Croatian Operational Research Review
CRORR **16:1**(2025), 85–98

A profit maximization model for sustainable inventory under preservation technology and linear time-dependent holding cost

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Abstract. This paper presents sustainable inventory model with constant deterioration rate when holding cost is time dependent. Demand rate is price and stock dependent. Preservation technology and carbon emission are considered for more sustainable approach. Shortages are allowed and are fully backlogged. In addition, a mathematical model is constructed to maximize the total profit function and the concavity is shown using three-dimensional graph. A numerical experimentation is carried to compute the total profit and the order quantity. To validate the proposed model, the sensitivity analysis is conducted for the total profit function as well as the order quantity. Results and observations are also discussed along with managerial insights.

Keywords: carbon emission, inventory, preservation technology, shortages, sustainable;

Received: September 15, 2024; accepted: December 9, 2024; available online: February 4, 2025

DOI: 10.17535/crorr.2025.0008

Original scientific paper.

1. Introduction

One of the greatest gifts we can give to next generation is a life worth living. A sustainable approach means using the world resources by thinking of future. An inventory model is not possible without carbon emission but it can be sustainable using preservation technology to prevent deterioration and wastage. [9] introduced an inventory model with time varying demand and holding cost whereas [17] formed a perishable inventory model with stochastic demand supply but the effect of carbon emission and deteriorations was missing. In this paper, preservation technology investment is made to counter the effects of deterioration on inventory with time and stock dependent demand. Carbon emission cost is also considered to make this model more environment friendly. In present world scenario, with everything becoming uncertain sustainability is the answer for the times ahead.

Different types of technologies are being discussed to make more energy efficient systems. A study by [10] showed that by investing in energy efficient technologies earlier, may result in more economic benefit with the flow of time. Many types of articles associated with preservation technologies investment (PRT) have been published recently. [25] discussed an Economic Order Quantity (EOQ) model with PRT and partial advance payments whereas [19] proposed a production model under PRT with salvage value.

The upcoming part of the paper is systematically divided into various Sections. Section 2 of this paper contains the literature review of the study whereas symbols and assumptions are showed in Section 3. Model development is constructed in Section 4 with the solution algorithm. A numerical experimentation is carried out in Section 5 with 3D graph. Sensitivity analysis, results and managerial insights are shown respectively in Sections 6 and 7 of this paper. The proposed study is concluded in Section 8.

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2. Literature review

[8] introduced an inventory model with constant demand rate. They found the minimal cost for inventory model with deteriorating items with variable rate of deterioration and known demand. [7] showed that inventory level is also an important parameter in demand rate of inventory management whereas [1] stated that pricing also plays an impactful part in determining demand rate. Deterioration is imminent and it always results in loss of profit which is why it became an important part of inventory management. [9] formulated a deteriorating inventory model with time dependent demand rate and holding cost to minimize the total cost. [21] considered two parameter Weibull distribution deterioration rate with stock and price dependent demand rate in EOQ model.

[15] developed an inventory model with constant holding cost and PRT. Completely and partially backlogged shortages was considered with two cases of demand function dependent on price and stock. [23] proposed an inventory model with price-time dependent demand and non-linear holding cost. [12] formulated an inventory model with parabolic holding cost and salvage value was also considered to reduce the wastage and total cost. [29] extended the work of [15] with carbon tax policy and carbon cap-trade policy. [5] optimized the delivery pattern to balance the workload of transport and warehouse.

[13] developed an inventory model for imperfect product with carbon emissions and investment in green technology. [18] discussed an inventory model with price and stock dependent demand rate to maximize the return on inventory management expense. To reduce the effect of global warming, several types of carbon emission norms are introduced by regulatory bodies. [16] optimized an inventory model for price dependent demand rate with carbon emission. [6] examined the learning effect on EOQ model with deteriorated items under carbon emission effect. [2] generated a multi-item inventory model with partially backlogged shortages to explore the effects of reliability.

[4] studied an inventory model with time and price dependent demand rate, non-instantaneous deterioration rate following the three-parameter Weibull distribution to find the optimal selling price. [24] proposed a model with price and stock dependent demand rate to increase the profit with more greening efforts. [26] generated an inventory model with time dependent quality demand under PRT by keeping in mind of dairy products. [28] introduced price based PRT in an inventory model to reduce carbon emissions. [3] discussed an interval valued inventory model with price dependent demand rate and buy now pay later scheme for payments.

This paper correlates time dependent holding cost with deteriorating items and PRT is made to counter deterioration and for a sustainable approach. The summary of recent work is represented in Table 1 below.

