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BROKEN SU(6) AND THE VECTOR MESON MASSES*

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Within a broken $SU(6)$ approach to the mass matrix we derive a set of experimentally well supported vector meson linear mass relations. We then determine the masses of the B^* vector mesons. We also investigate the average vector meson mass dependence on the number of quarks and use it to predict a new $t\bar{t}$ mass value and to confine the T^* vector meson mass spectrum within a plausible range.

1. Introduction

The history of the attempts to theoretically determine the vector meson masses goes way back to the first appearance of broken $SU(3)$ and the consequent Gell-Mann-Okubo mass formula[1]. Unlike the case of baryons where this formula produced experimentally well satisfied linear mass relations, the mesons, both pseudoscalar and vector, seemed to favour the analogous square-mass relations.

The observed vector meson masses (then, a nonet of K^*, ϱ, ω, Φ) required an additional consideration known as the $\omega - \Phi$ mixing, proposed by Sakurai[2] and later elaborated by many authors[3]. A variety of approaches converged to the conslusion that ω and Φ are nearly ideally mixed which, in the quark language, amounts to saying that Φ is the $s\bar{s}$ bound state.

The appearance of Ψ vector meson and the outstanding success of the charmonium sheme testified to the $c\bar{c}$ quark composition, and analogously, the detec-

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tion of Υ vector meson led to the $b\bar{b}$ quark composition. Hence our assumption, in this paper, that vector mesons form a basis of the 6×6 direct product representation of $SU(6)$.

The broken $SU(4)$ mass formulae were developed by Mathur, Okubo and Borchardt[4], with both linear and quadratic options, and the broken $SU(5)$ was considered by S. Ono, S. Sawada and also by Baghi at al.[5].

An interesting approach to the mass sum rules, in which $SU(3)$ subgroups of the broken symmetry group are investigated, instead of the full $SU(n)$ algebra, has been introduced by Dattoli et al.[6]. Unlike our paper this approach avoids explicit use of the $SU(6)$ representations and the corresponding algebraic machinery, but instead makes use of suitable assumptions on the $SU(3)$ transformation properties of the mass operator.

Group theoretical implications for the meson mass spectrum were also extensively studied in terms of broken chiral symmetries, realised in the Nambu-Goldstone manner[7], in particular $SU(3) \times SU(3)$ and $SU(4) \times SU(4)$. This approach, however, depends on current algebra techniques, partial conservation rules, gauge invariance etc., and generally produces sum rules in which the meson mass spectrum appears involved with other parameters, such as form-factors or decay constants. Combined with the quark-lagrangean model the chiral symmetry approach produces the current quark mass ratios and their connection with the observed meson masses makes an interesting study.

From our point of view the most exciting situation is created by the failure so far to experimentally detect the long expected $t\bar{t}$ bound state, i. e. the $SU(6)$ missing vector meson. To the current flood of theoretical conjuctures[8], our paper via an empirical mass recurrence relation adds a new $t\bar{t}$ mass estimate, higher than the recent folklore belief of 28—29 GeV[9].

2. *Masses of the heavy vector mesons*

In a recent paper by Aubrecht and Scott[10] a broken $SU(6)$ internal symmetry is used, supplemented by an empirical rule, to determine a quark mass spectrum. In our paper an analogous approach is applied to the vector meson masses.

We make the following assumptions about the physical states:

There are six quarks q_α, with $\alpha = 1, \ldots, 6$, conventionally named u, d, s, c, b, t and they make a basis of an $SU(6)$ six-dimensional representation IR-6, in which the generators F_{pq} are chosen as the 35 linearly independent, Hermitian, traceless matrices[11]:

$$\left.\begin{aligned}
(F^{(6)}_{pq})_{\alpha\beta} &= \frac{1}{2}\,(\delta_{p\alpha}\,\delta_{q\beta} + \delta_{q\alpha}\,\delta_{p\beta}), \qquad 1 \le p < q \le 6,\\
(F^{(6)}_{pq})_{\alpha\beta} &= \frac{i}{2}\,(\delta_{p\alpha}\,\delta_{q\beta} - \delta_{q\alpha}\,\delta_{p\beta}), \qquad 1 \le q < p \le 6,\\
(F^{(6)}_{p})_{\alpha\beta} &= \frac{1}{\sqrt{2p\,(p-1)}}\left(\sum_{k=1}^{p-1}\delta_{k\alpha} - (p-1)\,\delta_{p\alpha}\right), \qquad 2 \le p \le 6.
\end{aligned}\right\} \tag{1}$$

