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NONLEPTONIC EFFECTIVE WEAK HAMILTONIAN AND *QCD* CORRECTIONS

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We present a systematic calculation of the nonleptonic effective weak Hamiltonian based on the Weinberg-Salam model containing four or six quarks. We write out explicitly and discuss the following sectors of the effective weak Hamiltonian: the $\Delta S = 1$, $\Delta C = 0$ sector (including CP violation), the parity-violating $\Delta S = 0$, $\Delta C = 0$ sector and the $\Delta S = 1$, $\Delta C = 1$ sector. Our presentation includes a simple rule for the calculation of quantum-chromodynamics renormalisation effects when flavour-symmetry breaking is considered. We also discuss the theoretical foundation of quantum-chromodynamics corrections and experimental consequences.

1. *Introduction*

It has been argued that the quantum-chromodynamics (*QCD*) renormalisation of the nonleptonic weak Hamiltonian might play an important role in the understanding of nonleptonic weak processes[1]. *QCD* effects are modified when flavour-

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symmetry breaking (*FSB*), associated with unequal quark masses, is taken into account[2]. Such a phenomenon might also play a role in the understanding of *CP* violation in the framework of the Kobayashi-Maskawa model[3–5].

The main purpose of this paper is to present a systematic calculation of *QCD* corrections with *FSB*, which results in an effective Hamiltonian for nonleptonic weak processes. We also include a comparison with experiments and a discussion of the importance of *QCD* corrections for nonleptonic weak processes. *FSB* effects were helpful to calculate nonleptonic hyperon decays[2,6–8] in the description of Ω^- decay branching ratios[9,10] and of the $\Sigma^+ \to p + \gamma$ decay[11]. The effective weak Hamiltonian including *QCD* corrections can also be used to gain some insight into $\Delta S = 0, \Delta I = 0, 2$ parity-violating (*PV*) vector-meson exchanges [12–18]. Penguin diagrams, induced by *FSB*, predict large *PV* effects due to pion exchange[14–16].

However, an analogous approach does not work for $\Delta S = 1$, $\Delta C = 1$ D-meson decays[19]. As we shall show later on, *QCD* corrections including *FSB* make the disagreement between theory and experiment even worse.

While the *QCD* renormalisation of the weak Hamiltonian is clearly described[1] for zero-mass quarks, the inclusion of *FSB* is either decsribed in an intuitive way[2] or calculated in quite a complicated and involved scheme[4]. The original paper[2] presents the solution of the renormalisation-group equation (*RGE*)[20] without calculational details. We will show how the *QCD* renormalisation including *FSB* can be carried out in a mathematically well-defined way, giving results which are exact in the leading-logarithm approximation. It turns out that the Shifman-Vainstein-Zakharov[2] (*SVZ*) Hamiltonian agrees with such a calculation.

We perform our calculation in two steps:

i) We have to calculate renormalisation constants for various operators appearing in the short-distance expansion of the weak Hamiltonian.

ii) We use renormalisation constants as an input in the Callan-Symanzik-Weinberg[20] *RGE*, which we use to find the effective weak Hamiltonian.

In Sec. 2 we illustrate the calculation of renormalisation constants in the leading-logarithmic approximation. Owing to symmetry breaking, one is not allowed to calculate only the divergent parts of various diagrams. Therefore, we propose a rule which simplifies the calculation of the finite leading-logarithmic parts. (This rule is to some extent analogous to the Appelquist-Carazzone theorem[21].)

We can then find a solution of the *RGE* in a straightforward way. We show this in Sec. 3. We also discuss the approximations used in handling this problem. An important point following from our calculation is that *FSB* is not felt only through the so-called penguin diagrams. Any diagram containing at least one »heavy«-quark line is influenced by *FSB**.

In Sec. 4 we write explicitly various sectors of the effective weak Hamiltonian based on the Weinberg-Salam model containing either four or six quark flavours. Section 5 contains a discussion of the theoretical viability and of the experimental consequences of the *QCD* renormalisation procedure. In the Appendix we rederive the *QCD* renormalised $\Delta S = 1$, $\Delta C = 0$ effective weak Hamiltonian of Ref. 2 in which *FSB* was taken into account for the first time.

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*This is also felt in $\Delta S = 1$, $\Delta C = 1$ charmed-particle decays[1]

2. *Renormalisation*

We illustrate the renormalisation procedure for the Weinberg-Salam-Glashow-Iliopoulos-Maiani (*WS-GIM*) model[22]. To illustrate how *FSB* influences the renormalisation of operators which do not mix with penguin-type operators, we present in this section a detailed calculation for the $\Delta S = 1$, $\Delta C = 1$ sector of the *WS-GIM* model. However, *FSB* is particularly important in the $\Delta S = 1$, $\Delta C = 0$ sector which is responsible for nonleptonic hyperon decays, and we rederive the effective weak Hamiltonian for this sector in the Appendix. The *SVZ* result coincides with our result if we omit heavy quarks (c etc.) in our expressions.

The dominant $\Delta C = 1$, $\Delta S = 1$ operator, corresponding to the piece of the »bare« Hamiltonian which does not suffer Cabibbo-angle suppression, is

$$\bar{u}_L d_L \bar{s}_L c_L. \tag{2.1}$$

Here we have used the usual abbreviations

$$\bar{u}_{\substack{L\\R}} d_{\substack{L\\R}} = \bar{\Psi}_u \gamma_\mu \frac{1}{2} (1 \mp \gamma_5) \Psi_d. \tag{2.2}$$

The operator (2.1) has parts transforming as 84- and 20- representations of the flavour $SU(4)$ group; these are as follows:

$$S^{84} = \bar{u}_L d_L \bar{s}_L c_L + \bar{u}_L c_L \bar{s}_L d_L, \tag{2.3a}$$

$$S^{20} = \bar{u}_L d_L \bar{s}_L c_L - \bar{u}_L c_L \bar{s}_L d_L. \tag{2.3b}$$

Anomalous dimensions of the operators (2.3) can be found by the insertion of these operators into the one-loop diagrams shown in Fig. 1. These diagrams are usually renormalised at the symmetric point*

$$p^2 = p'^2 = r^2 = r'^2 = -\mu^2,$$

$$2pr = 2p'r' = -2pp' = \frac{2}{3}\mu^2.$$

Here p, p', r and r' are momenta associated with the external quark legs.

It is well known that the mass m_c of the charmed quark c is lighter than the intermediate vector-boson mass M, but much heavier than the masses m_q of the u, d and s quarks[23]. A direct one-loop calculation shows that the values of renormalisation constants depend on the relative magnitudes of the renormalisation mass μ and of the quark masses. Moreover, the mass of the charmed quark can be hea-

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*This choice makes sense when $\mu \gg m^q$ (m^q correspond to the masses of quarks appearing in the external legs of the diagram). This is less obvious for $\mu \sim m$.

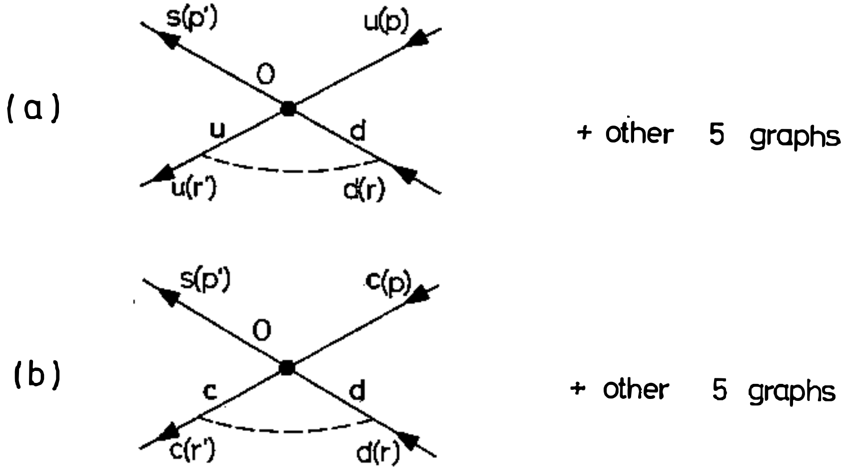

Fig. 1. Feynman diagrams for renormalisation matrices. Heavy dots indicate operator insertion Full lines are quarks and dashed lines are gluons.

vier than μ_0, which determines the effective coupling constant $g(\mu_0)$ appropriate for a particular weak process$^{2)}$. Then, the renormalisation procedure has to be performed separately for two regions, i. e.

$$\begin{aligned} &\text{(i)}\ m_q < \mu < m_c, \\ &\text{(ii)}\ m_c < \mu < M. \end{aligned} \tag{2.4}$$

The aim of the calculation is to find the leading $\ln \mu^2$ dependence for the diagrams shown in Fig. 1. When flavour symmetry is broken, one has to calculate systematically both divergent and finite contributions.

In calculating diagram (b) in Fig. 1, for example, (arguments are essentially the same for all integrals), we encounter a typical integral of the form

$$M \to I = \int_0^1 2y\,\mathrm{d}y \int_0^1 \mathrm{d}x \ln\frac{\Lambda^2}{\Phi^2}, \tag{2.5}$$

where

$$\begin{aligned} \Phi^2 = {} & (m_c^2 + \mu^2)\,xy + (m_q^2 + \mu^2)(1-y) - \\ & - \mu^2\left[(1-y)^2 + x^2y^2 + \frac{2}{3}xy(1-y)\right], \end{aligned} \tag{2.6}$$

and Λ is some gauge-invariant cut-off.