Literature	**Demand Rate**	**Holding Cost**	**Shortages**	**PRT**
[27]	Time dependent	Fixed	Partially backlogged	✓
[9]	Time dependent	Linear	Partially backlogged	×
[14]	Time dependent	Linear	Partially backlogged	×
[15]	Price-stock dependent	Fixed	Completely \ Partially backlogged	✓
[12]	Time dependent	Parabolic	Without shortages	×
[11]	Stock dependent	Fixed	Partially backlogged	×
[20]	Time dependent	Fixed	Without shortages	×
[22]	Time dependent	Fixed	Completely backlogged	×
[3]	Price dependent	Fixed	Without shortages	×
This paper	Price-stock dependent	Linear	Completely backlogged	✓

Table 1: *Summary of recent research.*

3. Symbols and assumptions

The employed symbols in the model are summarized below in Table 2.

Symbols	Meaning of the symbols	Symbols	Meaning of the symbols
n	Ordering frequency	A	Ordering cost ($/cycle)
t_1	Time when shortage occurs (unit time)	D_τ	Deteriorated products (units)
T	Cycle length (unit time)	S	Maximum inventory level (units)
$\lambda(\alpha)$	Deterioration after PRT (units/time unit)	Q	Order Quantity (units)
λ_0	Deterioration rate (units/time unit)	SR	Sales revenue ($/time unit)
δ	Sensitivity parameter for PRT	PC	Purchase cost ($/time unit)
β	Stock dependent parameter for the demand function	HC	Holding cost ($/time unit)
c	Cost price ($/unit)	SC	Storage cost ($/time unit)
p	Selling price ($/unit)	DC	Deterioration cost ($/time unit)
d	Deteriorating cost ($/unit)	OC	Ordering cost ($/time unit)
α	PRT cost ($/unit/ time unit)	PTC	PRT cost ($/time unit)
c_h	Holding cost ($/unit/time unit)	CEC	Carbon emission cost ($/time unit)

Table 2: *List of Symbols.*

The following assumptions are made in the inventory model.

(i) Lead time is zero.

(ii) Demand rate ($D(p,t)$) depends up on price and stock [15, 29], as follows:

$$D(p,t) = \begin{cases} D(p) + \beta I(t), & 0 \le t \le t_1 \\ D(p), & t_1 \le t \le \frac{T}{n} \end{cases}, \tag{1}$$

where $D(p)$ is demand function dependent on price and $I(t)$ is inventory level at time t. Here, $D(p) = \frac{a}{p^b}$, where a and b are non-zero positive real numbers, called the demand scale, and the price sensitive parameter respectively.

(iii) Holding cost C_h is linearly time dependent [9]

$$c_h = a_0 + a_1 t \tag{2}$$

where, a_0, $a_1 > 0$ are holding scale parameters.

(iv) The deterioration rate after preservation technology [15] is

$$\lambda(\alpha) = \lambda_0 e^{-\delta\alpha} \tag{3}$$

where, λ_0 is deterioration rate in the absence of preservation technology investment and δ is preservation technology parameter and λ_0, $\delta > 0$. The effect of preservation technology on deterioration rate is shown graphically in Figure 1.

(v) Carbon emission is considered and calculated by dividing it into two factors, fixed and variable. A tariff on carbon emission is applied to calculate the carbon emission cost.

(vi) Shortages are completely backlogged.

(vii) Single item is considered.

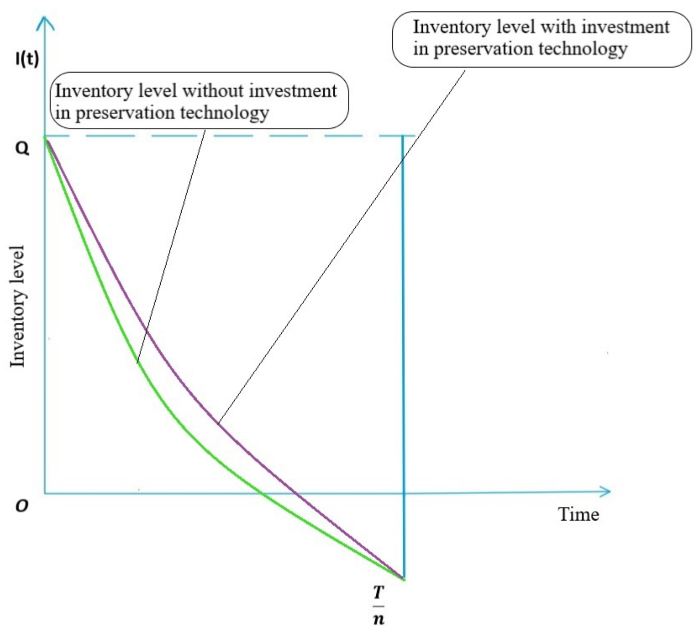


Figure 1: *Effect on inventory level with preservation technology.*

The behaviour of the inventory model is shown graphically in Figure 2.

