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A NEW GENERALIZED VIBRATIONAL RULE (GVR)

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The asymptotic complementarity of anharmonic contributions to the energies and transition moments is derived for one-phonon multiplet in odd-A nuclei. The new rule, to be referred to as the Energy-B ($E2$) rule, arises as a consequence of nuclear Ward identities. Systematic experimental verification of the new rule is suggested.

Let us consider in the framework of the particle anharmonic vibration coupling[1,2] one-phonon multiplet $|j, 12; I\rangle$ based on the single-particle state $|j\rangle$. We include leading diagrams, up to the second order, for the energies $E(j, 12; I)$ and for the transition moments $\sqrt{B(E2)(|j, 12; I\rangle \rightarrow |j\rangle)}$. These diagrams are presented in Fig. 1. The key in selecting this class of leading diagrams is such that neither the self energies nor the vertex corrections are included. In fact, the self-energies and the vertex corrections for the boson-fermion coupled system show a tendency towards cancellation (asymptotic nuclear Ward identity)[3-5], therefore their influence is reduced. Hence, the low- order diagrams dominate, although the particle-vibration coupling strength is not weak. The convergence of the diagrammatic expansion is especially improved in the case of only one single-particle configuration, as in the case of unique-parity states.

The diagrams considered here include both the usual particle-vibration vertex and a four-point vertex, which involves the collective matrix element.

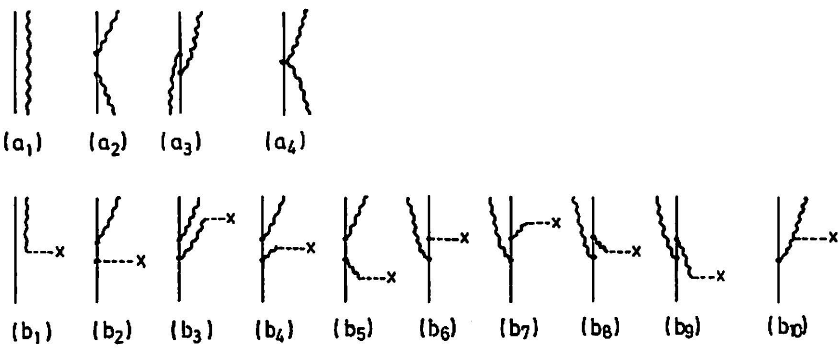

Fig. 1. Diagrams included for the energies of the multiplet states $E(j, 12; I)$ $(a_1 - a_4)$ and for the transition moments $\sqrt{(E2)(|j, 12; I\rangle \rightarrow |j\rangle)}$ $(b_1 - b_{10})$.

Total contributions from diagrams in Fig. 1 can be expressed in the form:

$$\frac{E(|j, 12; I\rangle)}{\hbar\omega_2} = 1 + \chi[\mathcal{J}_1 + \zeta\mathcal{J}_2] \tag{1}$$

$$\sqrt{\frac{B(E2)(|j, 12; I\rangle \rightarrow |j\rangle)}{B(E2)(2_1 \rightarrow 0_1)}} = 1 - \chi[\mathcal{J}_1(1 + \mu) + \zeta\mathcal{J}_2]. \tag{2}$$

Here

$$\chi = \left(\frac{a}{\hbar\omega_2}\right)^2 \frac{5}{4} \frac{(2j-1)(2j+3)}{2j(2j+2)} \tag{3}$$

$$\mathcal{J}_1 = \delta_{Ij} + (2j+1)\begin{Bmatrix} I & j & 2 \\ j & j & 2 \end{Bmatrix} \tag{4}$$

$$\mathcal{J}_2 = \sqrt{\frac{35}{2}}(-)^{I-j}\sqrt{2j+1}\begin{Bmatrix} 2 & 2 & 2 \\ j & j & I \end{Bmatrix} \tag{5}$$

$$\zeta = -\frac{1}{\sqrt{4\pi}} \frac{\hbar\omega_2}{|a|} \sqrt{\frac{2j(2j+2)}{(2j-1)(2j+3)}} \frac{\mathcal{O}(j)\,Q(2_1)}{\sqrt{B(E2)(2_1 \rightarrow 0_1)}} \tag{6}$$

$$\mu = \frac{2e^{sp}}{e^{pol}} \tag{7}$$

$$a = -\frac{\sqrt{4\pi}}{3}\langle k\rangle \frac{\sqrt{B(E2)(2_1 \rightarrow 0_1)}}{Z \cdot R_0^2} \tag{8}$$

$$\mathcal{O}(j) = \begin{cases} +1 & \text{if } |j\rangle \text{ is particle-like state} \\ -1 & \text{if } |j\rangle \text{ is hole-like state} \end{cases} \tag{9}$$

$$e^{\text{pol}} = \frac{40\,\pi}{9} \frac{B(E2)(2_1 \to 0_1)}{Z \cdot R_0^4 \cdot \hbar\,\omega_2} \langle k \rangle . \tag{10}$$

