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Deformation of the Cooling Vessel's Shell

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Abstract: This paper will present the procedure for calculating the internal volume obtained by deforming the outer shell of the cooling vessel. A nonlinear finite element analysis was conducted using the Ansys software. The material behavior when it transitions into the plastic deformation region was defined, and the deformed shape resulting from the plastic deformation of the outer shell of the cooling vessel was obtained. The volume of the deformed shape was determined with several simplifications: the profile of the outer shell was simplified into a circular arc, and the parameters of the circle were calculated. The formula for the volume of a rotational body was used to calculate the volume.

Keywords: Ansys software; cooling vessel; deformation; shell; volume

1 INTRODUCTION

The Finite Element Method is a technique based on the discretization of a continuum [1]. The continuous domain is divided into a finite number of subdomains called finite elements. The state within each element, such as displacement, deformation, stress, temperature, and other field variables, is described using interpolation functions. These functions must satisfy appropriate conditions to make the discretized model closely approximate the behavior of the continuous system. With the proper formulation of finite elements, the accuracy of the approximation to the exact solution increases as the number of elements increases. During the derivation of equations, we distinguish between the following procedures:

- the process of deriving the finite element equation based on solving a differential equation, i.e., the method of weighted residuals,
- Derivation of the finite element equation based on variational formulations (the principle of virtual displacements, the principle of minimum total potential energy, the principle of virtual forces, and the principle of minimum complementary energy).

If the independent variables are forces or stresses, the Finite Element Method is referred to as the force method. The other method is the displacement method, where the unknowns are the displacements at the nodes of finite elements. If the independent variables are displacements, stresses, and strains, it is a mixed formulation of finite elements. Due to the large number of equations and unknowns, the Finite Element Method uses computers to solve problems. Various software packages are based on the Finite Element Method, and in this work, Ansys software is used for finite element analysis.

We calculate the volume of the cooling vessel so that the vessel does not come under the state inspection. Previous research has not addressed this issue, so due to the lack of prior experience and for the sake of simplicity, this method of volume calculation has been chosen.

2 COOLING VESSEL

For the purpose of cooling food products and alcoholic beverages, vessels with double or triple shells are manufactured, known as duplicators in food technology [2]. Water or glycol flows through the double shell, cooling the contents of the vessel through the wall. In some designs, there is additional thermal insulation on the outside, along with the outer shell of the vessel that protects the insulation and the vessel's fittings. An example of such a vessel for beer is shown in Fig. 1.

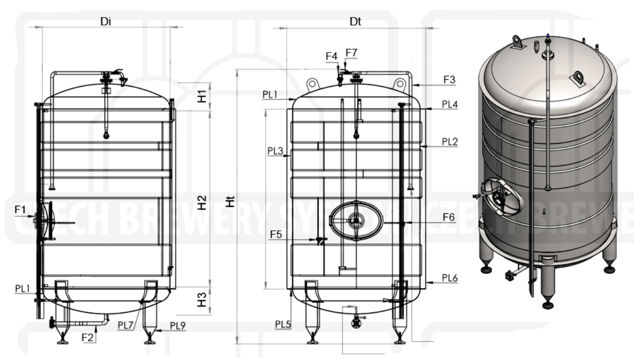

Figure 1 An example of a cooling vessel for beer [3]

One way to implement a double shell is to weld the thinner shell, and a pressure cleaner is connected through the provided opening to create pressure, causing the stretching of the outer, thinner shell. An example of simulating the outer shell of such a vessel is presented in this paper.

The material of such vessels is most commonly stainless steel, such as SS304 or AISI 304 (X5CrNi18-10 acc. to EN 10027-1). The typical behavior of SS304 stainless steel is shown in Fig. 2. The figure depicts the stress-strain relationship in a tensile test. This diagram is obtained from the manufacturer, and a similar characteristic is obtained depending on the production technology and parameters. If the manufacturer does not guarantee the properties, the mechanical properties of samples from a particular batch of material can be tested on a tensile testing machine to obtain the accurate characteristic.

