

LETTER TO THE EDITOR

THE PSEUDO- DIMENSION RULE AND  $\Upsilon'$  (10.0)-DECAY

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In this note the pseudo-dimension rule has been employed for the investigations of the two-body radiative and hadronic decays of the  $\Upsilon'$  (10.0). The selection rule concerned suggests that in  $\Upsilon'$  (10.0)-decay the two-body hadronic modes  $\pi\pi$ ,  $K\bar{K}$ ,  $D\bar{D}$ ,  $F\bar{F}$  will be strongly suppressed. It has also been discussed that the strong suppressions of the  $\pi\pi$  and  $K\bar{K}$  modes in  $\Upsilon'$  (10.0)-decay will imply that the particle concerned is an  $SU(3)$  singlet.

It may be recalled that the recently discovered<sup>1-3)</sup> vector resonance  $\Upsilon$  (9.4) and its excited states are interpreted<sup>4,5)</sup> as the bound states of a heavy quark (the fifth quark) and its antiquark. It may be noted that the upsilon-dynamics can be probed by investigating their decays. Their radiative decays, in particular, can offer a convenient platform for testing<sup>6-8)</sup> the predictions of *Quantum Chromodynamics* (QCD), the nonabelian gauge theory of quarks with flavour and colour interacting with massless coloured vector gluons<sup>2,8)</sup>. For these reasons the radiative and other two-body hadronic decays of the  $\Upsilon$  (9.4) have already been theoretically investigated<sup>6,7,9)</sup>. For the same reasons the radiative and other hadronic decays of the excited states of the  $\Upsilon$  (9.4) are also important. In this connection we may also note that the suppressions or otherwise of the theoretically expected modes of the upsilons will throw much light on the special features of the upsilon dynamics. In investigating the probable suppressions of some theoretically allowed hadronic modes of the upsilons the well known selection rules and, in particular, the quark duality diagram constraint i.e. the OZI rule<sup>10-12)</sup> are very useful. If, however, these selection rules are used along with the pseudo-dimension rule<sup>13-18)</sup> for particle decays, then, additional information regarding the probable suppressions of some theoretically expected decay modes of unstable particles are obtained. In a recent paper<sup>18)</sup> we have investigated the feasibility of occurrences of the radiative and other two-body hadronic decays of the  $\Upsilon$  (9.4). In this note we shall be concerned with the probable suppressions of some two-body hadronic modes

in  $\Upsilon'$  (10.0)-decay making use of the *OZI* rule along with the pseudo-dimension rule<sup>13-18)</sup> discussed below. To be specific, in this note we shall show that the  $\pi\pi$ ,  $K\bar{K}$ ,  $D\bar{D}$  and  $F\bar{F}$  modes in  $\Upsilon'$ -decay must be strongly suppressed. Needless to mention that the suppressions of the  $\pi\pi$  and  $K\bar{K}$  modes in  $\Upsilon'$ -decay will imply that the particle concerned is an  $SU(3)$  singlet.

The pseudo-dimensions of free fields as well as the pseudo-dimension rule have been discussed in a compact form in recent papers<sup>14-18)</sup> and the details are given in the original paper<sup>13)</sup>. The pseudo-dimension rule reads: All the allowed decays of an unstable particle must be governed by *one and only one* of the following two constraints

$$d_u > D \tag{1a}$$

$$\text{and, } d_u < D \tag{1b}$$

where  $d_u$  is the magnitude of the pseudo-dimension of the field of the unstable particle and  $D$  is the sum of the magnitudes of the pseudo-dimensions of the fields of the particles constituting a decay mode for the unstable particle concerned. The magnitudes of the pseudo-dimensions, denoted by  $d$ , of free fields are given by the following relations

$$d = 3\mathcal{J} \text{ for fields having the actual spin } \mathcal{J} \neq 0 \tag{2a}$$

$$d = 1 \text{ for fields having } \mathcal{J} = 0 \tag{2b}$$

$$d = 2 \text{ for the photon} \tag{2c}$$

As Eqs. (2a) — (2c) refer to free fields for the reasons discussed in previous papers<sup>13-18)</sup>, therefore, the quantity  $D$  appearing in relations (1a) and (1b) refers to decay modes *not* occurring through subreactions which necessitate interacting fields. For this reason, the selection rule stated above concerns itself, for example, with the  $3\pi$  mode but *not* with the quasi two-body mode  $\rho\pi$  (which results from the subreactions associated with the  $2\pi$  mode) in  $\varphi$  (1020)-decay.

