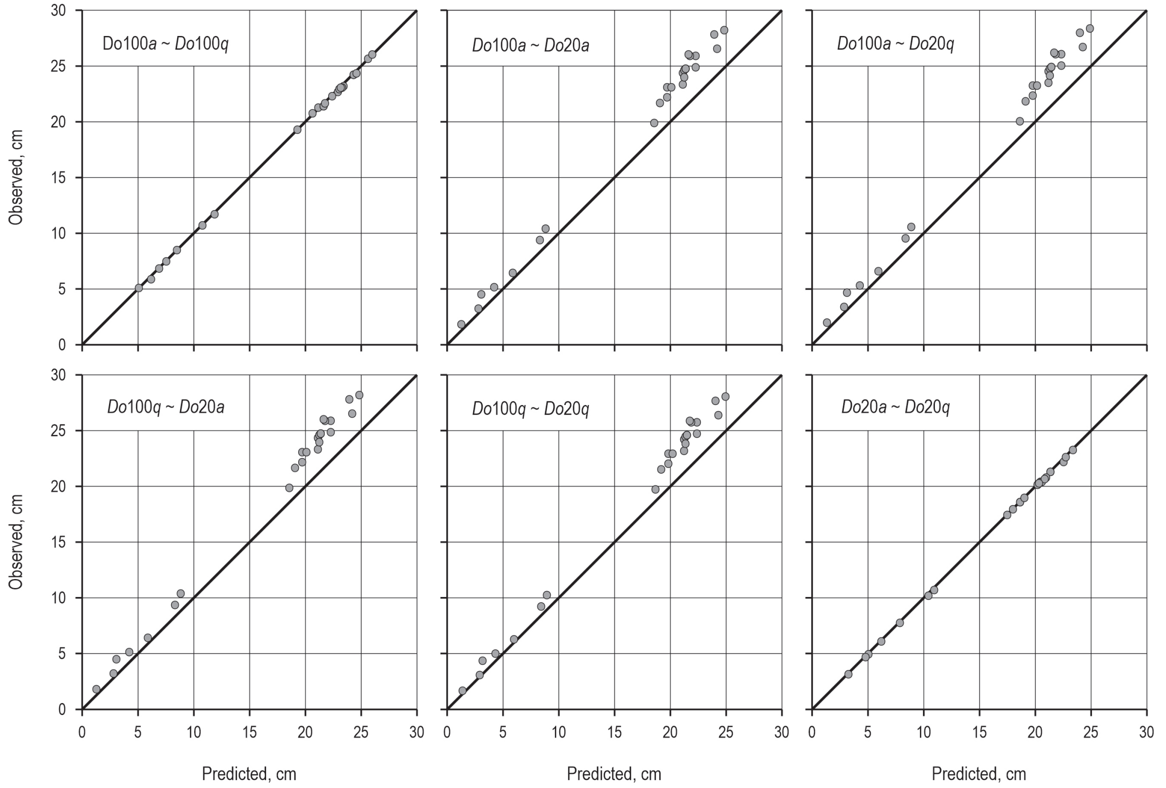

b. Relative deviations *Difi*

Fig. 3 Hybrid black poplar plantations

a. Comparison of differently defined dominant diameter values



b. Relative deviations *Difi*

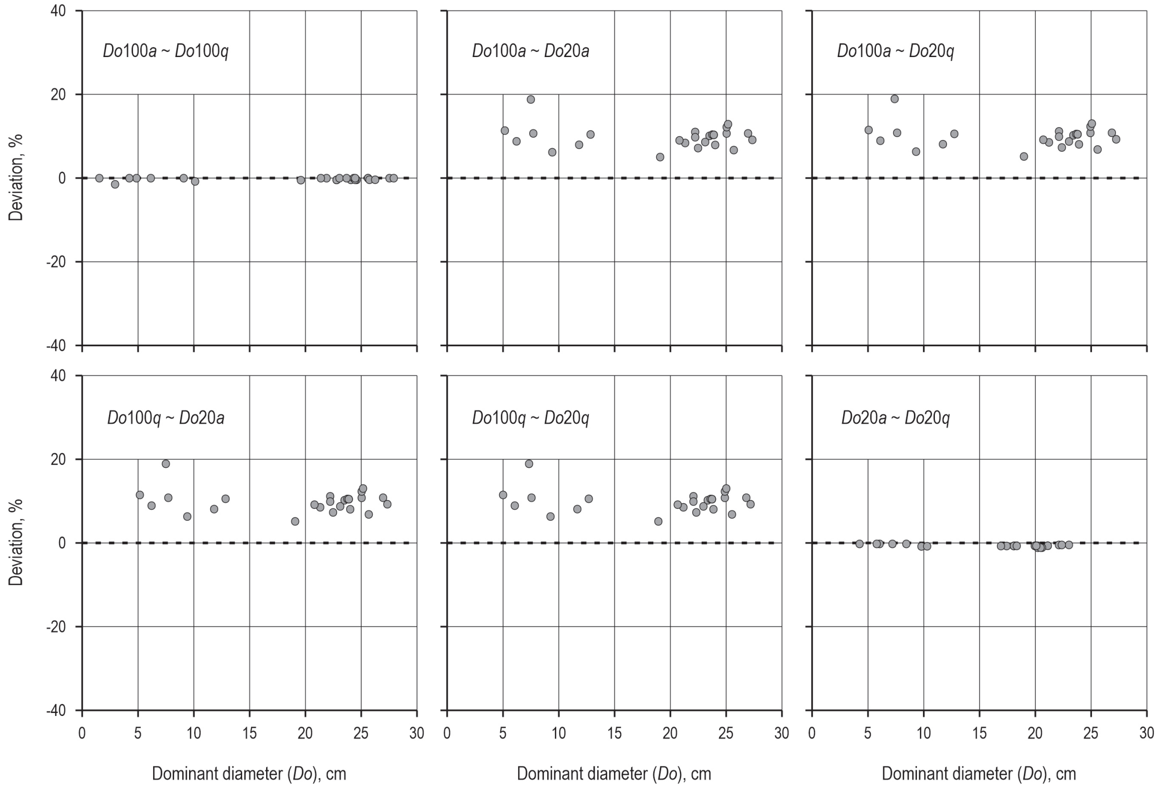


Fig. 4 Black locust plantations

Table 3 Dominant diameter prediction functions for Black pine plantations

<table>
<tr><td colspan="11">$ln(N) = a_0 + a_1Dq$ (LS)</td></tr>
<tr><td>Adj. R^2</td><td colspan="3">RMSE</td><td colspan="3">CF</td><td colspan="4">Regression Parameters</td></tr>
<tr><td rowspan="4">0.848</td><td colspan="3" rowspan="4">0.259</td><td colspan="3" rowspan="4">1.042</td><td colspan="2"></td><td>a_0</td><td>a_1</td></tr>
<tr><td colspan="2">Estimate</td><td>9.313</td><td>–0.091</td></tr>
<tr><td colspan="2">SE</td><td>0.060</td><td>0.003</td></tr>
<tr><td colspan="2">PRSE%</td><td>0.65</td><td>3.55</td></tr>
<tr><td colspan="11">$D_0100q = a_0 + a_1Dq + a_2G$ (GLS)</td></tr>
<tr><td>Adj. R^2</td><td colspan="6">Variance function</td><td colspan="4">Regression Parameters</td></tr>
<tr><td>0.951</td><td colspan="6">$(G^{\theta}Dq^{\eta})^2$</td><td></td><td>a_0</td><td>a_1</td><td>a_2</td></tr>
<tr><td></td><td colspan="6">Variance Function Parameters</td><td>Estimate</td><td>2.852</td><td>1.034</td><td>0.075</td></tr>
