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# LEPTONIC DECAYS OF VECTOR MESONS IN SIX-QUARK MODELS*

S. FAJFER and Z. STIPČEVIĆ

*Institute of Physics, University of Sarajevo,* 71000 *Sarajevo, Yugoslavia*

Within the six-quark sequential model and the assumption that heavy quarks do not mix we deduce the vector meson leptonic-decay ratios compatible with an empirical rule. A specified modification of Weinberg's first spectral function sum rule is implied.

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We consider six-quark models with a charge assignment (2/3, −1/3, −1/3, 2/3, −1/3, 2/3) for ($u$, $d$, $s$, $c$, $b$, $t$), respectively (This corresponds to what is usually refered to as the sequential model (Harari 1978[1]) and has gained wide acceptance through its connection with the sequential doublets of quarks and leptons in unified theories). The quarks are classified according to a 6-dimensional representation of $SU(6)$. Due to large mass differences among the quarks an interaction symmetry connected with this group would be badly broken. We assume, however, that the electromagnetic current $j_\mu^{em}$ transforms as components of a regular tensor operator $j_a^\mu$ of $U(6)$, i.e. of the direct product of the $SU(6)$ and the baryon number group $U(1)$. Since the electromagnetic current never changes quantum numbers we demand that $j_\mu^{em}$, as a linear combination of $j_a^\mu$, contains $a$ corresponding only to the labels of the $SU(6)\times U(1)$ diagonal generators, i.e. $a = 0, 3, 8, 15, 24, 35$. The quark charge assignment and the choice of $\lambda$-like matrices for the quark representation of the $SU(6)$ generators yields

$$j_\mu^{em} = j_\mu^3 + \frac{1}{\sqrt{3}} j_\mu^8 - \frac{2}{\sqrt{6}} j_\mu^{15} + \frac{2}{\sqrt{10}} j_\mu^{24} - \frac{3}{\sqrt{15}} j_\mu^{35} + \frac{1}{\sqrt{3}} j_\mu^0 \qquad (1)$$

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or

$$j_\mu^{em} = C_\mu^j j_\mu^a \tag{2}$$

with coefficients given in Table 1.

TABLE 1.

| $a$ | 0 | 3 | 8 | 15 | 24 | 35 |
|---|---|---|---|---|---|---|
| $C_a^j$ | $\frac{1}{\sqrt{3}}$ | 1 | $\frac{1}{\sqrt{3}}$ | $-\frac{2}{\sqrt{6}}$ | $\frac{2}{\sqrt{10}}$ | $-\frac{3}{\sqrt{15}}$ |

It is assumed that the vector mesons with vacuum quantum numbers: $\varrho^0$, $\omega$, $\varphi$, $\psi$, $\Upsilon$ and $\zeta$ form a unitary mapping of quark-antiquark pairs: $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, $c\bar{c}$, $b\bar{b}$ and $t\bar{t}$. This necessitates the inclusion of yet undetected but broadly discussed very heavy resonance $\zeta$ (e.g. Lichtenberg and Wills 1978)[2]. We sustain the assumption that $\varrho^0$, $\omega$ and $\varphi$ contain only the $SU(3)$ triplet quarks $u$, $d$ and $s$ with the $\omega$-$\varphi$ ideal mixing. The heavy quarks $c$, $t$ and $b$ compose heavy resonances but cannot mix, as notably evidenced by the success of the quarkonium approach. The quark in the $\Upsilon(9.46)$ is taken as $b$ on account of the recent measurements carried out by the PLUTO and DASP collaborations which indicate the charge assignment $-\frac{1}{3}$ (Berger et al. 1978, Darden et al. 1978)[3,4].

So we have

$$| V > = C_{qq}^V | q > | \bar{q} >, \tag{3}$$

with nonzero coefficients:

$$C_{uu}^\varrho = - C_{dd}^\varrho = C_{uu}^\omega = C_{dd}^\omega = \frac{1}{\sqrt{2}},$$

$$C_{ss}^\varphi = C_{bb}^\Upsilon = C_{tt}^\zeta = 1.$$

Making use of the $6 \times \bar{6}$ reduction coefficients of $SU(6)$ one can express vector mesons via the basis of the regular and the singlet representations. One gets

$$| V > = C_a^V | a >, \quad a = 0, 3, 8, 15, 24, 35, \tag{4}$$

with coefficients $C_a^V$ given in Table 2:

TABLE 2.

| | 0 | 3 | 8 | 15 | 24 | 35 |
|---|---|---|---|---|---|---|
| $\varrho^0$ | 0 | 1 | 0 | 0 | 0 | 0 |
| $\omega$ | $\frac{1}{\sqrt{3}}$ | 0 | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{6}}$ | $\frac{1}{\sqrt{10}}$ | $\frac{1}{\sqrt{15}}$ |
| $\varphi$ | $\frac{1}{\sqrt{6}}$ | 0 | $-\frac{2}{\sqrt{6}}$ | $\frac{1}{\sqrt{12}}$ | $\frac{1}{\sqrt{20}}$ | $\frac{1}{\sqrt{30}}$ |
| $\psi$ | $\frac{1}{\sqrt{6}}$ | 0 | 0 | $-\frac{3}{\sqrt{12}}$ | $\frac{1}{\sqrt{20}}$ | $\frac{1}{\sqrt{30}}$ |
| $\Upsilon$ | $\frac{1}{\sqrt{6}}$ | 0 | 0 | 0 | $-\frac{4}{\sqrt{20}}$ | $\frac{1}{\sqrt{30}}$ |
| $\zeta$ | $\frac{1}{\sqrt{6}}$ | 0 | 0 | 0 | 0 | $-\frac{5}{\sqrt{30}}$ |