The vector mesons are assumed to form a basis $q_\alpha \overline{q}_\beta$ of the reducible 6×6 direct product representation (e. g. $B_c^* = b\,\overline{c}$, $T_s^* = t\overline{s}$, $\Phi = s\overline{s}$, $\Psi = c\overline{c}$, $\Upsilon = b\,\overline{b}$, etc.). This single-pair assumption is supported by increasing experimental evidence, in particular by the outstanding success of quarkonium schemes. The two lightest vector mesons ϱ and ω make a well known exception, but that is here irrelevant since we neglect the $SU(2)$ mass symmetry breaking.

The broken $SU(6)$ symmetry is manifested by the transformation properties of the mass operator $\mathcal{M}$ which is assumed to be an $SU(6)$ scalar plus a linear combination of the 35 components of a tensor operator of the $SU(6)$ regular representation. In order to ensure the mass matrix to be diagonal in the basis of the physical states only those components are included which do not change the quantum numbers. Neglecting the electromagnetic mass differences ($\mathcal{M}_3 \approx 0$) this gives

$$\mathcal{M} = \mathcal{M}_0 + \sum_p a_p \mathcal{M}_p, \qquad 2 \leq p \leq 6, \tag{2}$$

where a_p are the mass operator symmetry breaking constants.

The mass of a vector meson V is given as the matrix element

$$m_V = \langle V | \mathcal{M} | V \rangle, \tag{3}$$

and can be brought, on account of (2) and the stated assumptions about the quark structure of the vector mesons, into the following form

$$m_V = M + \sum_{p=1}^{6} \{(1-p)(\delta_{p\alpha} + \delta_{p\beta}) + \sum_{k=1}^{p-1} (\delta_{k\alpha} + \delta_{k\beta})\} c_p, \tag{4}$$

where α, β indicate the quark-antiquark structure. The constants M and c_p designate five, $SU(6)$-independent, parameters.

Relation (4) yields the vector meson mass formulae given in Table 1.

TABLE 1

m_V \ V	ϱ	ω	Φ	K^*	Ψ	D^*	F^*	Υ	B^*	B_s^*	B_c^*	$t\bar{t}$	T^*	T_s^*	T_c^*	T_b^*
M	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
c_3	1	1	-2	$-\frac{1}{2}$	0	$\frac{1}{2}$	-1	0	$\frac{1}{2}$	-1	0	0	$\frac{1}{2}$	-1	0	0
c_4	1	1	1	1	-3	-1	-1	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{3}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{3}{2}$	0
c_5	1	1	1	1	1	1	1	-4	$-\frac{3}{2}$	$-\frac{3}{2}$	$-\frac{3}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-2
c_6	1	1	1	1	1	1	1	1	1	1	1	-5	-2	-2	-2	-2

Vector meson mass formulae

As seen from Table 1 relation (4) can be viewed as a dependent linear system of 16 equations in 5 unknowns M, c_3, c_4, c_5, c_6 and its consistency implies linear mass relations such as:

$$2\,K^* = \Phi + \varrho, \tag{5}$$

$$\Psi - \Phi = 2\,D^* - 2\,K^*, \tag{6}$$

$$F^* + \varrho = D^* + K^*. \tag{7}$$

Experimental values support these mass relations to high accuracy giving in (5): 1.78 $\cong$ 1.80 (GeV), in (6): 2.08 $\cong$ 2.24 (GeV), and in (7): 2.92 $\cong$ 2.90 (GeV).

Comment: Relation (5) could have been known already within the $SU(3)$ meson spectroscopy, but at the time of its development the quadratic mass relations were held in favour. Relations (6) and (7) are typical of the $SU(4)$ spectroscopy and some equivalent expressions appeared within the $SU(4)$ approach in Dattoli at al.[6] and within the chiral $SU(4) \times SU(4)$ approach in Das and Deshpande[7].