<tr><td>RMSE</td><td colspan="4">θ</td><td colspan="2">η</td><td>SE</td><td>0.387</td><td>0.022</td><td>0.010</td></tr>
<tr><td>1.689</td><td colspan="4">0.587</td><td colspan="2">–0.244</td><td>PRSE%</td><td>13.56</td><td>2.10</td><td>13.39</td></tr>
<tr><td colspan="11">$D_0100q = a_1Dq + a_2\ N(Dq)Dq^2$ (GLS)</td></tr>
<tr><td>Adj. R^2</td><td colspan="6">Variance function</td><td colspan="4">Regression Parameters</td></tr>
<tr><td>0.922</td><td colspan="6">$(\theta_1+Dq^{\eta_1})^2\ (\theta_2+(Dq^2\ N(Dq))^{\eta_2})^2$</td><td colspan="2"></td><td>a_1</td><td>a_2</td></tr>
<tr><td></td><td colspan="6">Variance Function Parameters</td><td colspan="2">Estimate</td><td>0.957</td><td>1.19x10^{-5}</td></tr>
<tr><td>RMSE</td><td>θ_1</td><td colspan="2">η_1</td><td colspan="2">θ_2</td><td>η_2</td><td colspan="2">SE</td><td>0.038</td><td>1.09x10^{-6}</td></tr>
<tr><td>2.149</td><td>9.78x10^{-8}</td><td colspan="2">–0.340</td><td colspan="2">1.51x10^{-21}</td><td>7.487</td><td colspan="2">PRSE%</td><td>4.00</td><td>9.15</td></tr>
<tr><td colspan="11">$D_020a = a_1Dq + a_2G$ (GLS)</td></tr>
<tr><td>Adj. R^2</td><td colspan="6">Variance function</td><td colspan="4">Regression Parameters</td></tr>
