SEEFOR SOUTH-EAST EUROPEAN FORESTRY

OPEN ACCESS
ISSN 1847-6481
eISSN 1849-0891

ORIGINAL SCIENTIFIC PAPER

DOI: https://doi.org/10.15177/seefor.24-16

Predicting Diameter Distributions in Mixed Forests in Southern Mexico

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Citation: Guzmán-Santiago JC, de los Santos-Posadas HM, Vargas-Larreta B, Gómez-Cárdenas M, Santiago-García W, Nava-Nava A, 2024. Predicting Diameter Distributions in Mixed Forests in Southern Mexico. *South-east Eur for* 15(2): 161-173. https://doi.org/10.15177/seefor.24-16.

Received: 18 Aug 2024; **Revised:** 19 Oct 2024; **Accepted:** 29 Oct 2024; **Published online:** 6 Dec 2024

ABSTRACT

Understanding the diameter structure of a stand is crucial for making informed decisions regarding silviculture and forest management. This is achieved by collecting forest inventory data and applying them to probability density functions. The objective of this study was to identify the optimal probability density function for describing the diameter distribution in a mixed temperate forest in Oaxaca, Mexico. Circular sampling plots of 1000 m² were established, and the diameter at breast height (DBH) and total height of 16,863 trees of the 25 most important timber species were measured. Four probability density functions (Weibull 2P, Log-Normal, Log-Logistic, and Burr) were fitted by using maximum likelihood approach for different silvicultural treatments. The predictive ability of the functions was evaluated using bias, root mean square error, Kolmogorov-Smirnov statistic and graphical analyses. In terms of accuracy and parsimony, the Burr function demonstrated the best performance, followed by the Log-Normal and Log-Logistic functions. Consequently, the Burr function is recommended for modeling the diameter distribution of stands under different treatments in the study site.

Keywords: Burr's function; diameter distribution; goodness of fit; uneven-aged forests; silvicultural treatment

INTRODUCTION

By employing a range of silvicultural techniques, forest management enables the planning of actions that modify forest structures in accordance with pre-established objectives. In this context, diameter distribution models serve as useful tools for projecting different diameter classes present in a stand. These models facilitate the disaggregation of volume and allow for the calculation of product distribution to be extracted, as well as the potential value of the harvest (Bettinger et al. 2022). Diameter distribution analysis is directly related to forest structure, age, species composition, and site index (Bailey and Dell 1973). It also provides insight into the ecological aspects of species (Dardengo et al. 2017), and regulatory control can be implemented by assessing basal area and volume (Zhang et al. 2001, Nord-Larsen and Cao 2006). The analysis of diameter distributions provides information on the typologies and intensity of natural regeneration of species and the measurement of growth (Scolforo 2006). This, in turn, facilitates the recommendation of appropriate silvicultural treatments for different tree growth conditions (Gorgoso-Varela et al. 2020).

The use of probability density functions (PDF) in forestry has a long history, dating back to the 1970s (Bailey and Dell 1973). PDF are highly recommended for stand characterization and harvest planning (Maltamo et al. 1995, Wang and Rennolls 2005, Pérez-López et al. 2019). They provide the estimation of tree frequency by diameter category. By relating its parameters to stand attributes such as height, diameter at breast height (DBH), quadratic mean diameter, and basal area, it is possible to dynamically predict the diameter distribution (Cao 2004, Quiñonez-Barraza et al. 2015, Vega et al. 2022).

Forestry literature reports the use of PDF in silviculture for both even-aged and uneven-aged forests (Maltamo et al. 1995), which facilitate the estimation of diameter

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distributions. These include the Weibull, Normal, Log-Normal, Beta, Johnson's SB, Gamma, and Log-Logistic functions. Among these, the Weibull function is the most commonly used, as evidenced by the research by Álvarez and Ruiz (1998), Fidalgo et al. (2009), and Ogana (2020).

There is a lack of such studies in Mexico, particularly in mixed forests with high diversity. This is a gap in knowledge that needs to be addressed since it hinders the proposal of functions for forests in the south of the country. The results of this study will provide valuable insights into the actual stocks, product distribution, and timber yield, significantly contributing to the sustainable management of forest resources.

The objective of this study was to compare four PDF and to identify the optimal probability density function for estimating the diameter distribution in a mixed temperate forest in southern Oaxaca, Mexico.

MATERIALS AND METHODS

Site

The study was carried out in mixed stands located in the municipality of San Juan Quiahije, in the southwest of the state of Oaxaca. According to the National Agrarian Registry (NAR), the municipality area comprises 24,914 ha. It is located at the extreme coordinates of 16°17'46" to 16°21'33" N and 97°17'54" to 97°25'53" W, with elevations ranging from 680 to 1970 m (INEGI 2023a) (Figure 1). According to environmental information from INEGI (2023b), the orography of the study site is rugged, with slopes ranging from 15% to 60%.

Data Collection with SiPlaFor

The data used in the study were obtained from the Inventory of the Forest Management Unit number 2012, where the Mexican Method Management for Uneven-aged Forests is applied. A total of 450 circular sampling plots, each of 1,000 m^2 size, were established using randomized (block) design (SiPlaFor 2015), covering an area of 3,043 ha. The sample size comprises 16,863 trees, representing 25 tree species. During the field campaign, on each plot tree height was measured with a Haga® hypsometer (Haga GmbH and Co. KG, Nuremberg, Germany) (Cho et al. 2009), and the tree DBH for trees with minimal DBH of 5 cm was measured with a diameter tape Model 283D.