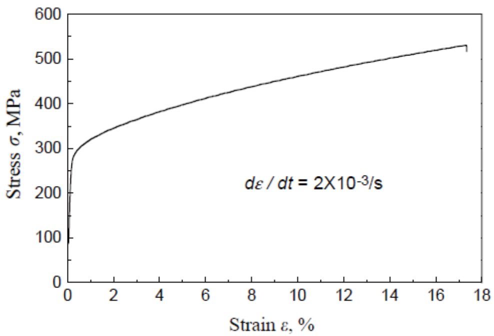

Figure 2 The deformation diagram for stainless steel SS304 [4]

Table 1 Material properties for SS304

Mechanical Properties	
Density (kg/m^3)	7930
Young's Modulus (GPa)	190-200
Poison' Ratio	0,29
Tensile Strength, Ultimate (MPa)	505
Tensile Strength, Yield (MPa)	215
Strain Hardening Coefficient, *n*	0,10-0,50
Chemical Composition	
Carbon, C	≤0,080 %
Chromium, Cr	18-20 %
Iron, Fe	66,345-74 %
Manganese, Mn	≤2,0 %
Nickel, Ni	8,0-10,5 %
Phosphorus, P	≤0,045 %
Silicon, Si	≤1,0 %
Sulfur, S	≤0,030 %

For the purposes of this analysis, and in the absence of an exact model for stainless steel, structural steel and material characteristics from the Ansys material database were used (Fig. 3). Material properties were used for nonlinear analysis, and the values are approximate. The work represents a calculation method rather than an exact solution.

Table 2 Material properties for Structural Steel

Mechanical Properties	
Density (kg/m^3)	7850
Young's Modulus (GPa)	0,3
Poison' Ratio	200
Tensile Strength, Ultimate (MPa)	460
Tensile Strength, Yield (MPa)	250

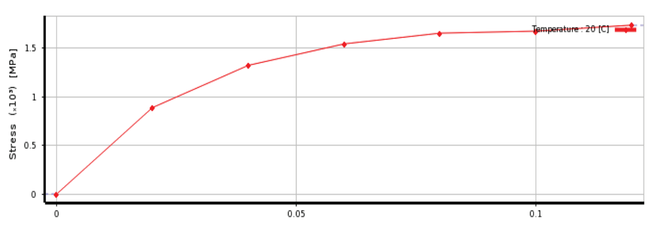

Figure 3 Material characteristics

2.1 The Outer Shell of the Cooling Vessel

The outer shell of the cooling vessel has dimensions: height 300 mm, diameter 1000 mm, and wall thickness of 1 mm (Fig. 4).

It is necessary to calculate the volume between the outer and inner walls of the vessel obtained through the plastic deformation of the outer shell. The shell is subjected to a pressure of 7 MPa and then relieved.

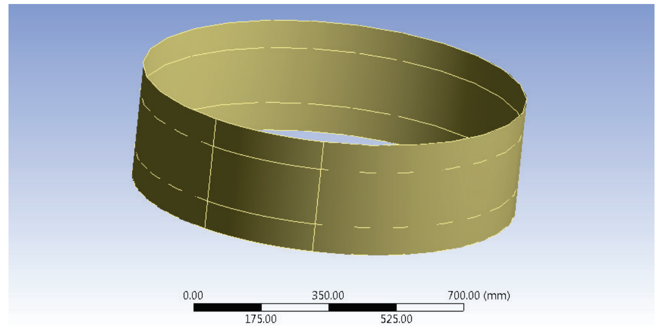

Figure 4 The appearance of the outer shell of the vessel

3 ANALYSIS USING SHELL ELEMENTS

In the first case, we are considering a model designed as a thin-walled structure. We defined a finite element mesh, using SHELL281 elements [5]. The geometry of this element is shown in Fig. 5. The SHELL281 element is suitable for analyzing thin to moderately thin structures. The element has 8 nodes, and each node has 6 degrees of freedom: translation in the *x*, *y*, and *z* directions, and rotation around the *x*, *y*, and *z* axes. When the membrane option is used, the element only has translational degrees of freedom. This element is suitable for linear problems, large rotations, and changes in shell thickness for nonlinear analysis. SHELL281 can be used for layered constructions to model composites or sandwich structures. The element's formulation is based on logarithmic strain and true stress. The element's kinematics allows for finite membrane deformation (stretching).