To exemplify how the pseudo-dimension rule explains<sup>13-18)</sup> the suppressions of the otherwise theoretically expected decay modes of the unstable particles, we consider  $\omega$  (783)-decay. For the observed<sup>19)</sup> decay  $\omega(783) \rightarrow 2\pi, 3\pi, \pi^0\gamma, e^+e^-$  we have for the  $\omega$  (which is a  $\mathcal{J} = 1$  particle),  $d_u = 3$  which follows from Eq. (2a) and for the  $2\pi$  mode  $D = d_\pi + d_\pi = 1 + 1 = 2$  as  $d_\pi = 1$  which is evident from Eq. (2b), for  $\pi^0\gamma$  mode,  $D = d_\pi + d_\gamma = 1 + 2 = 3$  since  $d_\gamma = 2$  which follows from Eq. (2c) and for the  $e^+e^-$  mode  $D = d_{e^+} + d_{e^-} = 3/2 + 3/2 = 3$  as  $d = 3/2$  for a spin  $-1/2$  particle as implied by Eq. (2a). Clearly, the observed decays  $\omega(d_u = 3) \rightarrow 2\pi(D = 2), 3\pi(D = 3), \pi^0\gamma(D = 3), e^+e^-(D = 3)$  reveal that the appropriate constraint for  $\omega(783)$ -decay is  $d_u > D$  which must have to be satisfied by all other theoretically expected modes (in order they are not forbidden and as such suppressed). This is so because according to the pseudo-dimension rule *all* the allowed decays of a given unstable particle must be controlled by one and the same constraint. It is interesting to note that the constraint  $d_u > D$ , which is found to hold in the observed decays of the  $\omega(783)$ , is *not* satisfied by the theoretically expected modes  $\pi^+\pi^-\gamma(D = 4), \pi^0\pi^0\gamma(D = 4), \pi^0\mu^+\mu^-(D = 4), 3\gamma(D = 6)$  as  $d_u = 3$  for the  $\omega(783)$ . These modes, needless to mention, have been found to be strongly suppressed<sup>19)</sup>. In this connection it may be recalled that an unstable particle, if it is a resonance, can have in principle at least electromagnetic and weak decay channels apart from the usual strong ones.

Obviously, then the strongly suppressed modes (mentioned above) of the  $\omega$  (783)-decay cannot be simply thrown away by hands as they are not strong decay modes. The suppressions of these theoretically not forbidden modes are elegantly understood in terms of the pseudodimension rule. We consider another example of the selection rule concerned. The observed decays  $\psi$  (3100,  $d_u = 3$ )  $\rightarrow e^+ e^-$  ( $D = 4$ ),  $\mu^+ \mu^-$  ( $D = 3$ ),  $4\pi$  ( $D = 4$ ),  $5\pi$  ( $D = 5$ ),  $6\pi$  ( $D = 6$ ),  $7\pi$  ( $D = 7$ ) and  $\psi'$  (3685,  $d_u = 3$ )  $\rightarrow e^+ e^-$  ( $D = 3$ ),  $\mu^+ \mu^-$  ( $D = 3$ ),  $\psi\eta$  ( $D = 4$ ),  $\psi\pi\pi$  ( $D = 5$ ),  $5\pi$  ( $D = 5$ ),  $\chi$  (3415)  $\gamma$  ( $D = 3$ ),  $\chi$  (3510)  $\gamma$  ( $D = 5$ ),  $\chi$  (3555)  $\gamma$  ( $D = 7$ ), ... suggest that the decays of both the  $\psi$  (3100) and  $\psi'$  (3685) are governed by the constraint  $d_u \leq D$  which is, however, not satisfied by the modes  $\pi^+ \pi^-$  ( $D = 2$ ),  $K^+ K^-$  ( $D = 2$ ). Therefore, according to the pseudodimension rule, the decays  $\psi, \psi' \rightarrow \pi^+ \pi^-, K^+ K^-$  are forbidden<sup>14)</sup> and as a consequence these decays should be suppressed. This is in conformity with the experimentally observed suppressions of the  $\pi^+ \pi^-$  and  $K^+ K^-$  modes in the decays of the  $\psi$  and  $\psi'$ . We have given above three examples and many such examples have been discussed in previous papers<sup>13-18)</sup>.