In region (ii), where $m_q^2 < m_c^2 < \mu^2$, we can write the integral (2.5) as

$$\int_0^1 2y\,\mathrm{d}y \int_0^1 \mathrm{d}x \left\{ \ln \frac{\Lambda^2}{\mu^2 y \left[x + (1-y) - x^2 y - \frac{2}{3} x (1-y) \right]} - \ln \left[1 + \frac{m_c^2 x y + m_q^2 (1-y)}{\mu^2 y \left[x + (1-y) - x^2 y - \frac{2}{3} x (1-y) \right]} \right] \right\}. \tag{2.7}$$

The first term behaves as

$$\ln \frac{\Lambda^2}{\mu^2} + \text{const},$$

while the second term contains no $\ln \mu^2$ contribution; it is of the order $0\,(m_c^2/\mu^2, m_q^2/\mu^2)$. This can be seen by detailed evaluation or through the following simple argument: Any $\ln \mu^2$ contribution can come only in the combinations $\ln (m_q^2/\mu^2)$ or $\ln (m_c^2/\mu^2)$. Since the second integral in (2.7) is finite for $m_q^2 = m_c^2 = 0$, it may contain no $\ln \mu^2$ dependence. Therefore, in region (ii), the integral (2.5) becomes

$$I = \ln \frac{\Lambda^2}{\mu^2} + \text{const} + 0 \left(\frac{m^2}{\mu^2} \right). \tag{2.8}$$

In region (i), where $m_q^2 < \mu^2 < m_c^2$, it is convenient to rewrite (2.5) as

$$I = \int_0^1 2y\,\mathrm{d}y \int_0^1 \mathrm{d}x \left\{ \ln \frac{\Lambda^2}{m_c^2 x y + \mu^2 y (1-y)} - \ln \left[1 + \frac{m_q^2 (1-y) + \mu^2 x y \left[1 - x y - \frac{2}{3} (1-y) \right]}{y \left[m_c^2 x + \mu^2 (1-y) \right]} \right] \right\}. \tag{2.9}$$

The first term can easily be calculated explicitly as follows:

$$\ln \frac{\Lambda^2}{m_c^2} + \text{const} + 0 \left(\frac{\mu^2}{m_c^2} \,;\, \frac{\mu^2}{m_c^2} \ln \frac{\mu^2}{m_c^2} \right). \tag{2.10}$$

The second term remains finite for $m_q = 0$:

$$\int_0^1 2y\,\mathrm{d}y \int_0^1 \mathrm{d}x \ln \left[1 + \frac{\mu^2 x \left[1 - x y - \frac{2}{3} (1-y) \right]}{m_c^2 \left[x + (1-y)\, \mu^2/m_c^2 \right]} \right]. \tag{2.11}$$

By expansion of the $\ln(1+\varepsilon)$ factor, the integration can be performed term by term, giving the $0\,(\mu^2/m_c^2)$ dependence. Hence, in region (i), the integral (2.5) behaves as

$$I = \ln\frac{\Lambda^2}{m_c^2} + \text{const} + 0\left(\frac{\mu^2}{m_c^2},\ \frac{\mu^2}{m_c^2}\ln\frac{\mu^2}{m_c^2},\ \frac{m_q^2}{\mu^2}\right) \tag{2.12}$$

and thus it does not contribute to anomalous dimensions in leading order.

In the leading-logarithm approximation, the renormalisation in region (2.4) (ii) yields $S \overset{R}{\rightarrow} Z\,S$*):

$$S^{84} \to \left(1 + 2\frac{q^2}{16\pi^2}\ln\mu^2\right) S^{84}, \tag{2.13}$$

$$S^{20} \to \left(1 - 4\frac{g^2}{16\pi^2}\ln\mu^2\right) S^{20}.$$

In region (2.4) (i) one obtains

$$S^{84} \to \left(1 + \frac{5}{3}\frac{g^2}{16\pi^2}\ln\mu^2\right) S^{84}, \tag{2.14}$$

$$S^{20} \to \left(1 - \frac{4}{3}\frac{g^2}{16\pi^2}\ln\mu^2\right) S^{20}.$$

Anomalous dimensions, which we will use as an input into the *RGE*, are the following $\left(\gamma = \dfrac{\partial \ln Z}{\partial \ln \mu}\right)$:

region (ii)

$$\gamma_2^{84}(g) = 4\frac{g^2}{16\pi^2} + \ldots,$$

$$\gamma_2^{20}(g) = -8\frac{g^2}{16\pi^2} + \ldots,$$

region (i) (2.15)

$$\gamma_1^{84}(g) = \frac{10}{3}\frac{g^2}{16\pi^2} + \ldots,$$

$$\gamma_1^{20}(g) = -\frac{8}{3}\frac{g^2}{16\pi^2} + \ldots.$$

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*)Here we have indicated the $\ln\mu^2$ dependence only, omitting some irrelevant mass and/or cut-off dependence.

A similar analysis can be performed for any operator or set of operators. The only complication, which is not essential for our consideration, is due to the operator mixing. One can easily overcome it by diagonalisation, as shown in the Appendix.

The analysis outlined in this section can be used as a theoretical basis for the formulation of the following rule (*R*):

> Omit all one-loop diagrams containing the propagator of the quark x whose mass m_x is larger than μ ($m_x > \mu$). In the remaining diagrams, the $\ln \mu$ dependence is (up to a sign) the same as the $\ln \Lambda$ dependence on the cut-off Λ. Accordingly, only the logarithmically divergent contributions ought to be calculated.

This rule determines the leading $\ln \mu$ dependence of any Green function renormalised at a symmetric point μ and considerably simplifies the renormalisation procedure. According to this rule, in region (i) defined by (2.4) one has to omit certain diagrams. As shown in the Appendix, in the $\Delta S = 1$, $\Delta C = 0$ sector this leads to the breaking of the *GIM* mechanism and to the apperance of the so-called penguin diagrams. In region (2.4) (ii), one has to consider all diagrams shown in Figs. 1, 2 and 3.

In some analyses of $\Delta S = 1$, $\Delta C = 1$ decays[24)], the renormalisation mass μ_0 was taken to coincide with the charmed quark mass m_c. Then the renormalisation procedure for four quarks is to be performed only in one region. The values of anomalous dimensions are the same as those for region (ii) given by (2.15). Both cases can be generalised to the model with six quarks[3)].

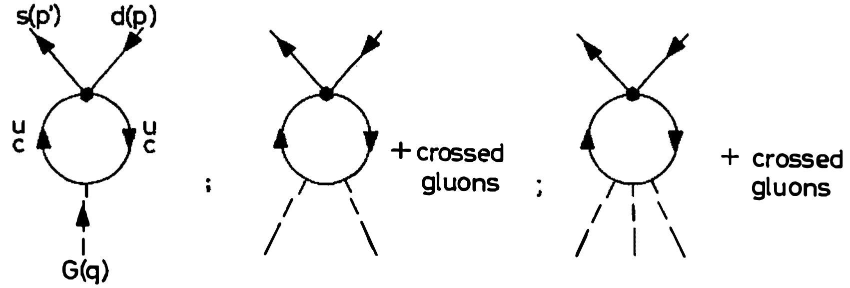

Fig. 2. Operator insertions in Green functions containing quarks and gluons.

When six quarks (u, d, s, c, t and b) are involved, one has to distinguish two possibilities:

$$\begin{aligned} &\text{a) } \mu_0 < m_c < m_b < m_t, \\ &\text{b) } m_c \leq \mu_0 < m_b < m_t. \end{aligned} \tag{2.16}$$

In case a), the renormalisation procedure has to be performed separately for four regions, i. e.

$$\begin{aligned}&\text{(i) } m_q < \mu < m_c,\\ &\text{(ii) } m_c < \mu < m_b,\\ &\text{(iii) } m_b < \mu < m_t,\\ &\text{(iv) } m_t < \mu < M.\end{aligned}\tag{2.17}$$

The values of anomalous dimensions of the operators for regions (i) and (ii) in (2.17) are the same as those given by (i) and (ii) in expression (2.15). Anomalous dimensions for regions (iii) and (iv) in (2.17) are equal to those found for region (ii). However, the rule *R* tells us that the running coupling constant entering the *RGE* (see Sec. 3) changes from region to region.

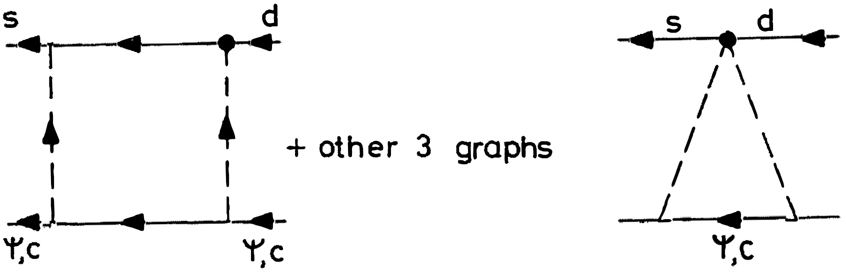

Fig. 3. Diagrams containing insertions of the quark-gluon operator R_7.

For case b) in (2.16), the renormalisation procedure has to be performed in three regions which coincide exactly with regions (ii), (iii) and (iv) in (2.17).

The $\Delta S = 1$ sector of nonleptonic weak interactions was discussed first[1,2] including four quark flavours and allowing for *SU* (4) symmetry breaking. Although all calculational details are given in the Appendix, it seems useful to make here some general remarks. First, one is dealing with two regions as defined by (2.4). In region (i), closed loops containing $c\bar{c}$ quarks do not contribute and the *GIM* cancellation is thus broken. A mixed quark-gluon operator of type R_7, defined by (A8), appears. Naively speaking, the gluon field from R_7 can be coupled to an additional quark-line leading to an effective four-quark contribution. The corresponding diagram is called in the literature a penguin diagram. However, the final results quoted in the Appendix and used in Sec. 3 do not depend on such a naive one-gluon-exchange picture. The gluon field from the operator R_7 is eliminated using the equation of motion. This equation links the operator R_7 to the operators R_5 and R_6 in which left and right components are mixed. No one-gluon approximation ought to be made and the result is exact in the leading $\ln \mu$ approximation.

3. *Solution of the renormalisation-group equation*

It is assumed that nonleptonic weak processes are described by the term which is of the second order in weak interactions[1,12]:

$$H^{(2)} \sim \int \mathrm{d}^4x\, D^{\alpha\beta}(x, M)\, T(\mathcal{J}^k_\alpha(x)\, \mathcal{J}^l_\beta(0)) + \text{h. c.} \tag{3.1}$$

Here we have introduced the intermediate-vector-boson (*IVB*) propagator *D*, the *IVB* mass *M* and the appropriate flavour-changing weak currents $\mathcal{J}^m$.