<tr><td>0.986</td><td colspan="6">$(Dq^{\eta_1})^2\ (\theta_2+G^{\eta_2})^2$</td><td colspan="2"></td><td>a_1</td><td>a_2</td></tr>
<tr><td></td><td colspan="6">Variance Function Parameters</td><td colspan="2">Estimate</td><td>1.163</td><td>0.039</td></tr>
<tr><td>RMSE</td><td colspan="2">η_1</td><td colspan="2">θ_2</td><td colspan="2">η_2</td><td colspan="2">SE</td><td>0.014</td><td>0.004</td></tr>
<tr><td>0.947</td><td colspan="2">0.703</td><td colspan="2">0.023</td><td colspan="2">–1.010</td><td colspan="2">PRSE%</td><td>1.162</td><td>10.964</td></tr>
<tr><td colspan="11">$D_020a = a_1Dq + a_2\ N(Dq)Dq^2$ (GLS)</td></tr>
<tr><td>Adj. R^2</td><td colspan="6">Variance function</td><td colspan="4">Regression Parameters</td></tr>
<tr><td>0.984</td><td colspan="6">$(\theta_1+\ (Dq^2\ N(Dq))^{\eta_1})^2\ exp(2\eta_2Dq)^2$</td><td colspan="2"></td><td>a_1</td><td>a_2</td></tr>
<tr><td></td><td colspan="6">Variance Function Parameters</td><td colspan="2">Estimate</td><td>1.109</td><td>4.40x10^{-6}</td></tr>
<tr><td>RMSE</td><td colspan="2">θ_1</td><td colspan="2">η_1</td><td colspan="2">η_2</td><td colspan="2">SE</td><td>0.021</td><td>5.60x10^{-7}</td></tr>
<tr><td>1.018</td><td colspan="2">1.46x10^{-17}</td><td colspan="2">-6.099</td><td colspan="2">9.49x10^{-3}</td><td colspan="2">PRSE%</td><td>1.890</td><td>12.780</td></tr>
</table>