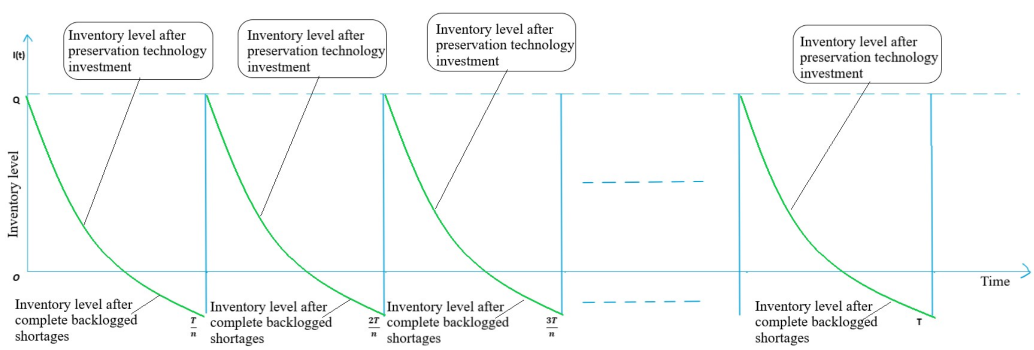


Figure 2: *Representation of inventory model with fully backlogged shortages.*

4. Model development and solution procedure

This model optimizes the value of total profit function by finding the optimal values of t_1 and T. For the inventory level (Figures 1-2), there are two differential equations: Equation (4) and Equation (5). Here, the Equation (4) holds when the inventory level is non-zero. On the other side, Equation (5) holds when the inventory level is replenished. Let $t = t_1$ when the inventory level replenishes. The concerned differential equations are presented by Equations (4) and (5) as follows

$$\frac{d(I(t))}{dt} + \lambda(\alpha)I(t)) = -D(p,t), \quad 0 \leq t \leq t_1 \tag{4}$$

$$\frac{d(I(t))}{dt} = -D(p), \quad t_1 \leq t \leq \frac{T}{n} \tag{5}$$

with the boundary conditions $I(0) = S$ when the inventory level is maximum, and $I(t_1) = 0$ when the shortage happens.

The solution of the above equations is given by

$$D(p,t) = \begin{cases} \frac{D(p)}{\lambda(\alpha)+\beta}(e^{(\lambda(\alpha)+\beta)(t_1-t)} - 1), & 0 \leq t \leq t_1 \\ D(p)(t_1 - t), & t_1 \leq t \leq \frac{T}{n} \end{cases}, \tag{6}$$

Applying the boundary conditions to Equation (9), the inventory reaches maximum level (S) which is determined as

$$S = I(0) = \frac{D(p)}{\lambda(\alpha)+\beta}(e^{(\lambda(\alpha)+\beta)t_1} - 1) \tag{7}$$

Products affected by deterioration (D_τ) in the interval $[0, t_1]$ is evaluated as

$$D_\tau = S - \int_0^{t_1} D(p,t)dt \tag{8}$$

The solution of Equation (8) is given by

$$D_\tau = \frac{D(p)}{\lambda(\alpha)+\beta}\left(e^{(\lambda(\alpha)+\beta)t_1} - (\lambda(\alpha)+\beta)t_1 + \frac{\beta}{\lambda(\alpha)+\beta} - \frac{\beta e^{(\lambda(\alpha)+\beta)t_1}}{\lambda(\alpha)+\beta} + \beta t_1 - 1\right) \tag{9}$$

Therefore, the order quantity per cycle can be computed as

$$Q = D_\tau + \int_0^{\frac{T}{n}} D(p,t)dt \tag{10}$$

The solution of Equation (10) is given by

$$Q = \frac{D(p)}{\lambda(\alpha)+\beta}(e^{(\lambda(\alpha)+\beta)t_1} - 1) + D(p)\left(\frac{T}{n} - t_1\right) \tag{11}$$

Here, the maximum number of backorder unit per cycle are as follows

$$S\left(\frac{T}{n}\right) = D(p)\left(\frac{T}{n} - t_1\right) \tag{12}$$

To find total profit function, first we calculate sales revenue and various types of costs.
Sales Revenue (SR) of the inventory model is

$$SR = np\int_0^{\frac{T}{n}} D(p,t)dt,$$

$$SR = \frac{np\beta D(p)}{(\lambda(\alpha)+\beta)^2}\left(e^{(\lambda(\alpha)+\beta)t_1} - (\lambda(\alpha)+\beta)t_1 - 1\right) + pD(p)T \tag{13}$$

whereas the purchase Cost (PC) is

$$PC = ncQ$$

substituting the value of Q from Equation (11)

$$PC = \frac{ncD(p)}{\lambda(\alpha)+\beta}(e^{(\lambda(\alpha)+\beta)t_1} - 1) + ncD(p)\left(\frac{T}{n} - t_1\right) \tag{14}$$