The largest measured tree dimensions belong to *Pinus maximinoi, P. douglasiana* and *P. oocarpa*, which exceeded 100 cm in DBH and 35 m in height, confirming them as dominant species. Most species have leptokurtic diameter distribution, while some such as *Quercus candicans* and *Q. castanea*, have platykurtic distribution. The positive

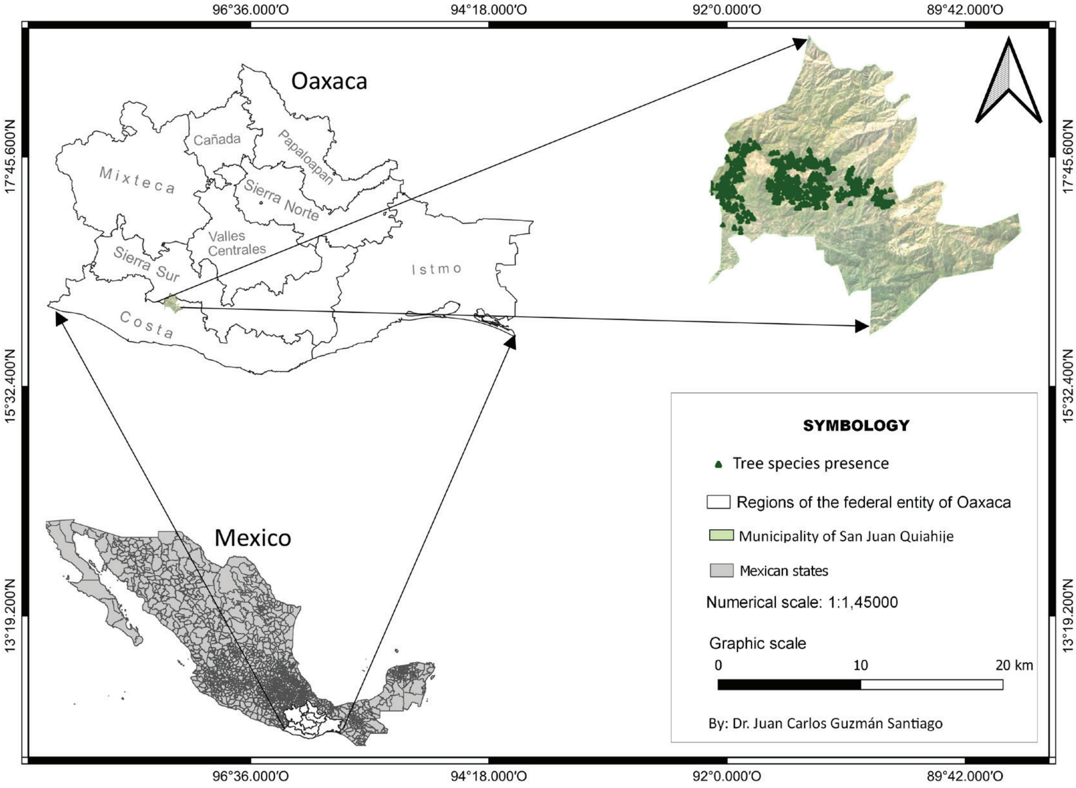

Figure 1. Location of the Oaxaca study site.

asymmetry in diameter distribution was found for all species, except for *Pinus ayacahuite*, showing the negative skew (Table 1).

Silvicultural Treatments in Stands

Silviculture consists of treatments such as regeneration cuts and intermediate cutting, which are essential for managing the diameter structure of forest species (Daniel et al. 1982, Nyland 2002). In the site area, the Mexican Method Management for Uneven-aged Forests was used, aimed at applying selective cutting to promote the maintenance of an uneven-aged structure composed of trees of different dimensions (Hernández-Díaz et al. 2008, Torres-Rojo et al. 2016). Given the presence of uneven-aged stands in the site area, silvicultural work has been implemented, including thinning activities (1st thinning, 2nd thinning, and 3rd thinning), which involve eliminating poor quality individuals without affecting the entire stand and thus favoring the most vigorous trees. Additionally, regeneration cutting (RGC) has been implemented to support thinning, eliminating competing trees that surround the selected ones, in addition to complementing with release cutting (RC) that allows light to enter the selected areas of the canopy and facilitates the natural regeneration of the species (Hernández-Díaz et al. 2008).

There are also unavailable forests (ND, segregated) intended to conserve and thus maintain ecological functions (biodiversity). Group selection cutting (SELG) is another important treatment. This method maintains the forest structure (age and size) by removing groups of tree species in a controlled manner, creating space for natural regeneration (Puettmann et al. 2016, Torres-Rojo et al. 2016).

In all treatments, the most dominant species were *Pinus maximinoi* and *P. oocarpa.* Those species were even more predominant in areas not segregated (ND) by moderate forest harvesting and in group selection cutting (SELG), where an average DBH of 23 cm was recorded. However, the SELG cutting resulted with a greater range of diameter class,

Table 1. Descriptive statistics of the main tree species used in the analyses.