Abbreviations: *GLS* – generalized least squares, *LS* – least squares, *Dq* – quadratic mean diameter (cm), *G* – stand basal area ($m^2 \cdot ha^{-1}$), *N* – stand density ($trees \cdot ha^{-1}$), D_020a – dominant diameter, estimated as the arithmetic mean of diameters of *n%*20 (20% of the thickest trees in the plot) thickest trees in the plot (cm), D_0100q – dominant diameter, estimated as the quadratic mean of diameters of *n*100 (number of trees in the plot corresponding to 100 thickest trees per hectare) thickest trees in the plot (cm), *CF* – ratio correction coefficient, *Adj.* R^2 – adjusted coefficient of determination, *RMSE* – residual standard error (cm), a_0, a_1, a_2, θ, η, θ_1, η_1, θ_2, η_2 – model parameters, *SE* – standard error, *PRSE*% — Parameter Relative Standard Error (%)

for the dominant diameter, based on the 20% of the largest trees (D_020a, D_020q) revealed higher accuracy than those for D_0100a and D_0100q. The goodness-of-fit of the predictions estimated for the quadratic and arithmetic mean by the same model were practically equivalent in all cases (Table 4). The regression models that included basal area as a second predictor showed slightly higher predictive potential, as indicated by the range and the magnitude of the relative errors and by the model efficiency assessed. Although manifestation of bias was registered for one of the regression equations, all models had very good accuracy, with model efficiency above 0.92, average absolute error below 8% (Table 4), with 90% of the relative errors less than 15% (5th percentile ≥–12%, 95th percentile ≤18%).

Table 4 Validation statistics of dominant diameter prediction functions for Black pine plantations

* Validated functions	ME	*MARE*%	*P*0	*P*25	*P*50	*P*75	*P*100	** *F* stat. (*df* = 2, *n*)
$D_0100q = 2.852 + 1.034Dq + 0.075G$	0.952	6.0	–16%	–6%	–1%	4%	23%	0.020^{NS}
$D_0100q = 0.957Dq + 1.19\text{x}10^{-5}\,N(\text{Dq})Dq^2$	0.922	7.1	–20%	–6%	0%	6%	36%	1.626^{NS}
$D_0100a = 2.852 + 1.034Dq + 0.075G$	0.952	6.0	–16%	–6%	–1%	4%	23%	0.018^{NS}
$D_0100a = 0.957Dq + 1.19\text{x}10^{-5}\,N(\text{Dq})Dq^2$	0.923	7.1	–20%	–6%	0%	6%	36%	1.456^{NS}
$D_020a = 1.163Dq + 0.039G$	0.986	3.4	–8%	–2%	0%	3%	25%	3.011’
$D_020a = 1.109Dq + 4.40\text{x}10^{-6}N(\text{Dq})Dq^2$	0.984	3.6	–9%	–3%	0%	3%	23%	0.839^{NS}
$D_020q = 1.163Dq + 0.039G$	0.985	3.5	–8%	–2%	0%	3%	26%	3.752*
$D_020q = 1.109Dq + 4.40\text{x}10^{-6}N(\text{Dq})Dq^2$	0.982	3.7	–9%	–2%	0%	3%	24%	2.002^{NS}

Abbreviations: *Dq* – quadratic mean diameter (cm), *G* – stand basal area ($\text{m}^2\cdot\text{ha}^{-1}$), *N* – stand density ($\text{trees}\cdot\text{ha}^{-1}$), D_020a – dominant diameter, estimated as the arithmetic mean of diameters of *n*%20 (20% of the thickest trees in the plot) thickest trees in the plot (cm), D_0100q – dominant diameter, estimated as the quadratic mean of diameters of *n*100 (number of trees in the plot corresponding to 100 thickest trees per hectare) thickest trees in the plot (cm), D_020q – dominant diameter, estimated as the quadratic mean of diameters of *n*%20 thickest trees in the plot (cm), D_0100a – dominant diameter, estimated as the arithmetic mean of diameters of *n*100 thickest trees in the plot (cm), *ME* – model efficiency, *MARE*% – average of absolute values of relative errors *ARE*%, *P*0, *P*25, *P*50, *P*75, *P*100 – 0th, 25th, 50th, 75th, and 100th percentile of relative errors *RE*%.