The holding Cost (HC) of the complete inventory is

$$HC = \int_0^{t_1} nc_h I(t)dt,$$

$$\begin{aligned} HC = &-\frac{na_0 D(p)t_1}{\lambda(\alpha)+\beta} - \frac{na_0 D(p)}{(\lambda(\alpha)+\beta)^2} + \frac{na_0 D(p)}{(\lambda(\alpha)+\beta)^2}e^{(\lambda(\alpha)+\beta)t_1} - \frac{na_1 D(p)t_1^2}{2(\lambda(\alpha)+\beta)} \\ &-\frac{na_1 D(p)t_1}{(\lambda(\alpha)+\beta)^2} - \frac{na_1 D(p)}{(\lambda(\alpha)+\beta)^3} + \frac{na_1 D(p)}{(\lambda(\alpha)+\beta)^3}e^{(\lambda(\alpha)+\beta)t_1} \end{aligned} \tag{15}$$

and shortage Cost (SC) of the inventory which is faced in the interval $\left[t_1, \frac{T}{n}\right]$ is

$$SC = ns\int_{t1}^{\frac{T}{n}} (-I(t))dt,$$

$$SC = nsD(p)\left(\frac{T^2}{2n^2} + \frac{t_1^2}{2} - \frac{Tt_1}{n}\right) \tag{16}$$

Ordering Cost (OC) and preservation technology Cost (PTC) of complete inventory model is

$$OC = nA \tag{17}$$

and

$$PTC = \alpha T, \tag{18}$$

Cost due to deterioration face by inventory model (DC) is

$$DC = ndD_\tau.$$

Putting the value of D_τ from Equation (9) in Equation (18) the value of (DC) is calculated as

$$DC = \frac{ndD(p)}{\lambda(\alpha)+\beta}\left(e^{(\lambda(\alpha)+\beta)t_1} - (\lambda(\alpha)+\beta)t_1 + \frac{\beta}{\lambda(\alpha)+\beta} - \frac{\beta e^{(\lambda(\alpha)+\beta)t_1}}{\lambda(\alpha)+\beta} + \beta t_1 - 1\right) \tag{19}$$

Carbon Emission Cost (CEC) of the inventory is calculated by variable and fixed carbon emission factor c_v and c_f, m be the weight of the product and γ is the tariff on the carbon emission done by the model. CEC is given by

$$CEC = (c_v mQ + c_f)\gamma \tag{20}$$

$$CEC = \frac{c_v m\gamma D(p)}{\lambda(\alpha)+\beta}(e^{\lambda(\alpha)+\beta)t_1} - 1) + c_f\gamma \tag{21}$$

Total profit function (TP) is evaluated by subtracting all costs of inventory from the sales revenue

$$TP = SR - (PC + HC + SC + DC + OC + PTC + CEC). \tag{22}$$

Putting all the required values from Equations (13), (14), (15), (16), (17), (18), (19) and (121), the total profit function is computed as follows

$$\begin{aligned} TP = {} & \frac{np\beta D(p)}{\lambda(\alpha)+\beta}\left(e^{(\lambda(\alpha)+\beta)t_1} - (\lambda(\alpha)+\beta)t_1 - 1\right) + pD(p)T - \frac{ncD(p)}{\lambda(\alpha)+\beta}(e^{(\lambda(\alpha)+\beta)t_1} - 1) \\ & -ncD(p)\left(\frac{T}{n} - t_1\right) + \frac{na_0 D(p)t_1}{\lambda(\alpha)+\beta} + \frac{na_0 D(p)}{(\lambda(\alpha)+\beta)^2} - \frac{na_0 D(p)}{(\lambda(\alpha)+\beta)^2}e^{(\lambda(\alpha)+\beta)t_1} + \frac{na_1 D(p)t_1^2}{2(\lambda(\alpha)+\beta)} \\ & + \frac{na_1 D(p)t_1}{(\lambda(\alpha)+\beta)^2} + \frac{na_1 D(p)}{(\lambda(\alpha)+\beta)^3} - \frac{na_1 D(p)}{(\lambda(\alpha)+\beta)^3}e^{(\lambda(\alpha)+\beta)t_1} - nsD(p)\left(\frac{T^2}{2n^2} + \frac{t_1^2}{2} - \frac{Tt_1}{n}\right) \\ & - \frac{ndD(p)}{\lambda(\alpha)+\beta}\left(e^{(\lambda(\alpha)+\beta)t_1} - (\lambda(\alpha)+\beta)t_1 + \frac{\beta}{\lambda(\alpha)+\beta} - \frac{\beta e^{(\lambda(\alpha)+\beta)t_1}}{\lambda(\alpha)+\beta} + \beta t_1 - 1\right) \\ & -nA - \alpha T - \frac{c_v m\gamma D(p)}{\lambda(\alpha)+\beta}(e^{\lambda(\alpha)+\beta)t_1} - 1) - c_f\gamma \end{aligned} \tag{23}$$