Tree species	n	Diameter at breast height - DBH (cm)						Total height - Ht (m)					
		Max	Mean	Min	Std. Dev.	Kurt	Ac	Max	Mean	Min	Std. Dev.	Kurt	Ac
Pinus pseudostrobus var. *apulcensis*	47	75.00	18.15	8.00	13.92	6.06	2.40	29.00	11.11	5.00	5.27	3.13	1.70
Pinus douglasiana	1535	120.00	26.93	6.00	17.14	3.54	1.73	45.00	15.64	2.00	6.81	1.05	0.90
Pinus maximinoi	5540	125.00	26.16	6.00	16.01	2.27	1.40	43.00	16.33	2.00	5.51	0.34	0.56
Pinus montezumae	79	72.00	32.01	11.00	15.54	-0.75	0.50	36.00	17.59	5.00	7.57	-0.94	0.15
Pinus ayacahuite	19	39.00	26.00	9.00	8.45	-0.13	-0.56	26.30	15.24	4.80	5.56	-0.18	-0.09
Pinus oocarpa	4105	115.00	26.00	6.00	15.38	2.73	1.48	37.50	14.38	3.00	5.86	0.10	0.61
Pinus devoniana	43	42.00	20.56	9.00	8.56	-0.10	0.92	19.00	10.65	4.00	3.69	-0.18	0.72
Pinus lawsonii	39	33.00	15.72	8.00	7.45	0.21	1.10	25.00	14.62	7.00	4.42	0.55	0.89
Pinus pringlei	280	84.00	20.13	7.00	13.22	3.63	1.77	30.00	14.65	5.00	5.28	-0.12	0.49
Pinus teocote	492	96.00	24.87	7.00	15.17	2.87	1.55	33.00	12.79	4.00	5.31	0.24	0.68
Quercus acutifolia	862	88.00	20.60	7.00	12.22	3.14	1.66	28.00	10.15	2.00	4.57	0.29	0.75
Quercus candicans	39	38.00	20.15	10.00	7.25	-0.11	0.69	17.00	11.95	4.00	3.18	-0.20	-0.45
Quercus castanea	216	50.00	21.31	8.00	9.08	-0.02	0.77	18.00	8.74	4.00	3.12	-0.06	0.58
Quercus conspersa	145	55.00	21.74	8.00	10.44	0.35	0.96	21.00	9.98	4.00	3.30	0.09	0.51
Quercus crassifolia	190	55.00	20.71	8.00	9.65	1.19	1.20	20.00	8.82	3.00	3.50	1.21	1.12
Quercus elliptica	600	90.00	22.31	8.00	13.87	3.43	1.73	29.00	10.32	2.00	4.99	1.01	1.21
Quercus frutex	58	50.00	20.02	8.00	10.94	0.82	1.27	19.00	10.78	5.00	3.76	-0.75	0.44
Quercus laeta	212	70.00	19.04	7.00	10.98	4.91	2.04	19.00	8.43	3.00	3.13	1.71	1.31
Quercus magnoliifolia	114	45.00	18.67	7.00	8.77	0.82	1.09	25.00	9.42	2.00	4.21	1.13	1.08
Quercus martinezii	31	28.00	11.94	8.00	4.12	6.70	2.20	12.00	8.29	5.00	1.88	-0.69	-0.23
Quercus ocoteifolia	85	40.00	15.95	9.00	7.20	2.41	1.66	17.00	10.78	6.00	2.46	-0.63	0.42
Quercus rugosa	102	60.00	20.25	8.00	9.57	2.79	1.46	18.00	8.17	4.00	3.02	1.18	1.13
Quercus scytophylla	1372	94.00	19.66	7.00	11.79	6.19	2.13	38.00	11.71	3.00	4.80	2.48	1.28
Arbutus xalapensis	44	53.00	17.00	8.00	9.80	4.07	1.91	20.00	9.45	4.00	4.41	0.03	1.01
Clethra mexicana	614	70.00	16.58	5.00	9.44	5.01	2.09	30.00	11.61	2.00	4.70	1.00	0.86

n – number of trees, Max – maximum; Min – minimum, Std. dev. – standard deviation, Kurt – kurtosis, Ac – asymmetry coefficient.

ranging from 6 to 125 cm. In this context, the majority of the forest stands are dominated by conifers across each of the silvicultural treatments, which was fundamental for the fitting and analysis of the functions (Table 2).

Probability Density Functions (PDF)

In the diameter distribution analysis, four PDF recommended in the studied literature (de Lima et al. 2017, Gorgoso-Varela et al. 2020, Ogana 2020, de Lima et al. 2023, Gorgoso-Varela et al. 2024) were fitted to describe the diameter distribution of the forest stands (Table 3).

Method for Fitting and Selection of the Functions

The maximum likelihood (MLE) method was used to estimate the parameters of each PDF using the fitdistrplus (Müller and Dutang 2015) and MASS (Venables and Ripley 2002) packages in the Rstudio programming environment (R Development Core Team 2023). The MLE approach consists of estimating the parameters by maximizing the probability of obtaining a sample from the theoretical distribution that matches the observed distribution (Bailey and Dell 1973, Ciceu et al. 2021, Gorgoso-Varela et al. 2021).

The analysis of the metrics to select the best function consisted of numerical and graphical comparisons of the bias (Bias, Equation 1) and the root mean squared error (RMSE, Equation 2), where it is expected that the values approach zero.

$$Bias = \frac{\sum_{i=1}^{n}\left(F(x_i) - \hat{F}(x_i)\right)}{n} \quad (1)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(F(x_i) - \hat{F}(x_i)\right)^2}{n-p}} \quad (2)$$

where $F_{(xi)}$ is the number of observed trees in diameter class *i*, $\hat{F}_{(xi)}$ is the number of estimated trees in diameter class *i*, *n* is the number of observations and *p* is the number of parameters of the PDF.

Bias and RMSE values were estimated for each PDF as the average of the relative frequencies of each diameter class (at 5 cm intervals) for each forest inventory plot, stand, and silvicultural treatment. The Kolmogorov-Smirnov statistic (D_n) was used to validate the similarity between the empirical cumulative $F_n(x_i)$ and theoretical $F_0(x_j)$ frequencies: $D_n{:}sup_x|F_n(x)\text{-}F_0(x)|$, where sup_x is the maximum of the sum of the set of distances. This value was calculated as follows (Equation 3, Cao 2004):

$$D_n{:}\,max\left\{\max_{1\le i\le ni}\left[F_n(x_i) - F_0(x_j)\right], \max_{1\le i\le ni}\left[F_0(x_j) - F_n(x_{i-1})\right]\right\} \quad (3)$$

The following expression was used to compare the obtained value (D_n) with the critical value proposed by Miller

Table 2. Situation after silvicultural treatments using the Mexican Method Management for Uneven-aged Forests.