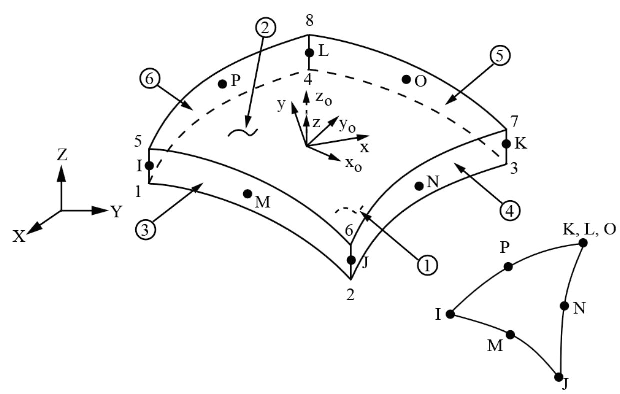

Figure 5 The geometry of the SHELL281 element [5]

The vessel is modeled as a thin-walled structure, and SHELL281 elements have been used. A finite element mesh has been generated (meshing performed), as shown in Fig. 6.

After applying boundary conditions, the outer edge and lower edge are defined as a fixed support, with zero displacements in all axes and zero rotations around all axes (Fig. 7).

The vessel is subjected to a compressive load (Fig. 8), and the load is defined as a time-dependent table (Fig. 9).

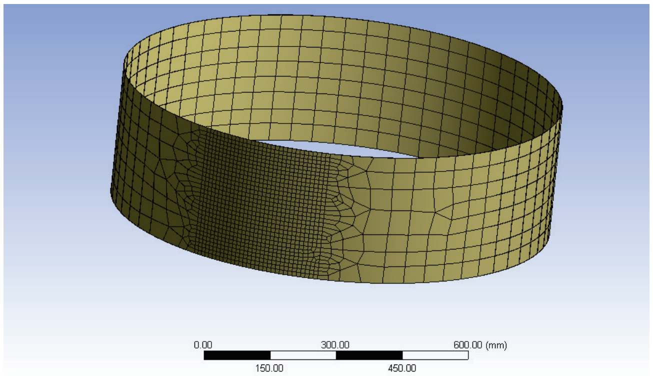

Figure 6 The finite element mesh

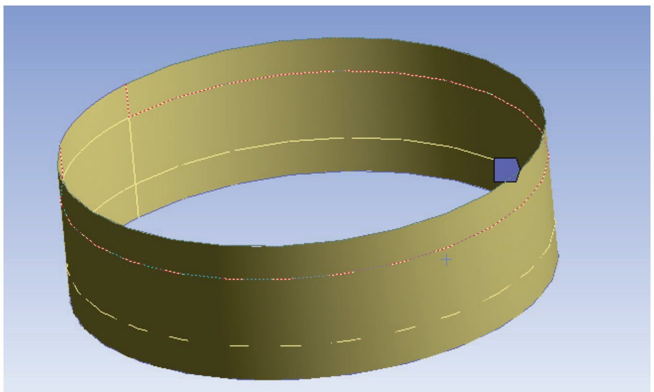

Figure 7 Restraint of the upper and lower edges

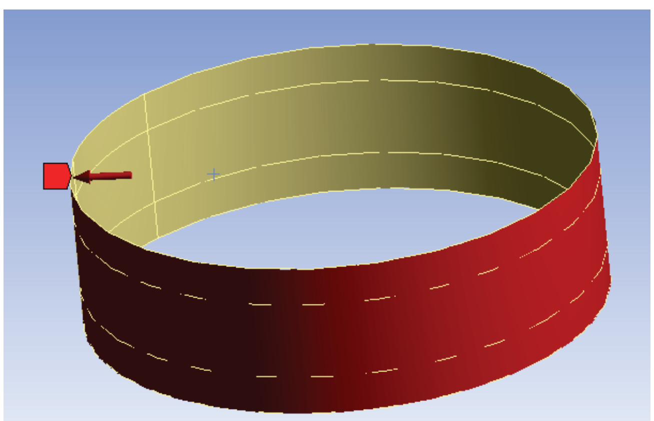

Figure 8 The load on the vessel

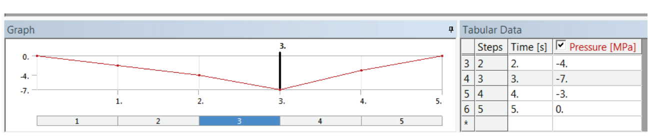

Figure 9 Defined load

Table 3 Defined load in Fig. 9

Steps	Time, s	Pressure, MPa
2	2	–4
3	3	–7
4	4	–3
5	5	0

After all parameters were defined, a finite element analysis was conducted, and results were obtained. Initially, data for the overall deformation throughout the vessel's volume were obtained.