We have to keep in mind that weak currents contain fields described in the Heisenberg picture from the point of view of strong *QCD* interactions. Thus, *QCD* effects are, in principle, included in (3.1) in an exact way. These effects can be calculated using the *RGE*[20]. For our purpose, we are mainly interested in the $\Delta S = 1$, $\Delta C = 1$ terms from (3.1). If strong interactions were absent, we should find up to the order $0(1/M^2)$

$$H^{(2)} \to \hat{H}_{eff}(\Delta S = \Delta C = 1) = 2\sqrt{2}\, G_F \cos^2\Theta_C\, \bar{u}_L d_L \bar{s}_L c_L + \text{h. c.} =$$

$$= \sqrt{2}\, G_F \cos^2\Theta_C\, (S^{84} + S^{20}). \tag{3.2}$$

Here we have omitted Cabibbo-suppressed terms. With *QCD* switched on and using Wilson's operator expansion, we can write (3.1) up to the terms $0(1/M^2)$ in the form

$$H^{(2)} \to H_{eff} = \sqrt{2}\, G_F \cos^2\Theta_C\, [A^{84} S^{84} + A^{20} S^{20} + \sum_i A^i S^i]. \tag{3.3}$$

The summation in (3.3) runs over all operators that are eventually mixed with S^{84} and S^{20} by the renormalisation procedure. (As we have illustrated in (2.13)—(2.14), no further operators are needed for the $\Delta S = 1$, $\Delta C = 1$ sector.) The coefficients *A* are dimensionless functions of quark masses, the *IVB* mass *M* and of the renormalisation mass μ. With unbroken flavour symmetry, the currents in (3.1) have zero anomalous dimensions; therefore, (3.1) and (3.3) have to be independent of μ. This is possible only if the μ dependence of the coefficients *A* corresponds to the μ dependence of the renormalisation constants for the operators *S*. When flavour symmetry is broken, the currents containing heavy quarks can acquire anomalous dimensions γ_J (otherwise $\gamma_J = 0$). These dimensions can be calculated to leading order by using the rule *R* from Sec. 2. In the example considered here, only the current containing the *c* quark has an anomalous dimension different from zero, i. e.

$$\gamma_{\bar{u}_L d_L} = 0; \quad \gamma_{\bar{s}_L c_L} = \begin{cases} 0 & \mu > m_c \\ \dfrac{4}{3}\dfrac{g^2}{16\pi^2} + \dots & \mu < m_c \end{cases} \tag{3.4}$$

Therefore, the coefficients A have to satisfy the *RGE*

$$\left(\mu \frac{\partial}{\partial_\mu} + \beta(g) \frac{\partial}{\partial g} + \sum_r \delta_r(g)\, m_r \frac{\partial}{\partial m_r} - \hat{\gamma}^a\right) A^a \left(\frac{M}{\mu}, \frac{m_1}{M}, \dots, g\right) = 0. \quad (3.5)$$

Here the summation goes over all heavy quarks. For the model with four quarks, the relevant functions are as follows:

$$\beta = \mu \frac{\partial}{\partial \mu} g,$$

$$\delta = \frac{\mu}{m_c} \frac{\partial m_c}{\partial \mu}, \quad (3.6)$$

$$\hat{\gamma}^a = \gamma^a - \gamma_{\bar{u}_L d_L} - \gamma_{\bar{s}_L c_L}.$$

The quantities γ_a appearing in (3.6) are anomalous dimensions of the operators calculated in Sec. 2. In the general case of operator mixing (see the Appendix), γ_a can be found by the diagonalisation of the anomalous-dimension matrix. All quantities in (3.6) can be calculated as a function of running coupling constant for a particular value of μ. As an example let us again consider $\Delta S = 1$, $\Delta C = 1$ transitions using the model containing four quarks. In regions (i) and (ii) defined by (2.4) in the preceding section, the functions (3.6) have the following terms:

$$\beta_i = b_i g_i^3 + \dots \quad \gamma_i = c_i g_i^2 + \dots \quad \delta_i = d_i g_i^2 + \dots \quad (3.7)$$

$i = 1, 2$ for (i) and (ii), respectively.

The running coupling constant g_i depends on the region, as do the constants b_i, d_i and c_i:

$$b_1 = -\frac{1}{16\pi^2} 9, \qquad b_2 = -\frac{1}{16\pi^2} \frac{25}{3},$$

$$d_1 = -\frac{1}{16\pi^2} 8, \qquad d_2 = 0, \quad (3.8)$$

$$c_1^{20} = -\frac{1}{16\pi^2} \frac{8}{3}, \qquad c_2^{20} = -\frac{1}{16\pi^2} 8,$$

$$c_1^{84} = +\frac{1}{16\pi^2} \frac{10}{3}, \qquad c_2^{84} = +\frac{1}{16\pi^2} 4.$$

It is well known[20,25] that the solution of Eq. (3.5) for a given $\mu = \mu_1$ can be connected with the solution for $\mu = \mu_2$ by

$$A^a\left(\frac{M}{\mu_1}, \frac{m_c}{M}, g\right) = A\left(\frac{M}{\mu_2}, \frac{1}{M}\bar{m}_c\left(m_c, g, \ln\frac{\mu_2}{\mu_1}\right), \bar{g}\left(g, \ln\frac{\mu_2}{\mu_1}\right)\right) \times \tag{3.9}$$

$$\times \exp\left(-\int\limits_{\bar{g}(g,0)}^{\bar{g}(g, \ln \mu_2/\mu_1)} \frac{\hat{\gamma}_a(z)\,\mathrm{d}z}{\beta(z)}\right).$$

Here $\bar{g}$ and $\bar{m}$ are the effective (running) coupling constant and the mass, respectively.

In the approximation that all quark masses are negligible, i. e. $m_c = \bar{m}_c = 0$, one can select $\mu_1 = \mu_0$ and $\mu_2 = M$ and thus recover the well-known expression of Ref. 1. If one deals with the situation described by (3.8), then one has to find a solution first in region (i) by introducing $\mu_1 = \mu_0$ and $\mu_2 = m_c$. Then one moves to region (ii) and uses $\mu_1 = m_c$ and $\mu_2 = M$. In this way, one connects stepwise the solution for $\mu = \mu_0$ with the solution for $\mu = M$ and writes

$$A^a\left(\frac{M}{\mu_0}, \frac{m_c}{M}, g\right) = A^a\left(1, \frac{1}{M} m_c^{(2)}\left(m_c^{(1)}, g_1, \ln\frac{M}{m_c}\right), g_2\left(g_1, \ln\frac{M}{m_0}\right)\right) \times$$

$$\times \exp\left(-\int\limits_{g_1(g,0)}^{g_1(g, \ln m_c/\mu_0)} \frac{\hat{\gamma}_1^a(z)\,\mathrm{d}z}{\beta_1(z)}\right) \times$$

$$\times \exp\left(-\int\limits_{g_2(g_1,0)}^{g_2(g_1, \ln M/m_c)} \frac{\hat{\gamma}_2^a(z)\,\mathrm{d}z}{\beta_2(z)}\right). \tag{3.10}$$

As one has

$$\begin{aligned} g_1(g, 0) &= g, \\ g_2(g_1, 0) &= g_1\left(g, \ln\frac{m_c}{\mu_0}\right), \end{aligned} \tag{3.11}$$

the running coupling constant is a continuous function of μ. In the leading-logarithmic approximation, it is of the form

$$g_j\left(g_i, \ln\frac{\varrho}{\sigma}\right) = \frac{g_i}{1 - 2\,g_i\,b_j \ln\frac{\varrho}{\sigma}}. \tag{3.12}$$

The essence of asymptotic freedom[26)] is that the function A^a on the r. h. s. of (3.10) can be adequately approximated by its free-field value A^a_F. Using (3.7) and (3.8), we can easily evaluate the integrals in (3.10):

$$A^a\left(\frac{M}{\mu_0}, \frac{m_c}{M}, g\right) = A^a_F\,(\varkappa_1)^{\hat{c}^a_1/2b_1}\,(\varkappa_2)^{\hat{c}^a_2/2b_2},$$

$$\varkappa_1 = 1 - 2\,b_1\,g^2 \ln \frac{m_c}{\mu_0}, \tag{3.13}$$

$$\varkappa_2 = 1 - 2\,b_2 \frac{g^2}{\varkappa_1} \ln \frac{M}{m_c}.$$

($\hat{c}^a$ corresponds to $\hat{\gamma}^a$ in expression (3.6).)

Now we apply steps (3.9) to the $\Delta\ S = 1, \Delta\ C = 1$ coefficients A^{84} and A^{20}. For region (2.4) (i), we find

$$A^{20} = (\varkappa_1)^{2/9}(\varkappa_2)^{0.48};\ A^{84} = (A^{20})^{-1/2}. \tag{3.14}$$

In deriving (3.14), we have used the values quoted in (3.4) and (3.8).

For the $\Delta\ S = 1, \Delta\ C = 0$ sector, such an approach, with $\hat{\gamma}^a$ diagonalised in an appropriate way, recovers the SVZ values for A^a (see the Appendix).

In the model with six quarks, the integral (3.11) has to be subdivided into 4 (or 3) parts. A straightforward repetition of the procedures outlined by (3.9)—(3.12) leads to the following results:

Possibility a) (see (2.16))

$$A^a = A^a_F \times \prod_{i=1}^{4} (\varkappa_i)^{\hat{c}^a_i/2b_i}, \tag{3.15}$$

where

$$\varkappa_1 = 1 - 2\,b_1\,g^2 \ln \frac{m_c}{\mu_0},$$

$$\varkappa_2 = 1 - 2\,b_2 \frac{g^2}{\varkappa_1} \ln \frac{m_b}{m_c},$$

$$\varkappa_3 = 1 - 2\,b_3 \frac{g^2}{\varkappa_1\,\varkappa_2} \ln \frac{m_t}{m_b}, \tag{3.16}$$

$$\varkappa_4 = 1 - 2\,b_4 \frac{g^2}{\varkappa_1\,\varkappa_2\,\varkappa_3} \ln \frac{M}{m_t}.$$

Possibility b)

$$A^{a} = A^{a}_{F} \times \prod_{i=2}^{4} (\varkappa_{i})^{\hat{c}^{a}_{i}/2b_{i}}, \tag{3.17}$$

where

$$\begin{aligned} \varkappa_2 &= 1 - 2\, b_2\, g^2 \ln \frac{m_b}{\mu_0}, \\ \varkappa_3 &= 1 - 2\, b_3 \frac{g^2}{\varkappa_2} \ln \frac{m_t}{m_b}, \\ \varkappa_4 &= 1 - 2\, b_4 \frac{g^2}{\varkappa_2\, \varkappa_3} \ln \frac{M}{m_t}. \end{aligned} \tag{3.18}$$

The coefficients b_i on (3.15)—(3.18) are defined by

$$b_i = \frac{1}{16\,\pi^2} \left(\frac{2\,N_i}{3} - 11 \right), \tag{3.19}$$

where $N_1 = 3$, $N_2 = 4$, $N_3 = 5$ and $N_4 = 6$ denote the number of quark flavours of relevance.