Treatment type	Number of plots	Number of trees			Minimum DBH (cm)	Average DBH (cm)	Maximum DBH (cm)
		Total	**Pine** (%)	**Broadleaf** (%)			
1Thin	47	1868	77	23	6.00	23.01	120.00
2Thin	42	1616	71	29	7.00	24.05	109.00
3Thin	31	1051	74	26	7.00	26.01	112.00
RC	31	1398	76	24	8.00	22.74	89.00
RGC	61	1978	71	29	7.00	25.48	115.00
ND	119	4147	72	28	6.00	24.36	110.00
SELG	119	4805	70	30	5.00	23.23	125.00

1Thin – first thinning; 2Thin – second thinning; 3Thin – third thinning; RC – release cutting; RGC – regeneration cutting; ND – not available (separate); SELG – group-selection cutting; DBH – diameter at breast height.

Table 3. Probability density functions (PDF) fitted to the data.

Name	Probability Density Function	Number
Weibull 2P	$f(x)=\left(\frac{\gamma}{\beta}\right)\left(\frac{x}{\beta}\right)^{\gamma-1}\exp\left[-\left(\frac{x}{\beta}\right)^{\gamma}\right]$	1
Log-Normal	$f(x)=\frac{1}{x\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\ln(x)-\mu}{\sigma}\right)^2\right]$	2
Log-Logistic	$f(x)=\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\left[1+\left(\frac{x}{\beta}\right)^{\alpha}\right]^{-2}$	3
Burr	$f(x)=\frac{\alpha k\left(\frac{x}{\beta}\right)^{\alpha-1}}{\beta\left(1+\left(\frac{x}{\beta}\right)^{\alpha}\right)^{k+1}}$	4

Weibull 2P: x – value of the random (independent) variable; β – scale parameter; γ – shape parameter. **Log-Normal**: x – value of the random variable; μ – average of x; σ – variance of x; π –the constant "pi" = 3.1416. **Log-Logistic**: x – value of the random variable; β – scale parameter; α – shape parameter. **Burr**: x – value of the random variable; k and α – shape parameter; β – scale parameter.

(1956); the probability error was determined to be 0.05 (Equation 4, Gorgoso-Varela et al. 2018):

$$D_{n,\alpha}: \sqrt{\frac{-ln\left(\frac{1}{2}\cdot\alpha\right)}{2\cdot n}} \tag{4}$$

where *ln* is the natural logarithm, α is the significance level, and *n* is the number of samples considered in each silvicultural treatment. If $D_n > D_{n,\alpha}$, the null hypothesis that the data distribution follows a theoretical distribution is rejected (Gorgoso-Varela et al. 2018).

RESULTS AND DISCUSSION

Evaluation of Probability Density Functions (PDF)

Table 4 shows the most commonly used goodness-of-fit metrics for each PDF by silvicultural treatment with a 95% confidence level. Reliable values were observed for both the bias and RMSE, with biases being consistently below zero.

The Burr function (see Table 4 and Figure 3) is considered the most suitable for describing the diameter distribution of the stands under different silvicultural treatments, followed by the Log-Normal and Log-Logistic functions (Table 4). The estimated parameters for each function and treatment are shown in Table A1 (Appendix A). While the Burr function is preferred, the Log-Normal and Log-Logistic functions are also viable alternatives, with the Log-Logistic slightly outperforming the Log-Normal in projections. The choice of the Burr PDF is supported by Sahin and Ercanli (2023), who found higher fit values among 14 PDF for northern Turkey forests, noting RMSE values between 27.60 and 71.06, even when using the same functions such as Weibull 2P and Log-Normal.

Similarly, Siipilehto and Mehtätalo (2013), Diamantopoulou et al. (2015) and Schmidt et al. (2020) reported significant

Table 4. Goodness-of-fit statistics of the probability density functions fitted to data from different silvicultural treatments.

PDF	Treatment	D*	$D_{5\%}$*	Rejected plots	Bias*	RMSE*
Weibull	1Thin	0.19	0.23	15	0.0252	4.11
	2Thin	0.17	0.23	6	0.0223	3.75
	3Thin	0.17	0.25	5	0.019	2.71
	RC	0.17	0.21	4	0.0161	3.60
	RGC	0.17	0.27	8	0.0261	3.26
	ND	0.18	0.25	23	0.0228	2.82
	SELG	0.18	0.23	28	0.0215	3.73
Log-Normal	1Thin	0.18	0.23	9	0.0182	3.23
	2Thin	0.16	0.23	4	0.0177	3.25
	3Thin	0.16	0.25	3	0.0147	2.13
	RC	0.14	0.21	4	0.0098	2.75
	RGC	0.16	0.27	2	0.0225	2.37
	ND	0.17	0.25	14	0.0175	2.24
	SELG	0.17	0.23	16	0.0171	2.99
Log-Logistic	1Thin	0.15	0.23	6	0.0183	3.01
	2Thin	0.15	0.23	1	0.0193	3.15
	3Thin	0.14	0.25	0	0.0155	1.98
	RC	0.13	0.21	3	0.0113	2.71
	RGC	0.15	0.27	2	0.0241	2.21
	ND	0.16	0.25	8	0.0195	2.19
	SELG	0.16	0.23	1	0.0188	2.83
Burr	1Thin	0.14	0.23	3	0.0177	2.57
	2Thin	0.15	0.23	2	0.0168	2.52
	3Thin	0.14	0.25	0	0.0144	1.92
	RC	0.13	0.21	2	0.0109	2.49
	RGC	0.15	0.27	1	0.0231	2.04
	ND	0.15	0.25	10	0.0179	1.92
	SELG	0.15	0.23	5	0.0168	2.50

*average values; RMSE – root mean square error; PDF – probability density function; D – Kolmogorov-Smirnov statistic; $D_{5\%}$ – critical value of the Kolmogorov-Smirnov test at a significance level of 5%. 1Thin – first thinning; 2Thin – second thinning; 3Thin – third thinning; RC – release cutting; RGC – regeneration cutting; ND – not available (separate); SELG – group-selection cutting.

modeling errors, highlighting the challenges in accurately modeling diameter distributions in forestry.