The quantities appearing in (1)—(10) are:

$\hbar\,\omega_2$ — the phonon energy, taken as the energy of 2_1^+ state in the nucleus vibrator, i. e. the neighboroing even-even nucleus;

$B(E2)(2_1^+ \to 0_1^+)$ — the $B(E2)$ value for the $2_1^+ \to 0_1^+$ transition in the nucleus-vibrator;

e^{sp} — the single-particle charge which includes also the polarization effect of the closed shells;

$$e^{\text{sp}} = \begin{cases} 0.5 & \text{for neutrons} \\ 1.5 & \text{for protons} \end{cases} ;$$

$Q(2_1)$ — the quadrupole moment of the 2_1^+ state from nucleus vibrator, a measure of anharmonicity;

$\langle k \rangle$ — the matrix element of the particle-vibration coupling strength; $\langle k \rangle \approx 40$ MeV.

R_0 — the nuclear radius; $R_0 = 1.2 A^{\frac{1}{3}}$ fm;

The Eqs. (1) and (2) will be referred to as Energy $B(E2)$ rule.

In the limit of strong collectivity there is $e^{\text{pol}} \gg e^{\text{sp}}$ and from (1), (2) it follows:

$$\frac{E(j, 12; I)}{\hbar\,\omega_2} + \sqrt{\frac{B\,|(E2)(|j, 12; I\rangle \to |j\rangle)}{B(E2)(2_1 \to 0_1)}} = 2 \tag{11}$$

This relation will be referred to as the Energy-$B(E2)$ sum rule. This relation is the same as the one for the zeroth-order processes, i. e. in the harmonic case; the contributions of all first and second-order diagrams in this sum cancel exactly, and the zeroth-order result is restored. On the other hand each of the ratios $\frac{E(j, 12; I)}{\hbar\,\omega_2}$ and $\sqrt{\frac{B(E2)(|j, 12; I\rangle \to |j\rangle)}{B(E2)(2_1 \to 0_1)}}$ sizeably deviates from the harmonic vibrational limit 1, but these deviations, $\chi\,[\mathcal{J}_1 + \zeta\,\mathcal{J}_2]$ and $-\,\chi\,[\mathcal{J}_1 + \zeta\,\mathcal{J}_2]$, respectively, are of the same magnitude and opposite sign.

In the limit $j \gg 1$ the following asymptotic Energy-B ($E2$) rule arises from (1) and (2):

$$\frac{E(j, 12; I)}{\hbar\,\omega_2} = -\sqrt{\frac{B(E2)(|j, 12; I\rangle \to |j\rangle)}{B(E2)(2_1 \to 0_1)}} = \chi \cdot \begin{cases} -\dfrac{12}{(2j)^2} - \xi, & \text{for } I = j-2 \\ \dfrac{12}{2j} + \dfrac{\xi}{2}, & \text{for } I = j-1 \\ \dfrac{1}{2j} + \xi, & \text{for } I = j \\ -\dfrac{12}{2j} + \dfrac{\xi}{2}, & \text{for } I = j+1 \\ -\dfrac{12}{(2j)^2} - \xi, & \text{for } I = j+2 \end{cases} \tag{12}$$

We note that the asymptotic rule (12) for $\zeta \approx +1$ and -1 generates the pattern characteristic of the »decoupled« and »normal« band, respectively.

The Energy-$BE2$ rule (1), (2), together with the collective (11) and the asymptotic limit (12) are derived in this letter. Experimental verification of these rules is desired. Several other generalized vibrational rules have been introduced previously[1-7].

A more complete account of this work will be published elsewhere[8].

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NOVO GENERALIZIRANO VIBRACIJSKO PRAVILO (GVR)

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Izvedena je asimptotska komplementarnost anharmonijskih doprinosa energiji i prijelaznim momentima za jednofononske multiplete u neparnim jezgrama. Novo pravilo, koje smo nazvali Energija-B ($E2$) pravilo, proizlazi kao posljedica nuklearnih Wardovih identiteta. Sugerirana je sistematska eksperimentalna verifikacija novog pravila.