This is a non-linear optimization problem with single objective. To find the optimal values of t_1 and T, the partial derivatives of objective function TP with respect to t_1 and T are solved by putting them equal to 0. The following equations are solved to find the optimal values.

$$\frac{\partial(TP)}{\partial t_1} = 0 \quad \text{and} \quad \frac{\partial(TP)}{\partial T} = 0 \tag{24}$$

$$\begin{aligned} \frac{\partial(TP)}{\partial t_1} = {} & \frac{np\beta D(p)}{\lambda(\alpha)+\beta}e^{(\lambda(\alpha)+\beta)t_1} - \frac{np\beta D(p)}{\lambda(\alpha)+\beta} - ncD(p)e^{(\lambda(\alpha)+\beta)t_1} + ncD(p) + \frac{a_0 D(p)}{\lambda(\alpha)+\beta} \\ & - \frac{a_0 D(p)}{\lambda(\alpha)+\beta}e^{(\lambda(\alpha)+\beta)t_1} + \frac{a_1 D(p)t_1}{\lambda(\alpha)+\beta} + \frac{a_1 D(p)}{(\lambda(\alpha)+\beta)^2} - \frac{a_1 D(p)}{(\lambda(\alpha)+\beta)^2}e^{(\lambda(\alpha)+\beta)t_1} \\ & -nsD(P)t_1 + sD(p)T - ndD(p)(\lambda(\alpha)+\beta)e^{(\lambda(\alpha)+\beta)t_1} + ndD(p)(\lambda(\alpha)+\beta) \\ & + ndD(p)\beta e^{(\lambda(\alpha)+\beta)t_1} - c_v m\gamma D(p)e^{(\lambda(\alpha)+\beta)t_1} = 0 \end{aligned} \tag{25}$$

and

$$\frac{\partial(TP)}{\partial T} = pD(p) - cD(p) - \frac{sD(p)T}{n} + sD(p)t_1 - \alpha = 0 \tag{26}$$

The optimal values t_1^* and T^* can be calculated by solving Equation (25) and (26). Maximum value of total profit function (TP^*) is calculated by using the newly found values of t_1^* and T^*. Furthermore, the necessary conditions for the maximization of objective function are

$$\left(\frac{\partial^2 TP}{\partial t_1^2}\right)\left(\frac{\partial^2 TP}{\partial T^2}\right) - \left(\frac{\partial^2 TP}{\partial t_1 \partial T}\right)^2 > 0 \;\; and \;\; \left(\frac{\partial^2 TP}{\partial t_1^2}\right)^2 < 0, \; \left(\frac{\partial^2 TP}{\partial T^2}\right)^2 < 0. \tag{27}$$

The partial derivatives to prove the necessary conditions are

$$\frac{\partial^2(TP)}{\partial t_1^2} = np\beta D\left(p\right)e^{(\lambda(\alpha)+\beta)t_1} - \left(\lambda\left(\alpha\right)+\beta\right)ncD\left(p\right)e^{(\lambda(\alpha)+\beta)t_1} - a_0 D\left(p\right)e^{(\lambda(\alpha)+\beta)t_1} + \frac{a_1 D\left(p\right)}{\lambda\left(\alpha\right)+\beta} - \frac{a_1 D\left(p\right)}{\lambda\left(\alpha\right)+\beta}e^{(\lambda(\alpha)+\beta)t_1} - nsD\left(P\right) - \left(\lambda\left(\alpha\right)+\beta\right)^2 ndD\left(p\right)e^{(\lambda(\alpha)+\beta)t_1} + \left(\lambda\left(\alpha\right)+\beta\right)ndD\left(p\right)\beta e^{(\lambda(\alpha)+\beta)t_1} - \left(\lambda\left(\alpha\right)+\beta\right)c_v m\gamma D\left(p\right)e^{(\lambda(\alpha)+\beta)t_1}, \tag{28}$$

$$\frac{\partial^2(TP)}{\partial T^2} = -\frac{sD\left(p\right)}{n}, \tag{29}$$

and

$$\frac{\partial^2(TP)}{\partial t_1 \partial T} = sD\left(p\right) \tag{30}$$

The concavity of the TP is shown numerically by satisfying the necessary conditions in Equation (27) in the next Section. The optimal values t_1^* and T^* is obtained using software Mathematica 11.

An algorithm to find the total profit function is shown in steps as follows:

Step 1: Input: Write all the parameters values in Equations (25) and (26).