Diameter Distribution with PDF

Diameter distribution is important for analyzing forest growth since it allows understanding the structure and dynamics of the tree population within an ecosystem through forest inventory. It provides detailed information on tree density (de Lima et al. 2023) and helps diagnose mortality, regeneration, and other events affecting the forest stand (Imaña-Encinas et al. 2009).

Our findings confirm the situation that most trees are grouped in the 5 to 30 cm diameter range (Figure 2), indicating effective regeneration due to forest management activities. Appropriate decision-making tools, such as PDF, are essential in this context. The Burr function was found to accurately describe all diameter classes for each treatment. However, it slightly overestimated diameter classes from 10 to 20 cm for the 1Thin and SELG treatments (Figure 3). While there have been studies on diameter distribution in Mexico (Corral-Rivas et al. 2015, Quiñonez-Barraza et al. 2015), southern Mexican forests are still not covered enough by this type of research (Pérez-López et al. 2019, Nava-Nava et al. 2024).

The Log-Logistic and Log-Normal PDF performed well in modeling the data, particularly for the 1Thin, RGC, ND, and SELG treatments. The Log-Normal function has also shown significant results in other studies (de Brito et al. 2022). However, Maltamo (1997) suggests that species-fitted functions often provide even better results.

The Weibull function, in its various forms, is known as flexible and has achieved significant results in diverse forest conditions (Reynolds et al. 1998, Liu et al. 2004, Lei 2008, Corral-Rivas et al. 2015, Pérez-López et al. 2019, Pogoda et al. 2019, Nava-Nava et al. 2024). Considering our research, the Weibull function did not perform as expected. Specifically, the two-parameter Weibull function tended to underestimate the measured values, particular in diameter classes from 20 to 30 cm. Sun et al. (2019) suggest that increasing sample area and plot size could improve PDF accuracy in even-aged stands. In contrast, Machado et al. (2009), Canuto-Amaral (2017), Medeiros et al. (2018), and de Lima et al. (2023) found that the two-parameter Weibull function performed poorly predicting diameter frequencies.

Uneven-aged natural forests often show an "inverted J" shape of diameter distribution, a characteristic of geometric series, as documented by Liocourt (1898) and de Lima et al. (2023). This unimodal distribution is positively skewed (de Lima et al. 2017, 2023). However, each species has a specific growth pattern and possesses unique adaptive traits, particularly during the juvenile stage, when rapid growth occurs.

The results show that *Pinus maximinoi* and *P. oocarpa* are the dominant species on all sampling plots, highlighting their ecological significance in southern Oaxaca. As pointed out by de Brito et al. (2022), when there is a concentration of young trees in a stand, there is a high probability that they will reach the adult stage, which guarantees the long-term sustainability of the species in the region.

Statistical Evaluation of PDF

We evaluated model performance by comparing the estimated inaccuracies in diameter class. Assessing bias and RMSE is essential for understanding the accuracy and

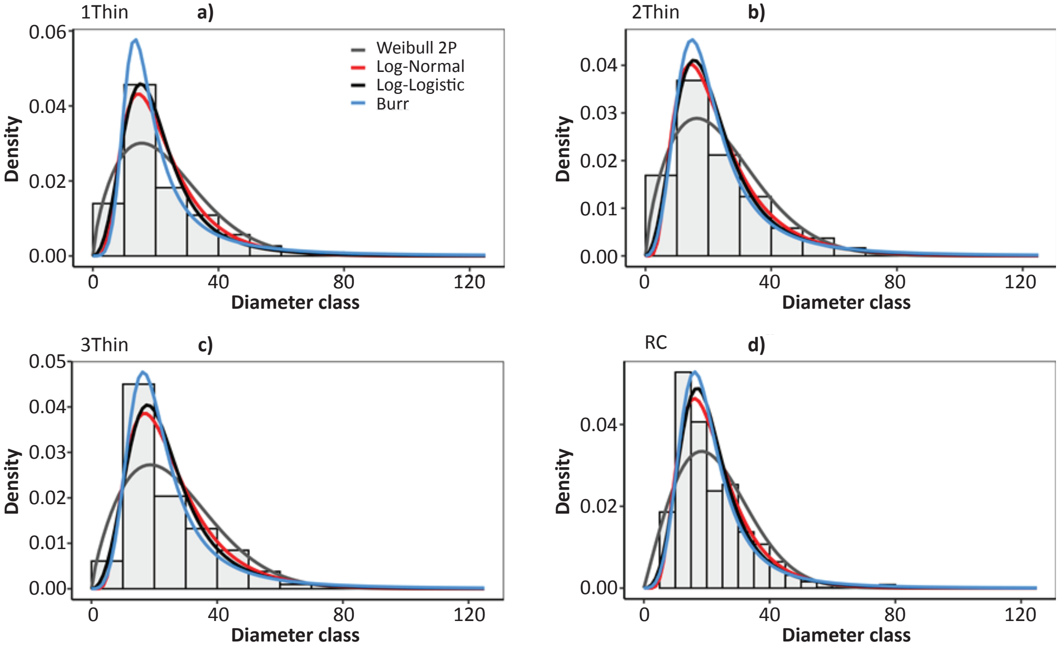

Figure 2. Observed diameter distributions and mean fitted curves of total values with the probability distribution functions used (1Thin – first thinning; 2Thin – second thinning; 3Thin – third thinning; RC – release cutting; RGC – regeneration cutting; ND – not available (separate); SELG – group-selection cutting).