4. *The effective weak Hamiltonian*

In the preceding section we have outlined the methods easily applicable to any weak Hamiltonian containing quarks and heavy intermediate vector bosons. First, we use them in the $\Delta\, S = 1$, $\Delta\, C = 1$ sector of the weak Hamiltonian containing four quarks. If we are not interested in the operators with c quarks, we can use the well-known *SVZ* Hamiltonian

$$H_W\, (\Delta\; S = 1) = \sqrt{2}\, G_F \sin \Theta_C \cos \Theta_C \sum_{k=1}^{6} C_k\, O_k,\; C_k = C_k\, (\varkappa_1, \varkappa_2), \tag{4.1}$$

where

$$\begin{aligned} O_1 &= (\bar{d}_L\, s_L)\, (\bar{u}_L\, u_L) - (\bar{d}_L\, u_L)\, (\bar{u}_L\, s_L), \\ O_2 &= (\bar{d}_L\, s_L)\, (\bar{u}_L\, u_L) + (\bar{d}_L\, u_L)\, (u_L\, s_L) + 2\, (\bar{d}_L\, s_L)\, (\bar{d}_L\, d_L) + 2\, (\bar{d}_L\, s_L)\, (\bar{s}_L\, s_L), \\ O_3 &= (\bar{d}_L\, s_L)\, (\bar{u}_L\, u_L) + (\bar{d}_L\, u_L)\, (\bar{u}_L\, s_L) + 2\, (\bar{d}_L\, s_L)\, (\bar{d}_L\, d_L) - 3\, (\bar{d}_L\, s_L)\, (\bar{s}_L\, s_L), \\ O_4 &= (\bar{d}_L\, s_L)\, (\bar{u}_L\, u_L) + (\bar{d}_L\, u_L)\, (\bar{u}_L\, s_L) - (\bar{d}_L\, s_L)\, (\bar{d}_L\, d_L), \\ O_5 &= \bar{d}_L\, t^a\, s_L\, (\bar{q}^{i}_{R}\, t^a\, q^{i}_{R}), \\ O_6 &= \bar{d}_L\, s_L\, (\bar{q}^{i}_{R}\, q^{i}_{R}), \end{aligned} \tag{4.2}$$

with the renormalisation coefficients given approximately by ($b = 25/3$):

$$C_1 = -\varkappa_2^{4/b}\,(0.98\,\varkappa_1^{0.42} + 0.01\,\varkappa_1^{0.80}) + 0.04\,\varkappa_2^{-2/b}\,(\varkappa_1^{0.42} - \varkappa_1^{-0.30}),$$

$$C_2 = 0.20\,\varkappa_2^{-2/b}\,(0.96\,\varkappa_1^{-0.30} + 0.03\,\varkappa_1^{-0.12}) - 0.02\,\varkappa_2^{4/b}\,(\varkappa_1^{0.42} - \varkappa_1^{-0.30}),$$

$$C_3 = \frac{2}{15}\,\varkappa_2^{-2/b}\,\varkappa_1^{-2/9}, \quad C_4 = 5\,C_3,$$

$$C_5 = 10^{-2}\,\varkappa_2^{4/b}\,(3.3\,\varkappa_1^{0.42} + 0.3\,\varkappa_1^{-0.30} - 3.9\,\varkappa_1^{0.80} + 0.30\,\varkappa_1^{-0.12} +$$

$$+ 10^{-2}\,\varkappa_2^{-2/b}\,(-0.1\,\varkappa_1^{0.42} + 2.9\,\varkappa_1^{-0.30} - 1.4\,\varkappa_1^{0.80} - 1.4\,\varkappa_1^{-0.12}),$$

$$C_6 = 10^{-2}\,\varkappa_2^{4/2}\,(4.8\,\varkappa_1^{0.42} - 0.6\,\varkappa_1^{-0.30} - 2.9\,\varkappa_1^{0.80} - 1.3\,\varkappa_1^{-0.12}) +$$

$$+ 10^{-2}\,\varkappa_2^{-2/b}\,(-0.2\,\varkappa_1^{0.42} - 5.8\,\varkappa_1^{-0.30} - 1.0\,\varkappa_1^{0.80} + 7.0\,\varkappa_1^{-0.12}), \tag{4.3}$$

$$\varkappa_2 = 1 + \frac{25}{6\pi}\,\frac{g^2(\mu_0)}{4\pi\,\varkappa_1}\,\ln\frac{M}{m_c},$$

$$\varkappa_1 = 1 + \frac{9}{2\pi}\,\frac{g^2(\mu_0)}{4\pi}\,\ln\frac{m_c}{\mu_0}.$$

Using analogous methods, we have found the $\Delta I = 1$, $\Delta S = 0$, $\Delta C = 0$ sector of the PV weak Hamiltonian including four quarks[15,16]:

$$H_W^{eff}(\Delta S = 0) = H_W(\text{charged}, PV) + H_W(\text{neutral}, PV),$$

$$H_W(\text{charged}, PV) = \sqrt{\frac{1}{2}}\,G_F \sin^2\Theta_C \left[-C_1\,\tilde{O}_1 + \frac{1}{5}\,C_2\,\tilde{O}_2 - \frac{2}{5}\,C_2'\,\tilde{O}_2' +\right.$$

$$\left. + C_5'\,\tilde{O}_5 + C_6'\,\tilde{O}_6\right],$$

$$H_W(\text{neutral}, PV) = \sqrt{\frac{1}{2}}\,G_F\,(1 - 2\sin^2\Theta_W)\left[-\bar{C}_1\,\tilde{O}_1 - \frac{1}{5}\,\bar{C}_2\,\tilde{O}_2 +\right.$$

$$\left. + \frac{2}{5}\,C_2'\,\tilde{O}_2' + C_5''\,\tilde{O}_5 + C_6''\,\tilde{O}_6\right] - \sqrt{\frac{1}{2}}\,G_F\,\frac{2}{3}\sin^2\Theta_W\,[C_5\,\tilde{O}_5 + C_6\,\tilde{O}_6 +$$

$$+ C_7\,\tilde{O}_7 + C_8\,\tilde{O}_8], \tag{4.4}$$

where $\sim$ denotes the PV part of O_i. The operators appearing in (4.4) can be defined as follows:

$$\begin{aligned}
O_1 &= (\bar{u}_L u_L)(\bar{s}_L s_L) - (\bar{u}_L s_L)(\bar{s}_L u_L) - (\bar{d}_L d_L)(\bar{s}_L s_L) + (\bar{d}_L s_L)(\bar{s}_L d_L),\\
O_2 &= 2(\bar{u}_L u_L \bar{u}_L u_L - \bar{d}_L d_L \bar{d}_L d_L) + (\bar{u}_L s_L)(\bar{s}_L u_L) + (\bar{u}_L u_L)(\bar{s}_L s_L) -\\
&\quad - (\bar{d}_L s_L)(\bar{s}_L d_L) - (\bar{d}_L d_L)(\bar{s}_L s_L),\\
O_2' &= (\bar{u}_L u_L)(\bar{u}_L u_L) - (\bar{d}_L d_L)(\bar{d}_L d_L) - 2(\bar{u}_L s_L)(\bar{s}_L u_L) - 2(\bar{s}_L s_L)(\bar{s}_L s_L) +\\
&\quad + 2(\bar{d}_L s_L)(\bar{s}_L d_L) + 2(\bar{d}_L d_L)(\bar{s}_L s_L),\\
O_5 &= (\bar{u}_L t^a u_L - \bar{d}_L t^a d_L)(\bar{q}_R^i t^a q_R^i),\\
O_6 &= (\bar{u}_L u_L - \bar{d}_L d_L)(\bar{q}_R^i q_R^i),\\
O_7 &= \frac{1}{2}(O_1 + O_2),\\
O_8 &= -\frac{4}{3} O_1 + \frac{2}{3} O_2.
\end{aligned} \tag{4.5}$$

In Table 1 we list the renormalisation coefficients C_i.

TABLE 1

	C_1	C_2	$\bar{C}_1$	$\bar{C}_2$	C_2'	C_5	C_6	C_7	C_8	C_5'	C_6'	C_5''	C_6''
without QCD	1	1	1	1	1	0	1	1	0	0	0	0	0
$QCD^{a)}$	2.71	0.61	2.71	0.61	0.61	0.85	1.37	1.40	– 0.68	0	0	0	0
$QCD^{b)}$	2.55	0.47	2.59	0.69	0.61	0.77	1.35	1.35	– 0.64	– 0.08	– 0.03	– 0.04	– 0.0
$SU(4)$ representation	20	84	20	84	84	15	15	15	15	20 + 84		20 + 84	

C_i constants for the $\Delta S = 0$, $\Delta C = 0$ weak Hamiltonian.

a) without flavour symmetry breaking

b) with flavour symmetry breaking

The operators appearing under the same number in expressions (4.1) and (4.4) are actually the $\Delta S = 1$ or $\Delta S = 0$ component of the same operator in the $SU(4)$-flavour classification.* The results listed in Table 1 can therefore be indicative for

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*Formulae (4.4) do not contain c quarks. These quarks do not contribute if the operators are applied to the $C = 0$ hadron states containing valence quarks only.