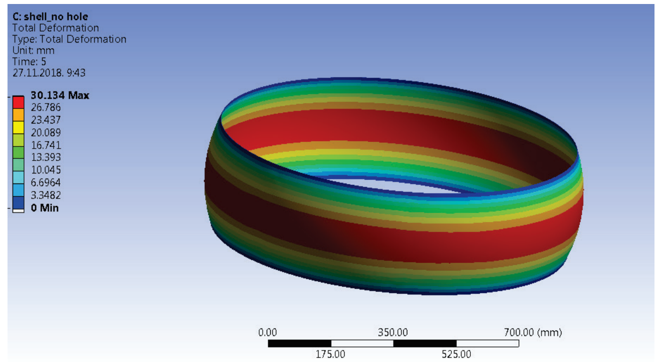

Figure 10 The deformed shape after unloading

From Tab. 1, it is evident that the maximum deflection increases as time increases, corresponding to the increasing pressure. In 3 seconds, the maximum pressure is 7 MPa, after which the pressure decreases, along with the deflection, until it comes to a stop. After unloading, the maximum deformation in the center is 30.134 mm. Deformation at the edges is zero, so the minimum deflection is 0. Tab. 3 displays the results for total deformation over time.

Table 4 Results of maximum total deformation over time for the thin-walled model

	Time, s	Maximum deformation, mm
1	0.2	2.21
2	0.4	4.31
3	0.7	7.17
4	1	9.70
5	1.2	11.26
6	1.4	13.17
7	1.7	15.94
8	2	18.50
9	2.2	21.07
10	2.4	23.86
11	2.7	28.16
12	3	33.11
13	3.2	32.76
14	3.4	32.41
15	3.7	31.87
16	4	31.34
17	4.2	31.08
18	4.4	30.82
19	4.7	30.46
20	5	30.13

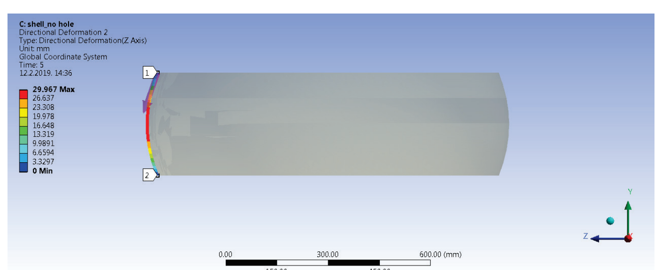

Figure 11 The profile of the deformed shape of the thin-walled shell

To determine the characteristics of the resulting vessel, it is necessary to calculate the internal volume of the vessel obtained in this way. To calculate the desired volume, a path is defined in the software, and the Finite Element Analysis results are analyzed along that path of cross-section.

Tab. 4 shows displacements in the *z*-axis direction, which in this case represents the horizontal axis. These displacements will be used to determine the profile function for calculating the volume of the rotational body.

Table 5 Results of deformation in the *z*-axis direction for the thin-walled model

	Length, mm	Deformation in the direction of the axis *z*, mm
1	0	0
2	6.25	2.16
3	12.5	4.63
4	18.75	6.81
5	25	8.83
6	31.25	10.81
7	37.5	12.70
8	43.75	14.50
9	50	16.22
10	56.25	17.86
11	62.5	19.40
12	68.75	20.85
13	75	22.19
14	81.25	23.43
15	87.5	24.56
16	93.75	25.58
17	100	26.49
18	106.25	27.30
19	112.5	28.00
20	118.75	28.60
21	125	29.08
22	131.25	29.47
23	137.5	29.75
24	143.75	29.91
25	150	29.97
26	156.25	29.91
27	162.5	29.74
28	168.75	29.47
29	175	29.08
30	181.25	28.59
31	187.5	27.99
32	193.75	27.29
33	200	26.49
34	206.25	25.57
35	212.5	24.55
36	218.75	23.43
37	225	22.19
38	231.25	20.84
39	237.5	19.40
40	243.75	17.86
41	250	16.22
42	256.25	14.50
43	262.5	12.70
44	268.75	10.81
45	275	8.83
46	281.25	6.80
47	287.5	4.63
48	293.75	2.16
49	300	0

The maximum deformation of the shell is on the central cross-sectional line at a height of 150 mm from the bottom. The difference between the total deformation and directed deformation occurs due to material stretching.