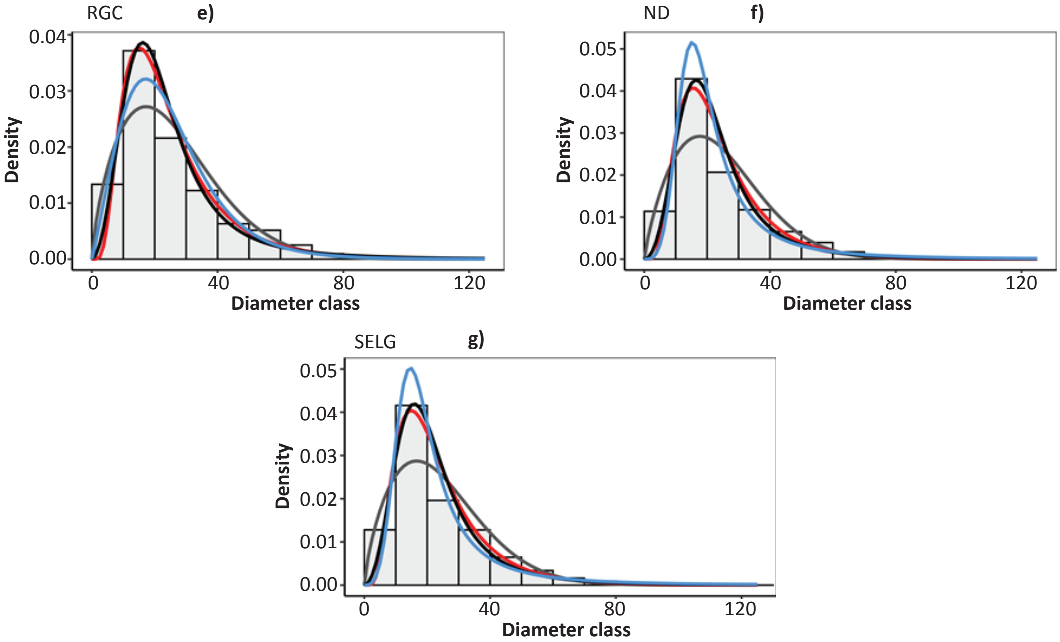

Figure 2. (continue) - Observed diameter distributions and mean fitted curves of total values with the probability distribution functions used (1Thin – first thinning; 2Thin – second thinning; 3Thin – third thinning; RC – release cutting; RGC – regeneration cutting; ND – not available (separate); SELG – group-selection cutting).

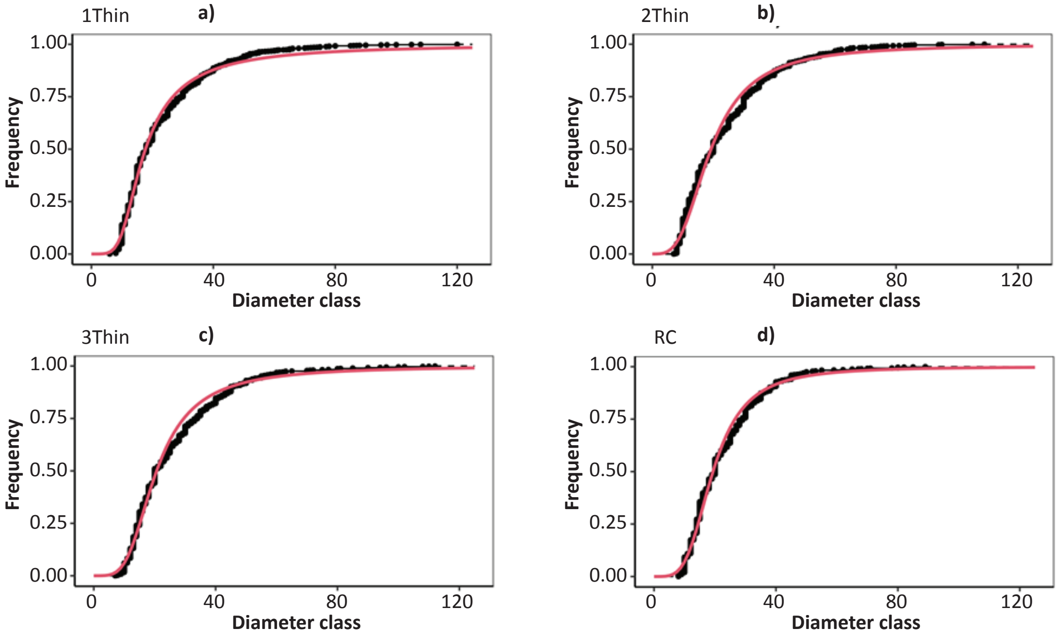

Figure 3. Empirical (black curve) and theoretical (red curve) cumulative diameter distribution means of total values with the Burr probability distribution function, (1Thin – first thinning; 2Thin – second thinning; 3Thin – third thinning; RC – release cutting; RGC – regeneration cutting; ND – not available (separate); SELG – group-selection cutting).