QCD and *FSB* effects in any sector (i. e. $\Delta S = 0, \Delta S = 1$) of the effective weak Hamiltonian. This conclusion is valid, however, only for that part of the weak Hamiltonian which is due to the product of charged weak currents (the entire $\Delta S = 1$ sector and a part of (4.4) that is proportional to $\sin^2 \Theta_C$). In these pieces, the operators O_5 and O_6 appear because $SU(4)$ symmetry breaking via quark masses renders the *GIM* mechanism[22] inoperable to a certain extent. Closed loops leading to quark-quark-gluon operators (A8) cancel only partially. The application of the equation of motion and the correct renormalisation procedure, as described in the Appendix, then lead to the final effective weak Hamiltonian. The appearance of the operators of the type O_5 and O_6, the matrix elements of which are known as »penguin« terms, is strong in the $\Delta S = 0$, $\Delta C = 0$ sector even without *FSB*. Moreover, as seen in Table 1, the operator O_6 is present even in the *QCD*-unrenormalised weak Hamiltonian*. *PV* pion exchange is thus suitable for testing the dynamical assumptions used to evaluate the matrix elements of the operators O_5 and O_6[2,14-16].

Now we turn to the effective Hamiltonian containing more than four quarks. At pressent, there is rather good evidence for the existence of a fifth quark. This is supported both by the observation of Y particles and preliminary evidence for the existence of a heavy B meson at 5.3 GeV.

The most natural model that contains six quark flavours and simultaneously incorporates the possibility of *CP* violation is the Kobayashi-Maskawa model[3]. This is a model of the standard type, i. e. the $SU(2) \otimes U(1)$ model, with three generations of quarks appearing in left-handed doublets and right-handed singlets.

The investigations of Pakvasa and Sugawara[27], Maiani and Ellis et al.[28] have shown that the predictions of this model for *CP* violation in K decays reduce to the superweak form of *CP* violation[29].

Charged currents in the six-quark model are of the form

$$\mathcal{J}_\mu^- = \overline{\Psi} \gamma_\mu (1 - \gamma_5) U \Psi. \tag{4.6}$$

Here $\overline{\Psi}$ is a six-flavour-component field $\overline{\Psi} = (\overline{u}, \overline{c}, \overline{t}, \overline{d}, \overline{s}, \overline{b})$ and U is a 6×6 matrix:

$$U = \begin{pmatrix} 0 & C \\ 0 & 0 \end{pmatrix},$$

where C is the 3×3 Kobayashi-Maskawa matrix[3]

$$C = \begin{bmatrix} c_1 & s_1 c_3 & + s_1 s_3 \\ - s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ - s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{bmatrix} \tag{4.7}$$

and $c_i = \cos \Theta_i$, $s_i = \sin \Theta_i$.

---

*This is due to the fact that neutral currents contain both the left- and right-handed pieces.

Effectively, the bare Kobayashi-Maskawa $\Delta S = 1$, $\Delta C = 0$ Hamiltonian is of the form

$$H_W = 2\sqrt{2}\, G_F\, [a_1\, \overline{d}_L u_L \overline{u}_L s_L + a_2\, \overline{d}_L c_L \overline{c}_L s_L + a_3\, \overline{d}_L t_L \overline{t}_L s_L], \tag{4.8}$$

where

$$\begin{aligned} a_1 &= c_1 s_1 c_3, \\ a_2 &= -s_1 c_2 [c_1 c_2 c_3 - s_2 s_3 e^{i\delta}], \\ a_3 &= -s_1 s_2 [c_1 s_2 c_3 + c_2 s_3 e^{i\delta}]. \end{aligned} \tag{4.9}$$

Because of the *GIM* mechanism, the quantities a_1, a_2 and a_3 are related by

$$a_1 + a_2 + a_3 = 0.$$

The Hamiltonian (4.8) can be written in terms of an irreducible representation of the $SU(6)$ flavour group as follows

$$H_W = 2\sqrt{2}\, G_F [O^{189} + O^{405}], \tag{4.10}$$

where

$$\begin{aligned} O^{189} &= \sum_{i=1}^{3} a_i \left[\frac{1}{3}\, \overline{d}_L q_{iL} \overline{q}_{iL} s_L - \frac{1}{4}\, \overline{d}_L t^a q_{iL} \overline{q}_{iL} t^a s_L \right], \\ O^{405} &= \sum_{i=1}^{3} a_i \left[\frac{2}{3}\, \overline{d}_L q_{iL} \overline{q}_{iL} s_L + \frac{1}{4}\, \overline{d}_L t^a q_{iL} \overline{q}_{iL} t^a s_L \right], \end{aligned} \tag{4.11}$$

with $q_1 = u$, $q_2 = c$, $q_3 = t$.

QCD corrections should, in principle, be calculated separately for the regions defined by (2.17). However, the results will become somewhat simpler if region (2.17) (iii) is omitted, i. e. if one takes $m_b \approx m_t$. The error is of about a few percent. Starting with (2.17) (iv), we can write the Hamiltonian, defined at the renormalisation point $\mu \sim m_t$, as follows:

$$H(\mu \sim m_t) = 2\sqrt{2}\, G_F\, [\varkappa_t^{4/7}\, O^{189} + \varkappa_t^{-2/7}\, O^{405}], \tag{4.12}$$

where (see (3.16) with $m_b = m_t$))

$$\varkappa_t = \alpha_s(m_t)/\alpha_s(M) = 1 + \frac{14}{16\,\pi^2}\, \frac{g^2}{\varkappa\, \varkappa_c} \ln \frac{M}{m_t}. \tag{4.13}$$

For further use, we also define the coefficients $\varkappa$ and $\varkappa_c$ as follows:

$$\varkappa_c = \frac{\alpha(m_c)}{\alpha(m_t)} = 1 + \frac{50/3}{16\pi^2}\frac{g^2}{\varkappa} \cdot \ln\frac{m_t}{m_c},$$

$$\varkappa = \frac{\alpha(\mu_0)}{\alpha(m_c)} = 1 + \frac{18}{16\pi^2} g^2 \ln\frac{m_c}{\mu_0}. \tag{4.14}$$

Going to the next region, which compresses (2.17) (iii) and (2.17) (ii), i. e. $m_t > > \mu > m_c$, one finds that the *GIM* mechanism is no more effective, since the t-quark loop is suppressed and the operators O^{189} and O^{405} get mixed with the other four operators O'_5, O'_6, O'_7 and O'_8:

$$O'_5 \sim a_3 (\bar{d}_L t^a s_L) \bar{\Psi}_R t^a \Psi_R,$$

$$O'_6 \sim a_3 (\bar{d}_L s_L) \bar{\Psi}_R \Psi_R,$$

$$O'_7 \sim a_3 (\bar{d}_L s_L) \bar{\Psi}_L \Psi_L,$$

$$O'_8 \sim a_3 (\bar{d}_L t^a s_L) \bar{\Psi}_L t^a \Psi_L.$$

It is important to note that the coefficient a_3 is explicitly given by

$$a_3 = -s_1 s_2 (c_1 s_2 c_3 + c_2 s_3 \cos\delta) - i s_1 s_2 c_2 s_3 \sin\delta. \tag{4.15}$$

Obviously, it contains besides a real part also an imaginary part which would produce *CP* violation. The real part is doubly Cabibbo-suppressed in comparison with the original Hamiltonian, so that the *GIM* mechanism is rather effective at this stage.

By diagonalising the renormalisation matrix, one obtains the Hamiltonian $H(\mu \sim m_c)$ and then proceeds to find the *QCD* correction in the region (2.17) (i), where $m_c > \mu = \mu_0$, with μ_0 being some typical hadronic mass in the range $m_\pi < \mu_0 < m_\varrho$. As a result, one obtains[5)] a rather complicated Hamiltonian in terms of the operators $O_1, \ldots, O_6$. Since the operator O_5 is a dominant penguin operator, one can write an approximate expression as follows:

$$H_W^{eff}(\Delta S = 1, \Delta C = 0) = \sqrt{2}\, G_F \{c_1 s_1 c_3 a_R + i s_1 c_2 s_2 s_3 \sin\delta\, a_I\} O_5, \tag{4.16}$$

with

$$a_R = -\varkappa_t^{0.57} \varkappa_c^{0.48} [0.039 \varkappa^{0.80} - 0.033 \varkappa^{0.42} - 0.003 \varkappa^{-0.12} -$$
$$- 0.003 \varkappa^{-0.30}] - \frac{1}{5} \varkappa_t^{-0.29} \varkappa_c^{-0.24} [0.068 \varkappa^{0.80} + 0.007 \varkappa^{0.42} +$$
$$+ 0.071 \varkappa^{-0.12} - 0.146 \varkappa^{-0.30}] \tag{4.17}$$

$$\begin{aligned} a_I = & -\{\varkappa^{0.57}[0.037\,\varkappa_c^{0.85} - 0.031\,\varkappa_c^{0.42} - 0.002\,\varkappa_c^{-0.13} - 0.004\,\varkappa_c^{0.35}] + \\ & + \varkappa_t^{-0.29}[0.012\,\varkappa_c^{0.85} + 0.003\,\varkappa_c^{0.42} + 0.014\,\varkappa_c^{-0.13} - 0.029\,\varkappa_c^{-0.35}]\} \times \\ & \times \{0.8509\,\varkappa^{0.80} + 0.0091\,\varkappa^{0.42} + 0.122\,\varkappa^{-0.12} + 0.018\,\varkappa^{-0.30}\}. \end{aligned}$$

Obviously, a_R is just the coefficient c_5 of the operator O_5 in the standard approach and a_I is the coefficient of the penguin operator responsible for CP violation. $\varkappa_c$ and $\varkappa$ are defined by (4.14). It follows from Eq. (4.17) that for $\varkappa = 1$ and $\varkappa_t = 1$ one obtains $a_R = 0$ and $a_I = 0$, respectively. This reflects the fact that »imaginary« penguins appear in the middle region and »real« penguins in the last region.

This fact also determines the different sensitivity of the coefficients a_I and a_R on the choice of the parameters μ, m_c and m_t.

Table 1 in Ref. 5 shows that the coefficient a_R is *very* dependent on the choice of the renormalisation point μ (this was first realised by the *SVZ*). However, the coefficient a_I is rather stable; it depends more on the value of the mass of the t quark.

5. *Uses of the effective weak Hamiltonian*

In this section we outline general ides of using the effective weak Hamiltonian (the various sectors of which are given in Sec. 4) in calculating nonleptonic weak processes. This outline should make our text selfcontained to a certain extent. Full calculational details, however, will be published in another paper.

Broadly speaking, nonleptonic processes can be divided in two classes:

i) processes involving both baryons and mesons and
ii) processes involving mesons only.