4 ANALYSIS USING SOLID ELEMENTS

In the second case, we are considering a model designed as a three-dimensional model. We defined a finite element mesh, using SOLID187 elements [6], as shown in Fig. 12.

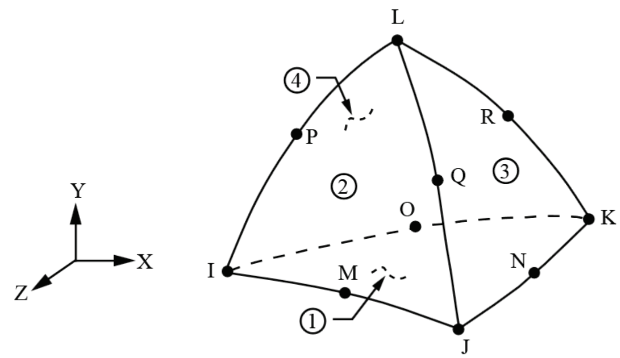

Figure 12 The geometry of the SOLID187 element [6]

The SOLID187 element is a higher-order element with ten nodes. It behaves according to quadratic displacement and is suitable for irregular meshes. Each node on the element has three degrees of freedom: displacement along the *x*, *y*, and *z* axes. The element has the capability to model plasticity, hyperelasticity, creep, strain-hardening, large deflections, and large deformations. It can handle mixed formulations for simulating deformations in nearly incompressible elastoplastic and fully incompressible hyperelastic materials.

The appearance of the finite element mesh is shown in Fig. 13. In the area where the path for the profile is defined, a finer mesh of elements was used. A function was applied to reduce the element size in a specific region, resulting in more accurate results with a smaller step.

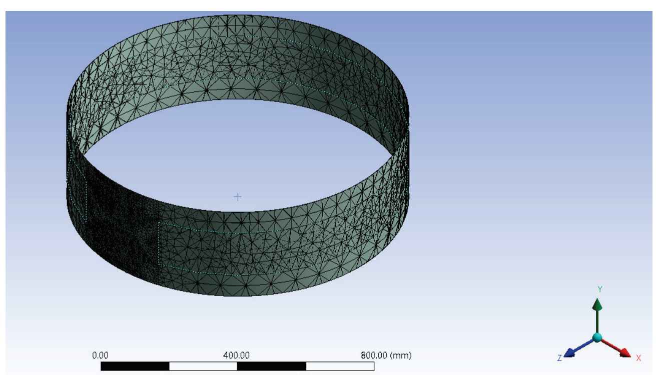

Figure 13 The finite element mesh of the three-dimensional model

Fig. 14 shows the total deformation throughout the volume of the body. The results are very similar to those shown in Fig. 10.

The maximum total deformation is calculated depending on the changing applied pressure. The results of total deformation over time are shown in Tab. 5.

In order to compare the analysis results using SHELL281 elements and SOLID187 elements, data was extracted for the unloaded state at the same cross-section, and a new profile for calculating the volume of the rotational body was obtained. The results are shown in Fig. 15 and Tab. 6.

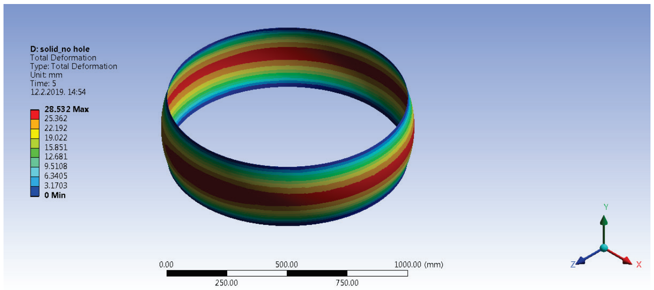

Figure 14 Total deformation of the three-dimensional model

Table 6 The total deformation over time for the three-dimensional model

	Time, s	Maximum deformation, mm
1	0.2	2.59
2	0.4	4.84
3	0.7	7.73
4	1	10.20
5	1.2	11.69
6	1.4	13.34
7	1.7	15.84
8	2	18.23
9	2.2	20.59
10	2.4	23.15
11	2.7	27.12
12	3	31.74
13	3.2	31.39
14	3.4	31.02
15	3.7	30.47
16	4	29.91
17	4.2	29.63
18	4.4	29.35
19	4.7	28.94
20	5	28.53