Parameters M, c_3, c_4, c_5, c_6 would be fully determined if masses of all the vector mesons with vacuum quantum numbers were known. In fact, denoting the masses of ϱ, ω, Φ, Ψ, Υ, $t\bar{t}$ as m_1, ..., m_6, respectively, we have

$$c_p = -\frac{m_p}{p} + \frac{1}{p\,(p-1)} \sum_{q=1}^{p-1} m_q, \tag{8}$$

and also

$$M = \frac{1}{6} \sum_{q=1}^{6} m_q. \tag{9}$$

Relation (8) is not the only way in which we can express the values of the constants c_p in system (4). The essential point, however, is that for each c_p we need the knowledge of the experimental mass value of at least one vector meson which contains the quark q_p. From (8) we have

$$c_3 = -\,80\ \text{MeV}, \qquad c_4 = -\,560\ \text{MeV}, \qquad c_5 = -\,1620\ \text{MeV}, \tag{10}$$

but cannot determine c_6 since there is no experimental mass value of $t\bar{t}$ vector meson.

The same reason prevents determination of M from (9) but Table 1 shows that within any subset of vector mesons, distinguished by the presence of a particular quark q_p, both M and c_6 can be eliminated when the mass value of any one member of the subset is known. In particular, the insertion of (10) into Table 1, the knowledge of Υ — mass value and subsequent elimination of M and c_6 gives for the B^* vector meson masses:

$$B^* = 5140\ \text{MeV}, \qquad B_s^* = 5260\ \text{MeV}, \qquad B_c^* = 6300\ \text{MeV}. \tag{11}$$

Therefore, we expect B^* vector mesons to have masses in the range 5—6.5 GeV.

Because, so far, there is yet no experimental knowledge about the mass of any t-qurak containing vector meson we can not use Table 1 to determine T^* vector meson masses unless we have some alternative method to obtain the value of M.

In this connection we consider formula (9) as the expression for the $SU(6)$ average vector meson mass, $M = M_6$, and introduce an $SU(n)$ generalization

$$M_n = \frac{1}{n} \sum_{p=1}^{n} m_p, \qquad 2 \leq n \leq 6. \tag{12}$$

The experimental data yield:

$$M_2 = 780 \text{ MeV}, \quad M_3 = 860 \text{ MeV}, \quad M_4 = 1420 \text{ MeV}, \quad M_5 = 3036 \text{ MeV}. \tag{13}$$

The values (13) fit well into a recurrence relation

$$\frac{1}{n} \cdot \frac{M_{n+1}}{M_n} = \frac{1}{n-1} \cdot \frac{M_n}{M_{n-1}}, \tag{14}$$

which gives

$$\frac{1}{2} \frac{M_3}{M_2} = \frac{1}{3} \frac{M_4}{M_3} = \frac{1}{4} \frac{M_5}{M_4} = k, \tag{15}$$

with $k = 0.55$, 0.55, 0.53, respectively.

Insertion of M_4, M_5 into (14) yields

$$M \leq M_6 = 8349 \text{ MeV}, \tag{16}$$

which, when introduced into the mass formula for Υ in Table 1 gives

$$c_6 = -\ 5400 \text{ MeV}, \tag{17}$$

and, consequently, from the $t\bar{t}$ mass formula in Table 1, predicts the mass value for the $t\bar{t}$ bound state as

$$m_{t\bar{t}} \approx 35 \text{ GeV}. \tag{18}$$

This is somewhat higher than the previous estimates, obtained by a variety of methods and by different authors, but the negative results of the experimental search so far, and also the forthcoming extension of the $e^+ e^-$ coliding beams energy range[12] to 30—40 GeV, qualifies result (18) as an interesting possibility.

To obtain the T^* meson masses we insert (10), (17) and (18) into Table 1 and get

$$\left.\begin{aligned} T^* &= 18029 \text{ MeV}, & T_s^* &= 18149 \text{ MeV}, \\ T_c^* &= 19189 \text{ MeV}, & T_b^* &= 22399 \text{ MeV}, \end{aligned}\right\} \tag{19}$$

thus indicating 18—23 GeV as a plausible T^* mass range.

3. Conclusions

The broken $SU(6)$ approach to the mass matrix gives rise to the experimentally well supported vector meson linear mass relations. Supplemented by an empirical recurrence relation it leads to an interesting estimate for the missing $t\bar{t}$ vector meson mass.

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SU(6) NARUŠENJE I MASE VEKTORSKIH MEZONA

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U okviru $SU(6)$ narušenja masene matrice dobiven je niz linearnih masenih relacija za vektorske mezone. One pokazuju dobro slaganje sa eksperimentalnim vrijednostima. U radu se dobiva i novo predviđanje vrijednosti mase očekivanog $t\bar{t}$ vezanog stanja.