Theoretical calculations of processes of class i) have been reasonably successful. One could reproduce both signs and magnitudes of parity-violating amplitudes (A amplitudes) for nonleptonic hyperon decays*[6, 7]. The relative signs (and magnitudes) of parity-conserving amplitudes (B amplitudes) come out also correctly. Their absolute magnitudes are always too small[6, 7]. There are three types of theoretical contributions:

(a) current commutator (CC) terms,
(b) pole (P) terms and
(c) separable (S) terms.

CC terms are found by reducing the pion field via $PCAC$ and current algebra (CA). One is left with the rotated Hamiltonian between baryon states; for example**[30],

$$\langle B_f\,\pi\,|H_W^{PV}|\,B_i\rangle \underset{PCAC+CA}{\rightarrow} \frac{1}{f_\pi}\langle B_f\,|H_W^{PC}|\,B_i\rangle + \dots \tag{5.1}$$

---

*Examples are $\Sigma \rightarrow N + \pi$, $\Lambda \rightarrow N + \pi$, etc. decays.

**Here PV and PC denote parity violation and parity conservation, respectively.

The effective weak Hamiltonian appearing in (5.1) is actually a sum of four-quark operators O_j defined in Sec. 4. One is thus considering products of the type

$$C_j(\mu^2)\,\langle B_f\,|O_j|\,B_i\rangle\,(\mu^2). \tag{5.2}$$

In such products, both C_j and the matrix element depend on the renormalisation mass μ. The product (5.2) is, however, invariant under the renormalisation group and should be μ-independent. This can be achieved only if the renormalisation mass is selected in such a way that the matrix element $\langle B_f\,|O_j|\,B_i\rangle$ carries a proper, though concealed, μ dependence. This can easily depend on the particular quark model for baryon states $|B_i\rangle$. Our procedure provides an explicit presentation of the μ dependence of the coefficients $C_j(\mu^2)$. Nothing is known about the μ dependence of the matrix elements of the operator O_j. It is usually believed that if the subtraction point μ is chosen around the mean momentum of the confined quarks, the effect of gluon corrections is relatively unimportant[1]. Some additional clues to such a selection of μ might come form Refs. 31 and 32.

A special situation is encountered in the Kobayashi-Maskawa model. In this model, the contribution from the penguin diagrams (4.16) is of the form

$$H\,(\text{penguin}) = (C_R + i\,C_I)\,O_5. \tag{5.3}$$

Explicit calculation shows that the μ dependence of C_R is different from the μ dependence of C_I. This seems contradictory since the μ dependence of the operator O_5 should be the same for both C_R and C_I. Careful analysis shows that the C_I coefficient is μ-sensitive in the region where α_S is relatively small, and C_R is very sensitive around the subtraction point μ_0 (at μ_0 one probably could not believe completely in the perturbation theory). The principal contribution to C_R comes precisely from the region going from m_c^2 to μ_0^2 and here nonleading contributions may be important. The region essential for C_I is, however, that going from m_t^2 to m_c^2. Furthermore, the operators O_j consist of current quarks. Without any transformation, they are used to act as operators in the space of valence quarks constituting baryon (or meson) states. It is not at all obvious that such matchings should be equally bad or good for any quark model of hadrons. It is surprising that two very different models, the *MIT* bag model[33] and the harmonic-oscillator model[34], yield very close results for nonleptonic hyperon decays[6, 7, 35 36]. Figure 4 shows schematically P terms used in the calculation of B amplitudes. One theoretical analysis used B-pole terms and separable contributions[6]. Another analysis included B-pole and K-pole terms [7, 36]. Separable terms follow from the fact that any four-quark operator can always be written as a product of two currents, one of which is an axial-vector current A. Thus,

$$\langle B_f\,\pi\,|O_j|\,B_i\rangle = \text{const}\,\langle B_f\,\pi\,|\mathcal{J}_\mu A_\mu|\,B_i\rangle = \text{const}\,\langle B_f\,|\mathcal{J}_\mu|\,B_i\rangle\,\langle\pi\,|A_\mu|\,O\rangle. \tag{5.4}$$

The matrix elements in (5.4) are usually well known (at least on the mass shell) from semileptonic experiments[30]. The inclusion of the K-pole, instead of separable contributions, does not drastically alter the overall conclusion[6, 7, 36]. However,

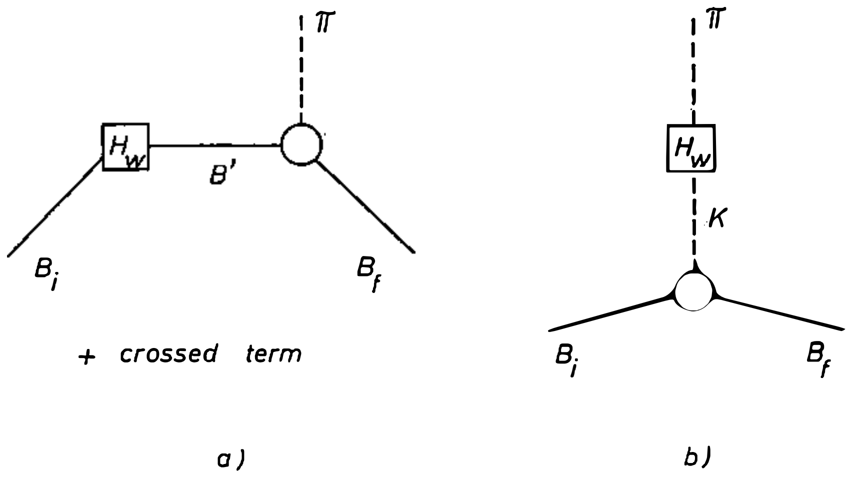

Fig. 4. Baryon-pole a) and meson-pole b) terms used in the calculation of the $B_i \to B_f + \pi$ $\Delta S = 1$ weak amplitude.

the contributions of the K-pole, estimated in the $>IT$ bag model, are generally smaller than S terms. Therefore, Refs. 7 and 36 assume that the coefficients in front of FSB terms, i. e. C_5 and C_6 can be used as free parameters for fitting experimental data. The same approach was adopted by Ref. 2, where no B-pole terms were included. Generally speaking, theoretical results for nonleptonic hyperon-decay amplitudes are not drastically changed by reasonable variations of the enhancement factors C_i and other semiempirical parameters such as current form factors and coupling constants of mesons.[36] As reasonable variations of form factors and coupling constants we understand changes made within the limits set by theoretical and/or experimental uncertainties (experiments concerned with other processes such as semileptonic transitions). The enhancement factors C_i can be varied by changing the masses M, m_q and μ in such a way that they do oscillate around theoretically indicated values (i. e. M from weak-interaction models, m_q around current quark masses, μ around the mean quark momentum).

An analogous analysis, including B-pole terms (but with the exchange of resonances) and S terms, has successfully described Ω^- decays, reproducing experimental amplitudes within 25%[10,36].

Of particular theoretical interest is the investigation of the $NN\pi$ PV amplitude; it can be studied in PV nuclear electromagnetic transitions. In the $\Delta S = 0$ sector of H_W (4.4), the »penguin« operators O_5 and O_6 appear even in the »bare« Hamiltonian. The strength of their contribution is not drastically changed by QCD corrections. Therefore, nuclear PV experiments can, in principle, be used to investigate dynamical approximations* employed in the calculation of the contributions of the operators O_5 and O_6[14–16].

The effective weak Hamiltonian, derived in Sec. 4, has also been succesfully used to calculate important contributions to the $NN\varrho$ PV amplitude[18] and to explain the $\Delta S = 1$ electromagnetic decay $\Sigma^+ \to p + \gamma$[11].

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*In our dynamical approximation/ they contribute to CC, P and S terms. S terms are by far the largest ones.

The Hamiltonian based on the Kobayashi-Maskawa model has been used to estimate CP violation in kaon decays[4, 5]. The prediction for the ratio of CP-violating parameters $|\varepsilon'/\varepsilon|$ is well within the present experimental bound. Experimental limits have to be pushed further down to test whether there is any departure from the superweak theory[29] in which the ratio $|\varepsilon'/\varepsilon|$ vanishes.

Kaon decays in two pions have also been analysed using (2.1) as the effective weak Hamiltonian and including appropriate CC and P term[2, 7]. The conclusion is that they can also be explained within such a calculational framework which works well for all other tested processes involving baryons. However, this does not work for all processes involving mesons only. Charm-changing decays of the D-meson should be described by the $\Delta C = 1$, $\Delta S = 1$ piece of the weak Hamiltonian (3.2) Measurements yield the decay ratio

$$R = \frac{\Gamma(D^0 \to K^- \pi^+)}{\Gamma(D^0 \to \bar{K}^0 \pi^0)} = 1.6 \pm 0.9. \tag{5.5}$$

The value for the decay ratio calculated using the weak Hamiltonian (3.3) is[24, 37]

$$R = 2 \frac{[2\, A^{84} + A^{20}]^2}{[2\, A^{84} - A^{20}]^2}. \tag{5.6}$$

When (3.14) (with the choice of parameters (A26)) is inserted in (5.6), the value for the decay ratio is found to be approximately 52.

It is not quite clear as yet what this starting discrepancy really signalizes. Two extreme possibilities immediately come to mind:

(i) QCD corrections leading to the effective weak Hamiltonian (3.4)—(3.13) have not been handled correctly.

(ii) The dynamical scheme in which this effective weak Hamiltonian serves as an input is either incomplete or incorrect.

Attempts[19, 38, 41] made to answer these questions have used combinations of both possibilities.

In the first approach[19], the mechanism called »quark coherence« was proposed. At the quark level, one obtains the cancellation of the terms proportional to A^{20} for D^+ decays and supposes that the cancellation is almost complete also at the hadronic level. In order to obtain values for the ratio of branching rates of D^+ to D^0 as large as experimental values, it was necessary to suppose that $A^{20} \gg A^{84}$, say $A^{20}/A^{84} \simeq 10$.

An alternative explanation supposes a large gluon content in the meson wave function[38, 40] or the contribution coming from real-gluon emission[41].