* $N(\text{Dq}) = 1.042exp(9.313 – 0.091Dq)$

** *F*-statistics and its significance with (2, *n*) degrees of freedom of the simultaneous test for slope equal to 1 and zero intercept of the linear regression relating the observed and predicted values. Levels of significance: *** – $P<0.001$, ** – $P<0.01$, * – $P<0.05$, ‘– $P<0.1$, *NS* – $P>0.1$

3.3 Objective 3: Goodness-of-fit of the Dominant Height Predictions, Based on Dominant – Quadratic Mean Diameter Relationships: Estimates for Scots Pine Plantations

Two adequate regression equations based on the quadratic mean diameter alone and two two-predictor models, including either basal area or stand density as a second independent variable, were developed to predict the D_0100q estimates for the Scots pine plantations (Table 5). The coefficients of determination assessed ranged between 0.94 and 0.955 and Root Mean Squared Errors between 1.72 and 1.99cm were estimated, with Percent Relative Standard Errors of the regression parameters being of magnitude below 17%.

When the predicted values of the dominant diameter were substituted into the height-diameter model with the validation data, all four regression equations yielded similar results in the dominant height predictions in terms of model efficiency that was as high as 0.95 and average absolute error that ranged from 5.2 to 5.6% (Table 6). The dominant height predictions from the basal area-including model were slightly biased, but the relative errors remained below 25% in all cases.

4. Discussion

The way of calculation of the number of trees, which are used for dominant diameter estimation, suggests that this number depends on stand density ($\text{trees}\cdot\text{ha}^{-1}$). For densities less than 500 $\text{trees}\cdot\text{ha}^{-1}$, *n*%20 obtains smaller values than *n*100, while the opposite is true for the denser stands. The dominant diameters based on the smaller subsample of trees will exceed those calculated from the bigger one; therefore, D_0100a and D_0100q will have larger values than D_020a and D_020q at densities higher than 500 $\text{trees}\cdot\text{ha}^{-1}$. Indeed, our results showed that the deviations Dif_2–Dif_5 obtained positive values for all, but the poplar data, collected in intensively managed plantations for timber, established at low stocking rates (Table 1). The study by Sharma et al. (2002) that compared differently defined dominant heights for loblolly pine plantations showed that the average height of the 100 thickest trees per hectare was significantly bigger than the average height of the 20% thickest trees at the 6 consecutive measurements of the unthinned stands taken over 15 years. In addition, at the time of the last measurement, the estimates of the dominant height according to the 100 thickest trees definition exceeded the values calculated by the other 6 definitions for both the thinned and unthinned stands (Sharma et al. 2002).

The tendency of decreasing deviations Dif_2–Dif_5 with the increase of diameter, observed for the coniferous plantations in our study, supports the idea of equivalency between the dominant diameter definitions based on differently defined tree subsamples at advanced age when the density decreases to around 500 trees per hectare and the 100 thickest trees account for 20% of the trees per hectare. Such equivalence, however, can rarely be observed with the short-rotation poplar plantations, with conventionally applied

Table 5 Dominant diameter prediction functions for Scots pine plantations

$D_0100q = a_1Dq + a_2N$ (LS)				
Adj. R^2		Regression Parameters		
0.948			a_1	a_2
		Estimate	1.299	6.21×10^{-4}
RMSE		*SE*	0.011	5.13×10^{-5}
1.849		*PRSE*%	0.813	8.256

$D_0100q = a_1Dq + a_2 N(\sim Dq)$ (LS)				
Adj. R^2		Regression Parameters		
0.946			a_1	a_2
		Estimate	1.298	7.641
RMSE		*SE*	0.011	0.664
1.895		*PRSE*%	0.848	8.686

$D_0100q = a_0 + a_1Dq + a_2G$ (GLS)								
Adj. R^2	Variance function				Regression Parameters			
0.955	$(\theta_1 + Dq^{\eta_1})^2 (\theta_2 + G^{\eta_2})^2$					a_0	a_1	a_2
	Variance Function Parameters				Estimate	3.371	1.047	0.062
RMSE	θ_1	η_1	θ_2	η_2	*SE*	0.336	0.022	0.010
1.719	0.032	–1.700	70.242	1.484	*PRSE*%	9.964	2.063	16.466