Step 2: Solve both Equations (25) and (26) using Mathematica 11 for optimal values t_1^* and T^*.

Step 3: Verify the concavity of the profit function by plotting a 3D graph.

Step 4: Find the maximum total profit function TP^* using values t_1^* and T^*.

Step 5: Output: The output is the total profit function.

A flowchart of the proposed solution procedure is presented in the following Figure 3.

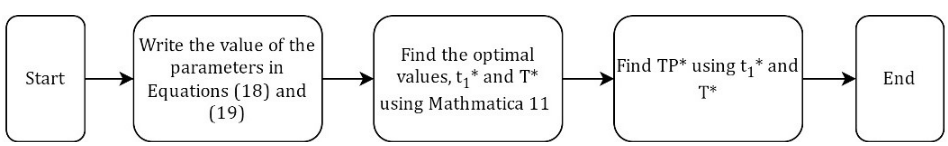


Figure 3: *Flowchart of the proposed solution procedure.*

5. Numerical experimentation

To illustrate the proposed mathematical model, a numerical example is taken and a 3D graph is modeled to show the concavity of the profit function along with the necessary conditions. Sensitivity analysis is also done for the numerical example.

Example: The parameter values in proper units are considered for this example are $n = 4$, $D\left(p\right) = \frac{a}{b^p}$ where $a = 50$, $b = 0.015$, $p = \$500$ /unit, $\alpha = \$2$ /unit/unit time , $\beta = 0.01$, $a_0 = 110$, $a_1 = 3$, $s = \$12$/unit , $d = \$0.5$/unit , $A = \$500$, $c = \$2$/unit , $\delta = 0.4$ and $\lambda_0 = 0.01$. Applying the above method the optimal solution for total profit.

Using the solution procedure, the optimal values are $t^* = 22.536$ and $T^* = 256$ whereas, total profit is \$1125440.

Based on the above data, the concavity is demonstrated numerically by using the conditions in Equation (27). The sufficient conditions are

$$\frac{\partial^2(TP)}{\partial t_1^2} = -5031.8521 < 0$$

$$\frac{\partial^2(TP)}{\partial T^2} = -60 < 0$$

$$\left(\frac{\partial^2 TP}{\partial t_1^2}\right)\left(\frac{\partial^2 TP}{\partial T^2}\right) - \left(\frac{\partial^2 TP}{\partial t_1 \partial T}\right)^2 = 302871.174 > 0$$

The variations of total profit function with respect to t_1 and T is shown graphically in the following Figure 4.

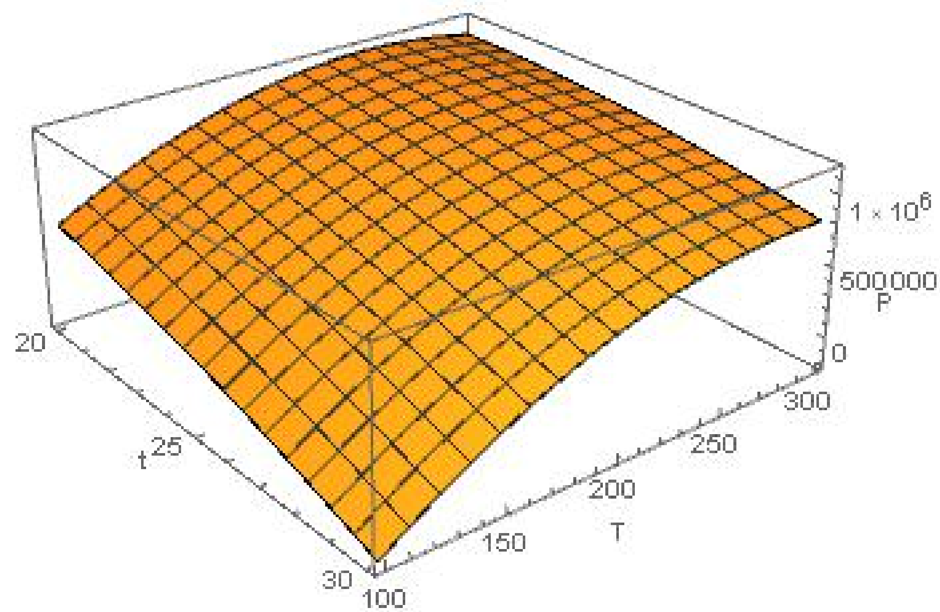


Figure 4: *Total profit function in relation with time t_1 and cycle length T.*

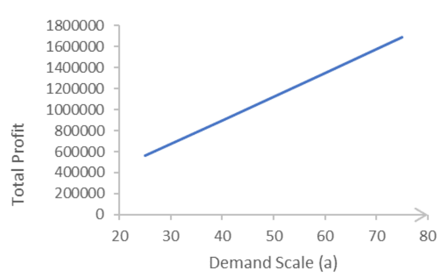
6. Sensitivity analysis

In order to demonstrate the effect of various parameters with respect to total profit and order quantity, the sensitivity analysis is carried out. In this analysis, at a time the value of one parameter is changed while remaining parameters are kept unchanged. All of this is summarized in the Table 3.