However, all these attempts have encountered difficulties[42]. It is not quite clear whether the interference in the »quark-coherence« picture persists for many-body decays[42, 43]. Besides, the mechanism for an additional enhancement of A^{20} over A^{84} is not known. On the other hand, the gluon admixture in the meson wave function should be rather large[38], say 30%, and both approaches[38, 41] require a rather large value for the decay constant f_D, say of the order of 500 MeV. However, recent estimates for f_D in the MIT bag model[44] show that it is rather smaller than f_π.

An attempt has recently been made[45] to estimate long-range effects such as real-gluon bremsstrahlung. The amplitude for the decay of the heavy quark Q into three light quarks + real gluon, $Q \to qqqg$, was added to the amplitude for $Q \to qqq$ + virtual gluons. A surprising result for the total rate in the lowest order in α_s was found. First of all, the leading log terms cancel in the total rate, so that to the order $0(\alpha_s)$ there is no short-distance enhancement factor. The result is infrared safe, but finite corrections go in the opposite direction, tending to suppress the total rate:

$$\Gamma = \frac{G_F^2 M_Q^5}{64 \pi^3} \left[1 - \frac{2}{3} \frac{\alpha_s}{\pi} \left(\pi^2 - \frac{31}{4} \right) \right].$$

However, a definite prediction is obtained for the charm decay where there are two leptonic channels. One has to the order $0(\alpha_s)$

$$\frac{\Gamma_c \to NL}{\Gamma_c \to SL} = \frac{3}{2} \left(1 + \frac{\alpha_s}{\pi} \right).$$

Some insight into these problems[42,46] might be obtained by looking for even heavier quarks[47,48,42]. The best candidate is the b-quark, as far as there is now rather good evidence for its existence[49,50]. It turns out[48] that QCD corrections with FSB induce new penguin operators which can alter the »natural« Cabibbo pattern of the Kobayashi-Maskawa model. Certain decays such as $B^- \to \overline{K}^0 \pi^-$, which are forbidden by the bare Hamiltonian, proceed with rates comparable with those for other $\Delta C = \Delta S = 0$ two-body decays, owing to the penguin operator

$$H_{NL}^{\text{Penguin}} = \sqrt{\frac{1}{2}}\, G_F \left(\frac{\alpha_s}{12} \ln \frac{m_t^2}{m_b^2} \right) (s_2 + s_3) \{ \overline{s}\, \gamma_\mu (1 - \gamma_5) \lambda_a\, b\, (\overline{q}^i \gamma^\mu \lambda_a q^i) \}$$

An experimentally more relevant prediction is[48] that the branching ratio is

$$\frac{\Gamma (B \to n \pi + K)}{\Gamma (B \to (n + 1) \pi)} \cong 0(1)$$

instead of being of the order of a few percent, as predicted by the bare Kobayashi-Maskawa Hamiltonian.

There is no doubt that pure mesonic decays have to be studied with the same intensity as applied to nonleptonic decays involving baryons.

Details of such investigations as well as details of quark dynamics will be explored in another paper.

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Appendix

Here we outline the *QCD* renormalisation of the $\Delta S = 1$, $\Delta C = 0$ part of the *WS* weak Hamiltonian$^{22)}$. We start from the second-order contribution which is of the form (3.1)*:

$$H \sim g_W^{eff} \int d^4 x \, D^{\mu\nu}(x) \, T(\mathcal{J}_\mu(x) \mathcal{J}_\nu(0)), \tag{A1}$$

and carry out the Wilson expansion for small x, as already explained in Sec. 3. An important contribution comes from the local four-quark operators Q_k, i. e.

$$H_W^{eff} = \sum_k \overline{A}_k (g, \mu, M, m_q, \ldots) \, Q_k. \tag{A2}$$

The expansion coefficients in (A2) depend on the quark-gluon (*QCD*) coupling constant g, the renormalisation mass μ, the *IVB* mass M and on the assorted quark masses m_q. When the *QCD* renormalisation procedure is switched off, the Hamiltonian H_W^{eff} takes the form**

$$H_W^{eff} = \frac{G_F}{2\sqrt{2}} \sin \Theta_C \cos \Theta_C \{2 \, \overline{s}u\overline{u}d - 2 \, \overline{s}c\overline{c}d\} + \text{h. c.}, \tag{A3}$$

where

$$\overline{a}b\overline{c}d \equiv \overline{a}^i \gamma_\mu (1 - \gamma_5) b^i \, \overline{c}^j \gamma^\mu (1 - \gamma_5) d^j, \tag{A4}$$

with the summation extending over colour indices i and j.

As explained in detail in Sec. 3, the *RGE* has to be solved as a function of continuously changing renormalisation mass, say μ, and as a function of the »running« (effective) *QCD* coupling constant $g(\mu)$. In this process, one has to distinguish several regions. When $\mu > m_c$, the leading $\ln \mu$ terms in the *RGE* do not »feel« *SU* (4) flavour symmetry breaking. In this region, the operators appearing in (A1) can be classified according to the *SU* (4) flavour group as follows

$$H_W^{eff} = \frac{G_F}{2\sqrt{2}} sc \, [\overline{A}_1 Q_1 + \overline{A}_2 Q_2 + \overline{A}_3 Q_3 + \overline{A}_4 Q_4], \tag{A5}$$

where

$$Q_1 = O_{8(1/2)}^{20} = (\overline{s}u\overline{u}d - \overline{s}d\overline{u}u) - (\overline{s}c\overline{c}d - \overline{s}d\overline{c}c) = R_1 + R_8,$$

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*In order to make this Appendix self-contained, we briefly repeat some steps already explained in the preceding sections of this paper.

**This form follows from the *WS-GIM* model assuming that the *IVB* is very heavy. Therefore, we have neglected the terms of order $1/M^4$ in expression (A1).

$$Q_2 = O^{84}_{8(1/2)} = \frac{1}{5}(\bar{s}u\bar{u}d + \bar{s}d\bar{u}u + 2\,\bar{s}d\bar{d}d + 2\,\bar{s}d\bar{s}s) - (\bar{s}c\bar{c}d + \bar{s}d\bar{c}c) \,-$$

$$= R_2 + R_9$$

$$\text{(A6)}$$

$$Q_3 = O^{84}_{27(1/2)} = \frac{2}{15}(\bar{s}u\bar{u}d + \bar{s}d\bar{u}u + 2\,\bar{s}d\bar{d}d - 3\,\bar{s}d\bar{s}s) = R_3,$$

$$Q_4 = O^{84}_{27(3/2)} = \frac{2}{3}(\bar{s}u\bar{u}d + \bar{s}d\bar{u}u - \bar{s}d\bar{d}d) = R_4.$$

(The indices in expressions (A6) denote $SU(4)$ structure, $SU(3)$ structure and isospin changes (ΔI). For later use, we distinguish parts without c quarks ($R_1 - - R_4$) from parts containing c quarks (R_8, R_9). As explained in Sec. 3, the expansion coefficients $\bar{A}_k$ are found by solving the *RGE*. This equation contains, as coefficients, anomalous dimensions of the operators Q_i (A6). Anomalous dimensions are to be found by the renormalisation procedure, as outlined below.

In the region $\mu > m_c$, the operators Q_k are multiplicatively renormalisable. This means that in the leading-logarithm approximation

$$Q_1 \to \left(1 - [4]\frac{g^2}{16\pi^2}\ln\mu^2\right) Q_1,$$

$$\text{(A7)}$$

$$Q_{2,3,4} \to \left(1 - [-2]\frac{g^2}{16\pi^2}\ln\mu^2\right) Q_{2,3,4}.$$

No further counter terms are needed in this region. The values in (A7) have been obtained from diagrams analogous to those in Fig. 1.

However, in the region $\mu < m_c$, the classification (A5, A6) is not »natural«. In this region, the renormalisation depends on the mass differences between the c quark and other lighter quarks. Although $SU(4)_{\mathrm{flavour}}$ symmetry is thus broken, its subgroups $SU(3)$ and $SU(2)$ are still useful. It is natural to consider in this region the operators R_i ($i = 1, \dots, 4$)* separately from the operators R_3 and R_9. All four-quark operators entering the renormalisation procedure are in the form of the local current-current product (A4). Each current, say $\bar{a}^i \gamma_\mu (1 - \gamma_5)\, b^i$, is usually taken to have zero anomalous dimension. However, this holds exactly only as long as quarks a and b have equal masses. Once $SU(4)$ symmetry is broken by quark mass differences, this is no longer true. For example, the operators R_8 and R_9 correspond to the current product $(\bar{s}c)(\bar{c}d)$. An anomalous dimension for each of these currents, say $\bar{s}c$ or $\bar{c}d$, has to be calculated and entered in the *RGE*.

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*The operators R_i ($i = 1, 4$) can be connected to the SVZ operators (see expression (4.2) as follows:

$\frac{1}{4}R_1 = - O_1$, $\frac{1}{4}R_2 = \frac{1}{5}O_2$, $\frac{1}{4}R_3 = \frac{2}{15}O_3$ and $\frac{1}{4}R_4 = \frac{2}{3}O_4$.