Table 7 Results of deformation in the *z*-axis direction for the three-dimensional model

	Length, mm	Deformation in the direction of the axis *z*, mm
1	0	−1.91E-14
2	6.25	1.82
3	12.5	3.73
4	18.75	5.60
5	25	7.43
6	31.25	9.22
7	37.5	10.98
8	43.75	12.71
9	50	14.40
10	56.25	16.03
11	62.5	17.61
12	68.75	19.15
13	75	20.63
14	81.25	21.85
15	87.5	22.96
16	93.75	23.96
17	100	24.84
18	106.25	25.62
19	112.5	26.30
20	118.75	26.86
21	125	27.32
22	131.25	27.68
23	137.5	27.93
24	143.75	28.08
25	150	28.14
26	156.25	28.09
27	162.5	27.94
28	168.75	27.70
29	175	27.34
30	181.25	26.90
31	187.5	26.33
32	193.75	25.68
33	200	24.92
34	206.25	24.06
35	212.5	23.08
36	218.75	21.99
37	225	20.79
38	231.25	19.42
39	237.5	17.97
40	243.75	16.44
41	250	14.83
42	256.25	13.14
43	262.5	11.37
44	268.75	9.52
45	275	7.63
46	281.25	5.72
47	287.5	3.79
48	293.75	1.85
49	300	0

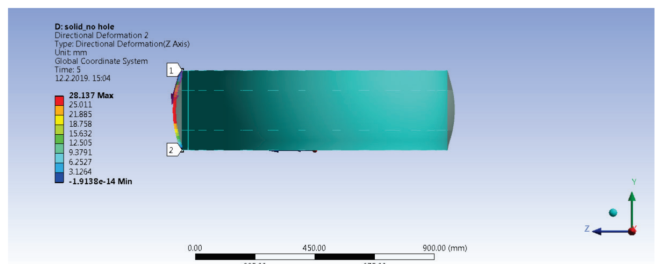

Figure 15 The directed deformation in the *z*-axis direction and the profile of the 3D model

5 THE CALCULATION OF THE VOLUME OF ROTATIONAL BODIES

The volumes of rotational bodies are calculated by taking into account that their cross-section at each level is a known circle, and its area is known [7].

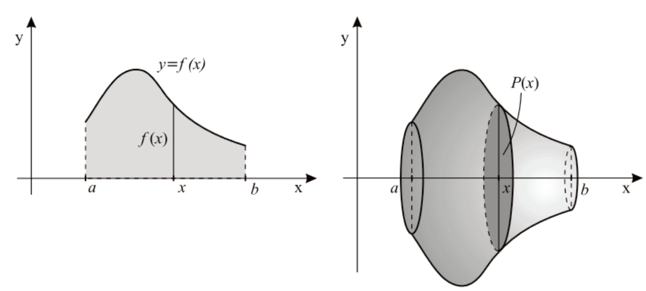

Figure 16 The volume of rotational bodies [7]

The volume of a rotational body when we have a known function $f(x)$:

$$\int_a^b \pi\left(f(x)\right)^2 \mathrm{d}x \tag{1}$$

The volume of a rotational body between two known functions $f(x)$ and $g(x)$:

$$\int_a^b \pi\left(\left(f(x)\right)^2-\left(g(x)\right)^2\right)\mathrm{d}x \tag{2}$$

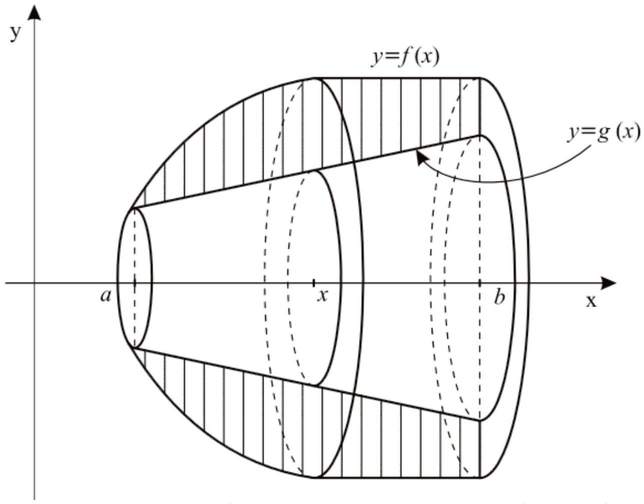

Figure 17 The volume of a rotational body between two given functions [6]