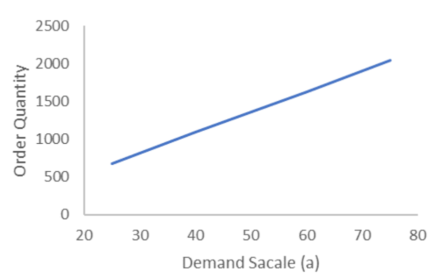
Parameters	**% Change**	**Total Profit** (TP)	**% Change in** (TP)	**Order Quantity** (Q)	**% Change in** (Q)
a	-50%	561460	-50.11	680	-50
	-20%	899487	-20.04	1088	-20
	20%	1351030	20.04	1632	20
	50%	1689420	50.11	2041	50.07
b	-50%	1802210	60.13	2177	60.07
	-20%	1351030	20.04	1632	20
	20%	956245	-15.03	1156	-15
	50%	730654	-35.08	884	-35
λ_0	-50%	1091720	-3	1348	-0.88
	-20%	1089450	-3.2	1356	-0.29
	20%	1081060	-3.94	1368	0.59
	50%	1074920	-4.49	1376	1.18
δ	-50%	1074490	-4.53	1376	1.18
	-20%	1081610	-3.89	1367	0.51
	20%	1088710	-3.26	1359	-0.15
	50%	1092680	-2.91	1353	-0.51
β	-50%	1109450	-1.42	1331	-2.13
	-20%	1095170	-2.69	1349	-0.81
	20%	1075530	-4.43	1375	1.1
	50%	1060270	-5.79	1395	2.57
α	-50%	1074740	-4.5	1376	1.18
	-20%	1081660	-3.89	1367	0.51
	20%	1088600	-3.27	1358	-0.15
	50%	1092420	-2.93	1353	-0.51
n	-50%	-769195	-131.65	2640	94.12
	-20%	505828	-55.06	1787	31.4
	20%	1482930	31.76	1104	-18.82
	50%	1709430	51.89	934	-31.32

Table 3: *Sensitivity analysis of numerical example.*

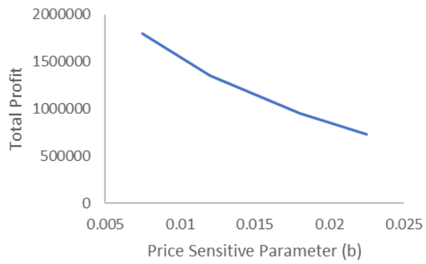
The variations in total profit and order quantity in relation with different parameters are represented graphically in Figure 5.



5(a)



5(b)



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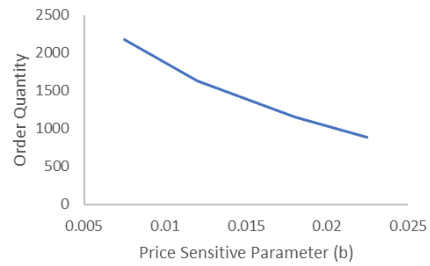
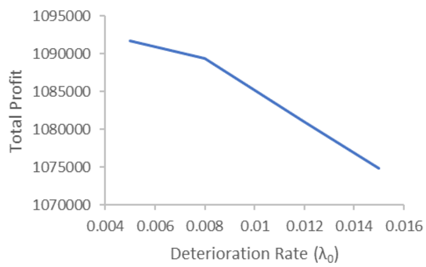
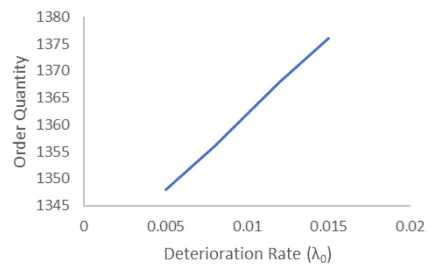


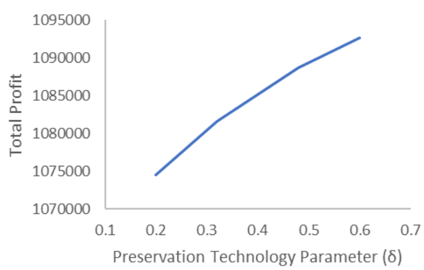
5(d)