To renormalise the operators R_i $(i = 1, \dots, 4)$ in the region $\mu < m_c$, we define the operator basis $\{R_i\}$. This basis contains besides the operators R_i $(i = 1, \dots, 4)$ also the operators

$$R_5 = \bar{s}\, t^a \gamma_\mu (1 - \gamma_5)\, d \times \sum_{u,d,s} \bar{\Psi}\, t^a \gamma^\mu (1 + \gamma_5)\, \Psi,$$

$$R_6 = \bar{s}\, \gamma_\mu (1 - \gamma_5)\, d \times \sum \bar{\Psi} \gamma^\mu (1 + \gamma_5)\, \Psi, \tag{A8}$$

$$R_7 = \frac{2}{g}\, \bar{s}\, t^a \gamma_\mu (1 - \gamma_5)\, d \left[D_\nu^{ab} F_b^{\mu\nu} + \frac{g}{2} \sum \bar{\Psi} \gamma^\mu t^a \Psi \right]^{*}.$$

The operator basis $\{R_i\}$ (A8) is now closed under the renormalisation[2)]. This means that no new operators appear when *QCD* corrections are calculated in the leading-logarithm approximation[**]. From diagrams analogous to those shown in Figs. 1, 2 and 3, one finds

$$\begin{bmatrix} R_3 \\ R_4 \end{bmatrix} \to \left(1 - \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \frac{g^2}{16\pi^2} \ln \mu^2 \right) \begin{bmatrix} R_3 \\ R_4 \end{bmatrix} \tag{A9}$$

and

$$\begin{bmatrix} R_1 \\ R_2 \\ R_5 \\ R_6 \\ R_7 \end{bmatrix} \to \left(1 - \begin{bmatrix} 34/9 & -5/9 & -1/6 & 0 & 1/3 \\ -2/9 & -23/9 & -1/6 & 0 & 1/3 \\ -4/3 & -10/3 & 6 & 16/3 & 2 \\ 0 & 0 & 3/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 9/2 \end{bmatrix} \frac{g^2}{16\pi^2} \ln \mu^2 \right) \begin{bmatrix} R_1 \\ R_2 \\ R_5 \\ R_6 \\ R_7 \end{bmatrix}. \tag{A10}$$

The operators R_8 and R_9 close upon themselves under the renormalisation in the region $\mu < m_c$. One finds

$$\begin{bmatrix} R_8 \\ R_9 \end{bmatrix} \to \left(1 - \begin{bmatrix} -1/2 & -1 \\ -1/2 & -1 \end{bmatrix} \frac{g^2}{16\pi^2} \ln \mu^2 \right) \begin{bmatrix} R_8 \\ R_9 \end{bmatrix}. \tag{A11}$$

Now we proceed to construct the effective *QCD* renormalised Hamiltonian at the point $\mu_0 = \mu < m_c$. For this purpose, we need as an input the bare H_W^{eff} and the

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[*] The operator R_7 has to be considered since in the renormalisatoin procedure we have to calculate also diagrams in Fig. 2. (These are the so-called »penguin« diagrams.) In the region $\mu > m_c$, the contributions from these diagrams cancel owing to the *GIM* mechanism, but this is no longer true when $\mu < m_c$. The *QCD* equation of motion turns R_7 into the current-current product: $R_7 = -\bar{s} t^a \gamma^\mu (1 - \gamma_5)\, d \bar{c} \gamma_\mu t^a c$.

[**] This is not strictly correct, because R_7 also generates counterterms, such as $\bar{s} \nabla \nabla \nabla d$ $(\nabla = \partial - igG \cdot t)$, and some other gauge-noninvariant terms. Since these do not belong to the class-*I* operators, they can be omitted[39)].

H_W^{eff} at the point $\mu > m_c$. They will provide boundary conditions needed to solve the *RGE*. We find

$$\left(\frac{G}{2\sqrt{2}}\right)^{-1} H_W^{eff}(g \to 0) = h(g \to 0) = Q_1 + Q_2 + Q_3 + Q_4 \tag{A12}$$

and (see Eq. (3.13))

$$h(\mu > m_c, g) = \sum_{k=1}^{4} \overline{A}_k \left(\frac{M}{\mu}, g\right) Q_k,$$

$$\overline{A}_k \left(\frac{M}{\mu}, g\right) = \overline{A}_k^F \left[\varkappa_2 \left(\frac{M}{\mu}, g\right)\right]^{\overline{a}_i}, \tag{A13}$$

$$\varkappa_2 \left(\frac{M}{\mu}, g\right) \approx 1 - 2\, b''\, g^2 \ln \frac{M}{\mu}.$$

As already mentioned, in this region the operators $Q_1, \dots Q_4$ are multiplicatively renormalisable and have anomalous dimensions proportional to $\overline{a}_k$. The constant A_k^F has to be determined from the »free-field« Hamiltonian (A3). Note that when *QCD* is switched off ($g \to 0$), expression (A13) coincides with expressions (A12) and (A3) provided that

$$\overline{A}_i^F \ (i = 1, \dots, 4) \equiv 1. \tag{A14}$$

The solution of the *RGE* for the region $\mu < m_c$ has to coincide with the solution of the region $\mu > m_c$ at the border $\mu = m_c$. The solution at the border for $\mu > m_c$ can be constructed using expressions (A5) and (A13). Thus, one obtains

$$h(\mu \to m_c, x) = \sum_{i=1}^{4} \left[\varkappa_2 \left(\frac{M}{m_c}, x\right)\right]^{\overline{a}_i} R_i + \sum_{i=1}^{2} \left[\varkappa_2 \left(\frac{M}{m_c}, x\right)\right]^{\overline{a}_i} R_{i+7}. \tag{A15}$$

In the region $\mu < m_c$, the renormalisation procedure requires introduction of additional operators, R_5, R_6 and R_7, as can be seen from expression (A10). One defines

$$h(\mu < m_c, g) = \sum_{i=1}^{7} A_i \left(\frac{M}{\mu}, \frac{M}{m_c}, g\right) R_i + \sum_{i=8}^{9} A_i \left(\frac{M}{\mu}, \frac{M}{m_c}, g\right) R_i. \tag{A16}$$

The coefficients A_i in (A16) should be determined from the *RGE* in such a way that expressions (A15) and (A16) coincide for $\mu \to m_c$. However, the operators R_i are not multiplicatively renormalisable, as can be seen from (A10). In order to be able to solve the *RGE*, one has to find another set of operators which are multiplicatively renormalisable.

This can be achieved by introducing a matrix β whose elements are so chosen as to diagonalise (A10). Then a new set of operators is obtained*

$$P_i = \sum_{j=1}^{9} \beta_{ij} R_j, \tag{A17}$$

Expression (A16) now transforms into the following expression:

$$h(\mu < m_c, g) = \sum_{k=1}^{9} D_k \left(\frac{M}{\mu}, \frac{M}{m_c}, g\right) P_k. \tag{A18}$$

The coefficients D_k can be found in a straightforward way by solving the *RGE*. They are as follows:

$$D_k \left(\frac{M}{\mu}, \frac{M}{m_c}, g\right) = D_k^0 \left(\frac{M}{\mu \to m_c}, \frac{M}{m_c}, \bar{g}\right) [\varkappa_1]^{a_k}. \tag{A19}$$

Here one has used the running coupling constant (see Sec. 3)

$$\bar{g} = g\left(g, \frac{m_c}{\mu}\right); g, (g, 1) = g, \tag{A20}$$

while (see 3.13)

$$\varkappa_1 \approx \left(1 - 2b' g^2 \ln \frac{m_c}{\mu}\right). \tag{A21}$$

All other quantities are as defined in Sec. 3. The value of D_k^0 is to be found from the boundary condition at $\mu \to m_c$. One can connect the coefficients D_k with A_k via the matrix β^{-1} whose matrix elements are β_{ij}^*. Then from expressions (A16) and (A8) one obtains

$$D_k = \sum_i A_i \beta_{ik}^*. \tag{A22}$$

Expression (A22) is sufficient to connect D^0 with the constants appearing in (A15).

The final result for H_W^{eff} is*

$$H_W^{eff}(\mu < m_c, g) = \frac{G}{2\sqrt{2}} \sin \Theta_C \cos \Theta_C$$

$$\times \left\{ \sum_{k=1}^{9} \left[\sum_{i,j} A_i^F \left[\varkappa_2 \left(\frac{M}{m_c}, g\right)\right]^{\bar{a}_i} \beta_{ij}^* [\varkappa_1]^{a_j} \beta_{jk} \right] R_k \right\} + \text{h. c.} =$$

$$= \frac{G}{2\sqrt{2}} \sin \Theta_C \cos \Theta_C \sum_{k=1}^{9} a_k R_k. \tag{A23}$$

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*Without diagonalisation one would have to consider a system of renormalisation-group equations.

Comparing expression (A23) with expression (A16) for $\mu \to m_c$, one obtains

$$A_i^F = 1, \qquad i = 1-4, 8, 9$$
$$A_i^F = 0, \qquad i = 5-7. \qquad \text{(A24)}$$

Using the equation of motion, one can eliminate the gluon field from the operator R_7, and one is left with the product of left-handed and right-handed currents,

$$R_7 = -1 \, \bar{s} \, t^a \gamma_\mu (1 - \gamma_5) d \, [\bar{c} \, t^a \gamma^\mu (1 - \gamma_5) c + \bar{c} \, t^a \gamma^\mu (1 + \gamma_5) c]. \qquad \text{(A25)}$$

The operators $R_1, \ldots, R_6$ correspond to the SVZ operators. The operators R_7, R_8 and R_9 containing the $\bar{c}c$ pair were not listed in Ref. 2. If hadrons are made mostly of valence quarks, such operators are not important in the $\Delta S = 1$ nonleptonic hyperon decays. However, they ought to be considered in the decays involving charmed hadrons (mesons).

To conclude, we list some numerical values for the coefficients a_k from expression (A23). Using, for example,

$$M_W = 100 \text{ GeV}, \qquad \mu = 0.5 \text{ GeV}, \qquad m_c = 2 \text{ GeV}, \qquad \text{(A26)}$$

one finds

$$\begin{aligned} a_1 &= 2.57, & a_2 &= 0.47, & a_3 &= 0.61, \\ a_4 &= 0.61, & a_5 &= -0.07, & a_6 &= -0.02, \\ a_7 &= 0.12, & a_8 &= 1.76, & a_9 &= 0.60. \end{aligned} \qquad \text{(A27)}$$

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*Keep in mind that the anomalous dimensions $\bar{a}_i$ are calculated for $\mu > m_c$, while the anomalous dimensions a_i are calculated for $\mu < m_c$.

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NELEPTONSKI EFEKTIVNI SLABI HAMILTONIJAN I *QCD* KOREKCIJE

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Prikazan je sustavni proračun neleptonskog efektivnog Hamiltonijana koji je zasnovan na Weinberg-Salamovom modelu i koji sadrži četiri ili šest kvarkova. Potanko su raspisana i raspravljena slijedeća područja efektivnog slabog Hamiltonijana: $\Delta S = 1, \Delta C = 0$ koji uključuje narušenje $C P$ refleksije), $\Delta S = 0, \Delta C = 0$ (u kojem se narušava paritet) i $\Delta S = 1$, $\Delta C = 1$ sektor. Naš prikaz uključuje jednostavno pravilo za proračun utjecaja kvantno kromodinamičke renormalizacije u slučaju da je slomljena »okusna« simetrija. Također su raspravljene teoretske osnove kvantno-kromodinamičkih korekcija, te eksperimentalne potvrde i posljedice.