The electromagnetic decay of vector mesons into a lepton pair is given, in the zero width approximation, by

$$\Gamma(V \to e^+ e^-) = \frac{4\pi\,\alpha^2}{3\,m_V^3}\, g_V^2\,(m_V^2) \tag{5}$$

with the vector meson form factor $g_V(p^2)$ defined via

$$< 0 \,|\, j_\mu^{em}(0) \,|\, V; \lambda, p > \; = \varepsilon_\mu^{(\lambda)}(p)\, g_V(p^2). \tag{6}$$

The assumed current structure (2), vector meson composition (4) and the Wigner--Eckart reduction

$$< 0 \,|\, \varepsilon \cdot j^\alpha \,|\, \beta > \; = \delta^\alpha_\beta\, f(m_V^2), \tag{7}$$

give

$$\Gamma(V \to e^+ e^-) = \frac{4\pi\,\alpha^2}{3\,m_V^3}\, C_V^2\, f^2\,(m_V^2), \tag{8}$$

with coefficients $C_V = \sum\limits_a C_a^V C_a^I$ given in Table 3.

TABLE 3.

| | $\varrho^0$ | $\omega$ | $\varphi$ | $\psi$ | $\Upsilon$ | $\zeta$ |
|---|---|---|---|---|---|---|
| $C_V$ | 1 | $\frac{1}{3}$ | $-\frac{\sqrt{2}}{3}$ | $\frac{2\sqrt{2}}{3}$ | $-\frac{\sqrt{2}}{3}$ | $\frac{2\sqrt{2}}{3}$ |

Examining the experimental decay widths

$$\left.\begin{aligned} \Gamma_{\varrho ee} &= 0.0067\ \text{MeV}, \\ \Gamma_{\omega ee} &= 0.0007\ \text{MeV}, \\ \Gamma_{\varphi ee} &= 0.0013\ \text{MeV}, \\ \Gamma_{\psi ee} &= 0.0047\ \text{MeV}. \end{aligned}\right\} \quad \text{(Particle Data 1978)}$$

and the recently established $\Gamma_{\Upsilon ee} = 1.32 \pm 0.09$ keV (DESY, Flügge 1979), which for comparison with the above values can be taken as $\Gamma_{\Upsilon ee} = 0.0013$ MeV, one concludes that satisfactory agreement with (8) implies the assumption that $f^2\,(m_V^2)/m_V^3$ is mass independent. Thus we have

$$\Gamma(V \to e^+ e^-) \sim C_V^2, \tag{9}$$

or, equivalently,

$$\Gamma_{\varrho ee} : \Gamma_{\omega ee} : \Gamma_{\varphi ee} : \Gamma_{\psi ee} : \Gamma_{\Upsilon ee} : \Gamma_{\zeta ee} = 9 : 1 : 2 : 8 : 2 : 8. \tag{10}$$

Noting the equality

$$\frac{1}{\sqrt{2}} \sum_a C_a^I C_a^V = \sum_q Q_q C_{qq}^V, \tag{11}$$

which connects quark charges $Q_a$ with the coefficients $C_a^I$, $C_a^V$ and $C_{qq}^V$, one obtains the empirical rule (Flügge 1979)[5)]

$$\Gamma_{\psi ee} : \Gamma_{\Upsilon ee} = Q_c^2 : Q_b^2. \tag{12}$$

We would like to make two comments. First, the expected decay width $\Gamma_{\zeta ee}$ should be comparable to the $\Gamma_{\psi ee}$. In case, however, that instead it happens to be roughly equal to $\Gamma_{\Upsilon ee}$ this would be an indication for $Q_t = -\frac{1}{3}$, and consequently the quark charge assignment (2/3, −1/3, −1/3, 2/3, −1/3, −1/3) for ($u, d, s, c, b, t$) which, within $SU(6)$ quark classification, presents an alternative possibility. The second comment concerns the implication of the mass independence assumption for $f^2\,(m_V^2)/m_V^3$, which indicates approximate validity of a Weinberg-type spectral functions sum rule

$$\int \frac{\varrho^a(m^2)}{m^3}\, \mathrm{d}m^2 = const. \tag{13}$$

An appropriate analysis of the field theoretic origin of (13) could add to our understanding of the high energy approach to exact symmetry.

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## LEPTONSKI RASPAD VEKTORSKOG MEZONA U MODELU ŠEST KVARKOVA

S. FAJFER i Z. STIPČEVIĆ

*Institut za fiziku, Univerzitet u Sarajevu,* 71000 *Sarajevo*

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U okviru sekvencijalnog modela šest kvarkova i pretpostavke o nemiješanju teških kvarkova izvedeni su omjeri leptonskih raspada vektorskih mezona koji su kompatibilni sa poznatim empirijskim pravilom. Rezultati impliciraju potrebu jedne specijalne modifikacije Prvog Weinbergovog sumacionog pravila za spektralne funkcije.