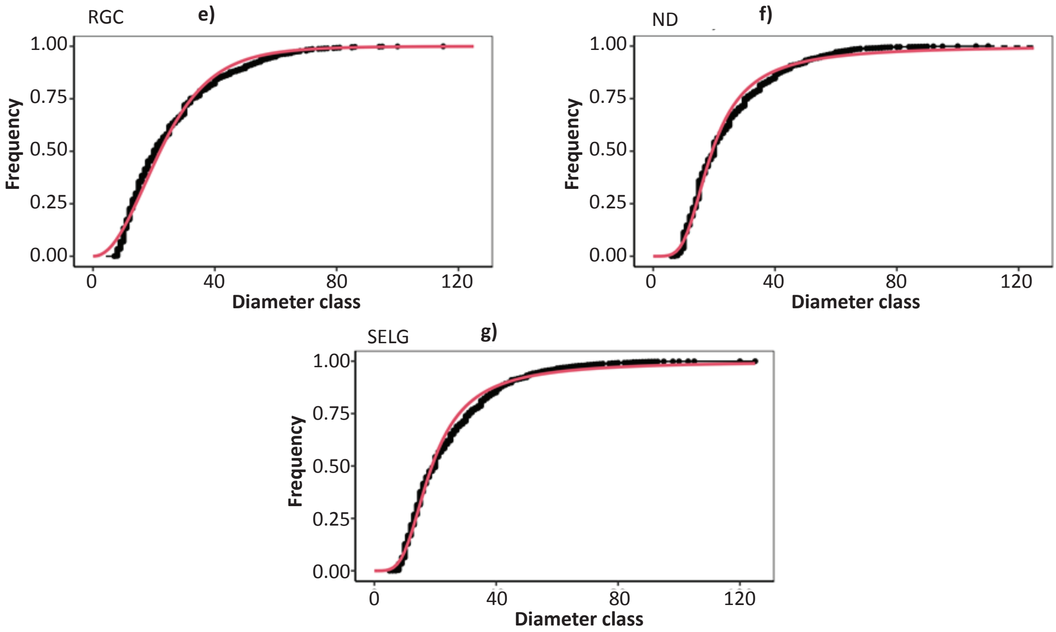

Figure 3. (continue) - Empirical (black curve) and theoretical (red curve) cumulative diameter distribution means of total values with the Burr probability distribution function, (1Thin – first thinning; 2Thin – second thinning; 3Thin – third thinning; RC – release cutting; RGC – regeneration cutting; ND – not available (separate); SELG – group-selection cutting).

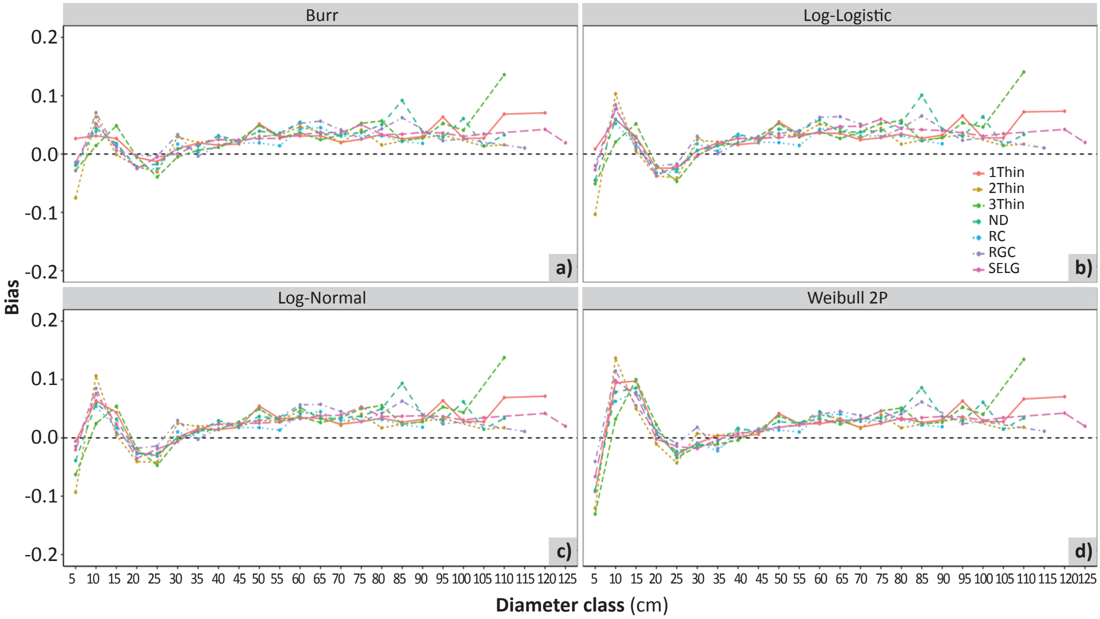

Figure 4. Mean of total values of the bias in the number of trees by diameter class obtained from the fitted four different PDF.

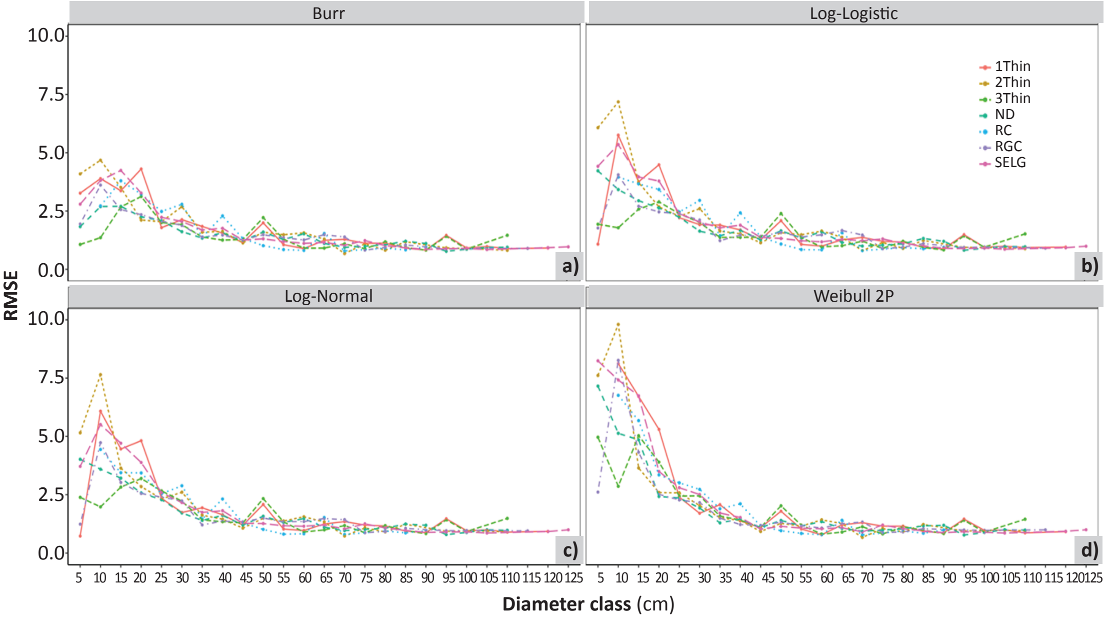

Figure 5. RMSE mean of total values for the number of trees by diameter class obtained from the fitted PDF.