For simplicity, we will assume that the profile line of the deformed shape is a segment of a circle. From the equation of a circle:

$$\left(x-x_0\right)^2+\left(y-y_0\right)^2=R^2 \tag{3}$$

A system of equations is set up to obtain the equation of a circle passing through three points, by which we approximate the deformed profile:

$$\left(x_1-x_0\right)^2+\left(y_1-y_0\right)^2=R^2 \tag{4}$$

$$\left(x_2-x_0\right)^2+\left(y_2-y_0\right)^2=R^2 \tag{5}$$

$$\left(x_3-x_0\right)^2+\left(y_3-y_0\right)^2=R^2 \tag{6}$$

5.1 The Volume of the Thin-Walled Model

The coordinates of three points on the deformed shell of the vessel have been obtained. It is known:

$x_1=0$, $y_1=500$
$x_2=150$, $y_2=530$
$x_3=300$, $y_3=500$

From the system of Eqs. (4), (5), (6), we obtain the parameters of the circle function:

$x_0=150$, $y_0=140$
$R=390$ mm

In this case, the first function describing the upper boundary of the rotational body $f(x)$ is:

$$f(x)=y_0+\sqrt{R^2-\left(x-x_0\right)^2}$$

$$f(x)=\sqrt{152100-\left(x-150\right)^2}+140 \tag{7}$$

The second function describing the lower boundary of the rotational body $g(x)$ is:

$$g(x)=500$$

The volume of the rotating body follows:

$$V=\int_0^{300} \pi\left(\left(f(x)\right)^2-\left(g(x)\right)^2\right)\mathrm{d}x=19457027{,}49\ \mathrm{mm}^3 \tag{8}$$

$$V=19{,}457\ \mathrm{dm}^3$$

The volume of the space inside the vessel obtained by deforming the outer shell calculated using SHELL281 elements amounts to 19,457 dm^3.

6 VOLUME OF THE THREE-DIMENSIONAL MODEL

The coordinates of three points on the deformed shell of the vessel have been obtained. It is known:

$x_1=0$, $y_1=500$
$x_2=150$, $y_2=528$
$x_3=300$, $y_3=500$

From the system of Eqs. (4), (5), (6), we obtain the parameters of the circle function:

$x_0=150$, $y_0=112{,}214$
$R=415{,}786$ mm

In this case, the first function describing the upper boundary of the rotational body $f(x)$ is:

$$f(x)=y_0+\sqrt{R^2-\left(x-x_0\right)^2}$$

$$f(x)=\sqrt{172877{,}760204-\left(x-150\right)^2}+112{,}214286 \tag{9}$$

The second function describing the lower boundary of the rotational body $g(x)$ is:

$$g(x)=500$$

The volume of the three-dimensional model body follows:

$$V=\int_0^{300} \pi\left(\left(f(x)\right)^2-\left(g(x)\right)^2\right)dx=18406357{,}997\ \mathrm{mm}^3 \tag{10}$$

$$V=18{,}406\ \mathrm{dm}^3$$

The volume of the space inside the vessel obtained by deforming the outer shell calculated using SOLID187 elements amounts to 18,113 dm^3.

7 CONCLUSION

Using the finite element method, it is easy to calculate an approximate volume that encloses the deformed shell of a welded cooling vessel. In this study, the deformed profile is approximated by a segment of a circle, and a more detailed analysis and nonlinear regression of cross-sectional point results could provide a more precise approximation. In that case, it would not be enough to only know the maximum deflection in the central part but also the exact arrangement of elements and the displacement of each element individually. This dependency would be represented as a function of the distance from the coordinate axis, yielding different results.

In this study, two analyses were performed using SHELL281 elements and SOLID187 elements, and the calculation results of the volume are similar, showing discrepancies at the level of 5 %. The results should be confirmed by theoretical calculations and volume measurements after the vessel is manufactured under the same loading conditions as in the simulation.

In the subsequent stages of the research, it will be necessary to create a physical model to validate the theoretical numerical simulation.

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