$D_0100q = a_1Dq + a_2Dq^2$ (GLS)						
Adj. R^2	Variance function			Regression Parameters		
0.940	$(\theta_1 + Dq^{2\eta_1})^2 (Dq^{\eta_2})^2$				a_1	a_2
	Variance Function Parameters			Estimate	1.679	–0.014
RMSE	θ_1	η_1	η_2	*SE*	0.029	0.001
1.986	2.70×10^{-39}	12.712	25.150	*PRSE*%	1.719	8.921

Abbreviations: *GLS* – generalized least squares, *LS* – least squares, *Dq* – quadratic mean diameter (cm), *G* – stand basal area ($m^2 \cdot ha^{-1}$), *N* – stand density ($trees \cdot ha^{-1}$), D_0100q – dominant diameter, estimated as the quadratic mean of diameters of n100 (number of trees in the plot corresponding to 100 thickest trees per hectare) thickest trees in the plot (cm), *Adj.* R^2 – adjusted coefficient of determination, *RMSE* – Residual Standard Error (cm), a_0, a_1, a_2, θ, η, θ_1, η_1, θ_2, η_2 – model parameters, *SE* – standard error, *PRSE*% – Parameter Relative Standard Error (%).

* $N(\sim Dq) = exp(-0.1Dq)$

planting schemes of growing space below 20 m² per tree.

Curtis and Marshal (2000) commented that, in stands of a narrow range of tree diameters, the difference between the quadratic and the arithmetic mean diameter is slight. Consequently, this should be even more valid when just the portion of the dominant trees is considered. Our study confirmed this conclusion since the difference between the quadratic and the arithmetic mean based on the same sample of trees did not exceed 2%. In addition, we found that the regression models derived for one of them predicted equally well the values of the other. Our results agree with those obtained by Ducey and Kershaw (2023), where D_0100a and D_0100q they investigated for complex stands were so similar that they were practically redundant. Therefore, in our opinion, quadratic and arithmetic mean dominant diameters, based on the same sample of trees, can be equivalently used for monospecific forest plantations, giving preference to the one that is easier to calculate.

Our study showed that the dominant stand diameter defined in various ways can be adequately expressed as a function of the quadratic mean diameter. The inclusion of the basal stand area as a predictor led to improved goodness-of-fit in some cases (e. g. with the Black pine data) but showed a tendency to produce biased estimates (*see* Tables 4 and 6). Fitting a simple linear regression to the quadratic mean diameter was an attractive alternative, but the estimated regression models showed a consistent violation of the requirement for normality of errors, with severely skewed

Table 6 Validation statistics for the dominant height based on the predicted dominant diameter for Scots pine plantations

Dominant height prediction function	Dominant diameter prediction function	ME	*MARE*%	*P*0	*P*25	*P*50	*P*75	*P*100	* *F stat.* (*df* = 2, *n*)
$H_0 100q = 1.3+(0.968H_m-0.748)\cdot e^{0.32(1-D_q/D_0\widehat{100q})+(1/D_q-1/D_0\widehat{100q})}$	$D_0\widehat{100q} = 1.299Dq + 6.21\text{x}10^{-4}N$	0.946	5.6	–24.3%	–5.8%	–2.2%	2.1%	23.0%	2.088^{NS}
	$D_0\widehat{100q} = 1.298Dq + 7.641exp(-0.1Dq)$	0.946	5.2	–16.4%	–4.0%	–0.5%	4.9%	24.8%	0.974^{NS}
	$D_0\widehat{100q} = 3.371 + 1.047Dq + 0.062G$	0.946	5.6	–22.9%	–5.9%	–2.3%	1.3%	22.4%	3.653*
	$D_0\widehat{100q} = 1.679Dq - 0.014Dq^2$	0.948	5.5	–20.3%	–5.9%	–2.6%	22.4%	6.9%	2.999’

Abbreviations: *Dq* – quadratic mean diameter (cm), *G* – stand basal area ($m^2 \cdot ha^{-1}$), *N* – stand density ($trees \cdot ha^{-1}$), D_0100q – dominant diameter, estimated as the quadratic mean of diameters of n100 (number of trees in the plot corresponding to 100 thickest trees per hectare) thickest trees in the plot (cm), H_0100q – dominant stand height (m) estimated as the Lorey's mean of heights of n100 thickest trees in the plot (m), *ME* – model efficiency, *MARE*% – average of absolute values of relative errors *ARE*%, *P*0, *P*25, *P*50, *P*75, *P*100 – 0th, 25th, 50th, 75th, and 100th percentile of relative errors *RE*%.