precision of predictions (Guzmán-Santiago et al. 2020). As shown in Figures 4 and 5, the Burr function had the lowest estimation errors for both bias and RMSE. However, it should be noted that the model tends to slightly underestimate trees with DBH ranging between 5 and 15 cm. These results correspond with findings from other studies (Vega et al. 2022, Sahin and Ercanli 2023), which identified the largest bias in the smallest diameter classes (5-15 cm). This is often related to silvicultural interventions and competition among young trees, as noted by Corral-Rivas et al. (2015) in conifer and broadleaf tree species in Durango, Mexico.

With the impact of these results on the region, a new base knowledge has been created for further studies on dynamic estimation modeling of diameter distribution and tree volume over time.

CONCLUSIONS

The Burr function proved to be the most suitable for describing diameter distribution in mixed stands across the seven silvicultural treatments evaluated in southern Mexico. The function is recommended to serve as an effective tool for describing the diameter distribution of stands relative to silvicultural prescriptions, supporting forest management planning and forest products distribution in the region.

Author Contributions

JCGS and ANN designed the research, performed the statistical analysis, and drafted the manuscript, HMSP, BVL, MGC, and WSG reviewed the writing. All authors read and approved the final manuscript.

Funding

This research received no external funding.

Acknowledgments

The first author is grateful to the Consejo Nacional de Humanidades, Ciencia y Tecnología (CONAHCYT) for granting the scholarship for the Postdoctoral stay at the Colegio de Postgraduados en Ciencias Forestales, Campus Montecillo, Texcoco, Mexico; as well as to the agrarian authorities of the community of San Juan Quiahije, Juquila, together with the forest technicians (Ing. Gregorio Morales and Bio. María del Sol), for the support in the field measurements.

Conflicts of Interest

The authors declare no conflict of interest.

Appendix A

Table A1. Estimated parameters of the probability distribution functions by silvicultural treatment.

PDF	Treatment	Parameter	Average	Minimum	Maximum
Weibull 2P	1Thin	γ	2.1845	1.2962	5.2504
		β	25.9157	18.5395	60.6537
	2Thin	γ	2.1884	1.2482	6.0886
		β	27.0123	15.8228	57.6832
	3Thin	γ	2.2295	1.2751	3.4953
		β	29.3317	20.6746	62.3924
	RC	γ	2.3431	1.3272	3.3268
		β	25.6487	16.5167	44.3235
	RGC	γ	2.2991	1.3632	6.7362
		β	28.6580	12.0096	60.5834
	ND	γ	2.2127	1.3775	7.3513
		β	27.4681	17.1157	69.7044
	SELG	γ	2.0907	1.2424	6.3472
		β	27.3075	18.3975	69.3897
Log-Normal	1Thin	μ	2.9729	2.6827	3.8702
		σ	0.4626	0.2241	0.6479
	2Thin	μ	3.0056	2.5944	3.8940
		σ	0.5143	0.2513	0.7745
	3Thin	μ	3.1084	2.7416	3.7428
		σ	0.4752	0.3151	0.8151
	RC	μ	3.0064	2.5904	3.5988
		σ	0.4289	0.3057	0.8246
	RGC	μ	3.0595	2.3164	4.0029
		σ	0.4842	0.2140	0.8395
	ND	μ	3.0396	2.6057	3.9964
		σ	0.4782	0.1880	0.7169
	SELG	μ	3.0204	2.6276	4.1454
		σ	0.5044	0.2007	0.8406
Log-Logistic	1Thin	α	4.0039	2.5409	7.4181
		β	19.9265	14.0275	46.7151
	2Thin	α	3.5369	2.0522	7.2897
		β	20.5133	11.7697	43.1212
	3Thin	α	3.8168	1.8605	5.4554
		β	22.6274	14.7618	35.8581
	RC	α	4.1669	1.9172	5.8365
		β	20.2162	12.8018	36.2662
	RGC	α	3.9457	1.8922	9.7715
		β	22.2136	9.8650	50.7575
	ND	α	3.7631	1.6896	8.7526
		β	21.1267	12.6003	48.5195
	SELG	α	3.5801	1.4572	8.7235
		β	20.6313	12.2191	54.3877

Table A1. (continue) - Estimated parameters of the probability distribution functions by silvicultural treatment.

PDF	Treatment	Parameter	Average	Minimum	Maximum
Burr	1Thin	k	0.9976	0.1829	4.4195
		α	5.4981	2.1662	8.6191
		β	14.5873	8.9370	69.8962
	2Thin	k	1.3568	0.1593	4.4204
		α	4.8357	1.6013	9.2310
		β	15.8798	7.5881	63.1124
	3Thin	k	1.5112	0.2248	4.5636
		α	4.3303	1.8430	8.1777
		β	19.9831	9.3729	84.4683
	RC	k	1.1389	0.1750	3.5155
		α	4.9388	2.5686	7.9740
		β	17.6885	9.1306	40.5087
	RGC	k	1.4981	0.1442	5.5913
		α	4.4671	1.6510	8.7076
		β	18.1160	8.5015	75.1849
	ND	k	1.2120	0.1630	5.3939
		α	4.6505	1.9116	8.1517
		β	17.4574	8.9460	70.6243
	SELG	k	1.2393	0.1343	5.8750
		α	4.8017	1.4492	9.9512
		β	16.3889	7.9126	95.0864

1Thin – first thinning; 2Thin – second thinning; 3Thin – third thinning; RC – release cutting; RGC – regeneration cutting; ND – not available (separate); SELG – group-selection cutting.

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