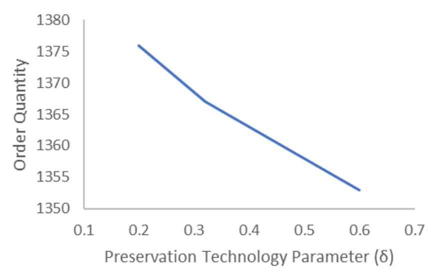
5(e)



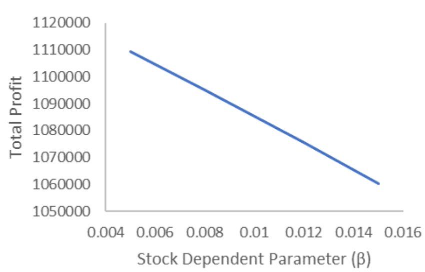
5(f)



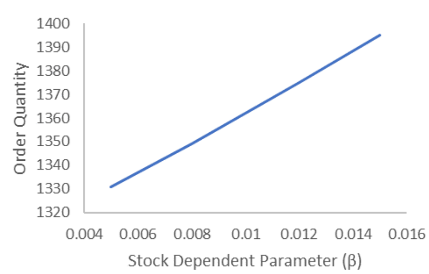
5(g)



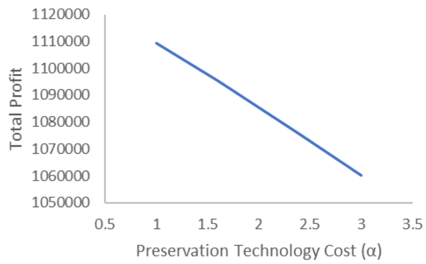
5(h)



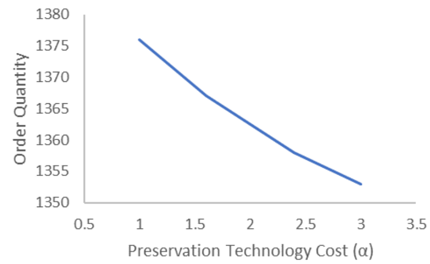
5(i)



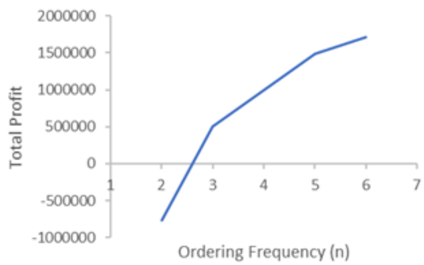
5(j)



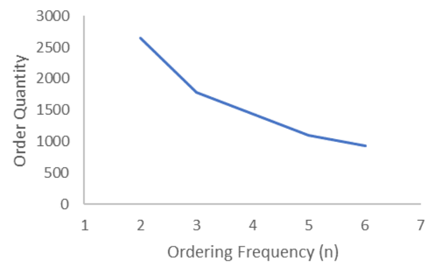
5(k)



5(l)



5(m)



5(n)

Figure 5: *Variations in total profit and order quantity.*

7. Results and managerial insights

Based on numerical example and sensitivity analysis, the following points are observed

a) Total Profit directly varies with a, δ, n as seen in Figures 5(a, g, m) respectively and it is inversely varying with the values of $b, \alpha, c, \lambda_0, \beta$ as noticed in Figures 5(c, e, i, k).

b) There is a sudden change in total profit with respect to ordering frequency n as shown in Figure 5(m)

c) Order Quantity directly varies with a, λ_0, β as seen in Figures 5(b, f ,j) and it is inversely varying with b, δ, α, n noticed in Figures 5(d, h, l, n).

d) Cost price c does not have any effect on order quantity.

e) PRT cost has very minimal effect on order quantity as seen in Figure 5(n).

The managerial insights of the inventory model are observed such that

- An increase in demand scale parameter a results in the increment of total profit as well as order quantity. Retailers should try to increase their demand for more profit.
- Increase in preservation technology parameter δ means high profit and less order quantity. Decision makers should increase their investment in preservation technology to get more profit with less order quantity.

8. Conclusions

In this paper, a profit optimization model was developed for sustainable inventory with demand rate dependent on price-stock considering the preservation technology. In addition, the shortages were allowed which was completely backlogged as they are very practical in nature. The total profit function was developed with sales revenue and various other types of costs. The main findings of this paper were summarized in terms of optimal values for the total profit and order quantity. In addition, it was observed that total profit varies directly with demand scale and price sensitive parameter. Whereas, increase in number of cycles result in increase of total profit and decrease in order quantity. This paper can be expanded in multiple directions. For example, one can consider the time dependent demand in place of price and stock dependent demand. Another possible extension is to consider the uncertainty in modal parameters, for instance fuzzy number, interval number, type-1 fuzzy sets, intuitionistic fuzzy sets, etc.

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