* *F*-statistics and its significance with (2, *n*) degrees of freedom of the simultaneous test for slope equal to 1 and zero intercept of the linear regression relating the observed and predicted values. Levels of significance: *** – $P<0.001$, ** – $P<0.01$, * – $P<0.05$, ‘– $P<0.1$, *NS* – $P>0.1$

residual distribution for all dominant diameter definitions and both datasets from pine plantations. The intercept of the line was alternatively expressed by exponential or quadratic term of the quadratic mean diameter (with the Scots pine data) or by their product (with the Black pine data), reflecting in this way the modifying effect of stand density.

The mean stand height is a stand-level attribute commonly used in the forest management practice in Bulgaria, while for dominant height estimation there is no officially established protocol. Duhovnikov (1972) preferred the dominant height definition as the height corresponding to the average diameter of the 20% thickest trees in the stand. Petrin (1987) and Tonchev (2022) favored the definition of the dominant height corresponding to the quadratic mean diameter of the 100 thickest trees per hectare. Shikov (1974) and Ferezliev and Tsakov (2010), on the other hand, gave their preference to the dominant height as an arithmetic average of the heights of the 100 thickest trees per hectare. Stankova et al. (2022) developed a height-diameter model for Scots pine plantations in Bulgaria based on tree diameter, average stand height and diameter, and validated the model for dominant stand height prediction from differently defined dominant diameters. Our study suggested a dominant diameter prediction from the average stand diameter, showing that its substitution into the height-diameter model yields accuracy comparable to that when the experimental dominant diameter values were used (*see* Table 4 in Stankova et al. 2022 and Table 6 of this study). Consequently, it can be concluded that the stand attributes of the monospecific plantations that are based on the dominant trees (i.e. dominant stand diameter and height) can be confidently estimated from the average stand parameters.

Wang et al. (2024) studied the influence of 37 different measures of stand height and diameter on the accuracy of estimation of stand volume from allometric equations. The authors found that, when density was removed from the equation, the height and diameter measures that best predicted stand volume shifted from moment estimators (i. e. the arithmetic and quadratic means) to the largest tree estimators (i. e. estimates based on largest tree collectives). They concluded that this outcome probably reflects the proportionally greater contribution of the large trees to the total volume than the small trees, which becomes particularly important when density is not known. However, their study showed that the best performing models without density caused errors 3–6 times greater than the best performing models with density. In line with these observations and in agreement with the notion of dominant stand height as the most common phytocentric measure of site productivity (Skovsgaard and Vanclay 2008, Weiskittel et al. 2011), our results suggest a practically applicable approach to estimate the site index, based on dominant height growth model, in Bulgaria (e. g. Stankova et al. 2024) using data, which are readily available in the stand inventory descriptions (i. e. average stand height and diameter).

5. Conclusions

Dominant stand diameter values, estimated as either arithmetic or quadratic mean from the same portion of the largest trees in the stand, are practically equal and can be used interchangeably. However, dominant stand diameters calculated from differently defined largest tree collectives may differ by as much as 35%. The difference decreases with advancing stand growth stage and with the progress of self-thinning, but not for intensively managed industrial plantations of density below 500 trees·ha^{-1}. Regardless of its definition, the dominant stand diameter can be adequately predicted from a function of the quadratic mean diameter alone or considering the stand basal area as a second independent variable. Its predicted value can be confidently used as a predictor in a height-diameter model to estimate the dominant stand height of a monospecific forest plantation, allowing the forecast of the stand attributes based on dominant trees when only average stand variables are known.

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