To know precisely which one of the two constraints, given by the relations (1a) and (1b), will be valid in  $Y'$  decay, the knowledge of its hadronic modes (which will determine the sign of the inequality in the appropriate constraint) are necessary. Since experimental identifications of the hadronic modes in  $Y'$  (10.0)-decay have not yet been reported, specification of the constraint appropriate for the decays of the particle concerned is not possible in a straightforward manner as in the decays of  $\omega$  (783),  $\psi$  (3100) and  $\psi'$  (3685) considered above. But, this difficulty can be bypassed by taking advantage of the *OZI* rule. It may be recalled that according to the *OZI* rule only those decays are allowed which are described by connected quark lines whereas the decays described by the disconnected diagrams are suppressed. The  $Y'$  (10.0) occurs below the «bottom» («beauty») threshold<sup>2)</sup> and as such all its decays occur by disconnected diagrams. However, the decays  $Y'$  (10.0)  $\rightarrow Y$  (9.4)  $\pi\pi$  can take place via *singly* disconnected diagrams and, therefore, they are first forbidden. Obviously, these decays are expected to dominate over those described by *doubly* disconnected diagrams. It may not be out of place if we mention that the decays  $\psi' \rightarrow \psi\pi\pi$  have been found to be the most dominant<sup>19)</sup> in  $\psi'$ -decay. Remembering the analogy<sup>20)</sup> between the  $\psi$  and  $Y$  and also the same between the  $\psi'$  and  $Y'$ , it is reasonable to expect that the decays  $Y' \rightarrow Y \pi\pi$  will be dominant. Further, the fact that the upsilons are produced in  $e^+ e^-$  annihilations suggests the nonvanishing *QED* amplitudes for the decays  $Y' \rightarrow e^+ e^-$ . Therefore, the decays  $Y'$  ( $d_u = 3$ )  $\rightarrow e^+ e^-$  ( $D = 3$ ),  $Y\pi\pi$  ( $D = 5$ ) indicate that  $Y'$ -decay must be governed by the constraint  $d_u < D$ ;  $d_u = 3$  for the  $Y'$  which is a  $J = 1$  particle as it couples to the photon. Obviously, the constraint  $d_u < D$  will allow the two-body modes like  $K\bar{K}^*$  ( $D = 4$ ),  $D(1865)\bar{D}^*$  (2010) ( $D = 4$ ),  $D^*$  (2010)  $\bar{D}^*$  (2010) ( $D = 6$ ),  $F\bar{F}^*$  ( $D = 4$ ),  $F^* \bar{F}^*$  ( $D = 6$ ), .....,  $X\gamma$  ( $D = 3$ , if  $X$  is a pseudo-scalar hadron,  $D = 4$  if  $X$  is a vector hadron and  $D = 5$  if  $X$  is a tensor hadron). It is interesting to note, however, that the above constraint is *not* satisfied by the two-body hadronic modes  $\pi\pi$  ( $D = 2$ ),  $K\bar{K}$  ( $D = 2$ ),  $D(1865)\bar{D}$  (1865) ( $D = 2$ ),  $F\bar{F}$  ( $D = 2$ ), and, therefore, these modes are forbidden by the pseudo-dimension rule according to which, we repeat to emphasize, only one of the two constraints can be valid in the decays of a given unstable particle. From what have been said so far it is clear that the two-body hadronic decay modes  $\pi\pi$ ,  $K\bar{K}$ , and

$F\bar{F}$  in  $\Upsilon'$ -decay are forbidden both by the *OZI* rule and the pseudo-dimension rule. Consequently, these modes will be strongly suppressed with respect to the two-body modes (considered above) which are *OZI*-forbidden but allowed by the pseudo-dimension rule. It may be emphasized here that the suppressions of the  $\pi\pi$  and  $K\bar{K}$  modes for  $\Upsilon'$ -decay are important for the  $SU(3)$  classification<sup>2)</sup> of the particle concerned. In this connection we may also note that the  $\Upsilon'(10.0)$  is produced in  $e^+e^-$  annihilations and, therefore, it can couple to the (virtual) photon. Therefore, from *QED* considerations one expects the decays  $\Upsilon' \rightarrow \pi^+\pi^-, K^+K^-, D^+D^-, F^+F^-$  which can occur via the one-photon intermediate state. However, inspite of the existence of the *QED* amplitudes for these decays, they will be strongly suppressed by the pseudo-dimension rule. The suppressions of the  $\pi^+\pi^-, K^+K^-$ -modes are also expected<sup>2)</sup> from the  $SU(3)$  considerations if we treat the  $\Upsilon'(10.0)$  as an  $SU(3)$  singlet.

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## PSEUDODIMENZIONALNO PRAVILO I $\Upsilon'(10.0)$ RASPAD

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Razmatra se primjena pseudodimenzionalnog pravila na dvočestični radijativni i hadronski raspad  $\Upsilon'(10.0)$  čestice. Odgovarajuće izborno pravilo sugerira da je raspad  $\Upsilon'(10.0)$  na dvočestične modove  $\pi\pi$ ,  $K\bar{K}$ ,  $D\bar{D}$ ,  $F\bar{F}$  potisnut. Također se zaključuje da jako potisnuti modovi  $\pi\pi$  i  $K\bar{K}$  u  $\Upsilon'(10.0)$  raspadu impliciraju da je raspadnuta čestica  $SU(3)$  singlet.