YU ISSN 0015—3206
FZKAAA 12 (3) 217 (1980)

QUANTIZATION OF THE DIRAC FIELD IN THE NEW CANONICAL FORMULATION*

J. BRANA and K. LJOLJE

Department of Physics, Faculty of Sciences,

University of Sarajevo, Sarajevo, Yugoslavia

Received 12 June 1980

UDC 530. 19

Original scientific paper

The new canonical formulation of the Dirac field is applied to the quantization of the field. It is found that the canonical pairs satisfy commutation relations. The scalar, energy-momentum and angular momentum constants of motion are evaluated. The diagonalization of the energy contains some divergencies. These divergencies indicate another approach to the Dirac field.

1. Introduction

Correct canonical formulation of the Dirac field has been recently developed[1,2,3]. The application of this formulation to the quantization of the massless Dirac field was subject of Ref. 4. In the present paper we apply this formulation to the quantization of the Dirac field with a mass term.

In Section 2 we reproduce in a short form the new canonical formulation of the Dirac field. We give general solution of the Lagrange's and the canonical equa-

* This work was supported by the SIZ of Science of SR Bosna and Hercegovina, Sarajevo.

tions in Section 3. The quantization of the Dirac field in the new canonical formulation is considered in Section 4. The scalar, energy-momentum and angular momentum constants of motion are evaluated in Section 5. The diagonalization of the energy operator is performed in Section 6. Conclusions are given in Section 7.

2. *Canonical formulation of the Dirac field*

Starting from the Lagrangian[1])

$$\mathscr{L} = K\overline{\Psi}\Psi = K(-\mathrm{i}\,\partial_\nu\overline{\Phi}\,\gamma^\nu - \varkappa\,\overline{\Phi})(\mathrm{i}\,\partial_\mu\,\gamma^\mu\,\Phi - \varkappa\,\Phi), \tag{1}$$

where Φ, $\Phi^\dagger$ are the Lagrange's variables of the field, $\overline{\Phi} = \Phi^\dagger\,\gamma^0$ and K is a real constant, and the Lagrange's equation

$$\frac{\partial\mathscr{L}}{\partial u} - \partial_\mu\frac{\partial\mathscr{L}}{\partial(\partial_\mu u)} = 0 \tag{2}$$

follow the Lagrange's equations for $\Phi^\dagger$, Φ:

$$(\mathrm{i}\,\partial_\nu\,\gamma^\nu - \varkappa)(\mathrm{i}\,\partial_\mu\,\gamma^\mu - \varkappa)\,\Phi = 0, \tag{3}$$

$$(-\mathrm{i}\,\partial_\nu\,\overline{\Phi}\,\gamma^\nu - \varkappa\,\overline{\Phi})(-\mathrm{i}\,\overleftarrow{\partial}_\mu\,\gamma^\mu - \varkappa) = 0. \tag{4}$$

The canonical momenta conjugate to Φ, $\Phi^\dagger$ are

$$\Pi_\Phi = \mathrm{i}\,K\,\Psi^\dagger = \mathrm{i}\,K(-\mathrm{i}\,\partial_\nu\overline{\Phi}\,\gamma^\nu - \varkappa\,\overline{\Phi})\,\gamma^0, \tag{5}$$

$$\Pi_{\Phi^\dagger} = -\,\mathrm{i}\,K\,\Psi = -\,\mathrm{i}\,K(\mathrm{i}\,\partial_\nu\,\gamma^\nu\Phi - \varkappa\,\Phi). \tag{6}$$

The Hamiltonian density

$$\mathscr{H} = \Pi_\Phi\,\dot{\Phi} + \dot{\Phi}^\dagger\,\Pi_{\Phi^\dagger} - \mathscr{L} \tag{7}$$

is then

$$\mathscr{H} = \frac{1}{K}\Pi_\Phi\,\beta\,\Pi_{\Phi^\dagger} - \Pi_\Phi(\partial_i\,a^i\,\Phi + \mathrm{i}\,\varkappa\,\beta\,\Phi) + (-\,\partial_i\,\Phi^\dagger\,a^i + \mathrm{i}\,\varkappa\,\Phi^\dagger\,\beta)\,\Pi_{\Phi^\dagger}, \tag{8}$$

where $\beta = \gamma^0$, $\beta\,a^i = \gamma^i$.

According to

$$\frac{\mathrm{d}F}{\mathrm{d}t} = \frac{\partial F}{\partial t} + \{F, H\}, \tag{9}$$

where $\{F, H\}$ is the Poisson bracket, the canonical equations are

$$\dot{\Phi} = \{\Phi, H\} = \frac{\delta H}{\delta \Pi_\Phi} = \frac{1}{K} \beta \Pi_{\Phi^\dagger} - \partial_i \alpha^i \Phi - \mathrm{i} \varkappa \beta \Phi,$$

$$\dot{\Phi}^\dagger = \{\Phi^\dagger, H\} = \frac{\delta H}{\delta \Pi_{\Phi^\dagger}} = \frac{1}{K} \Pi_\Phi \beta - \partial_i \Phi^\dagger \alpha^i + \mathrm{i} \varkappa \Phi^\dagger \beta, \tag{10}$$

$$\dot{\Pi}_\Phi = \{\Pi_\Phi, H\} = -\frac{\delta H}{\delta \Phi} = -\partial_i \Pi_\Phi \alpha^i + \mathrm{i} \varkappa \Pi_\Phi \beta,$$

$$\dot{\Pi}_{\Phi^\dagger} = \{\Pi_{\Phi^\dagger}, H\} = -\frac{\delta H}{\delta_{\Phi^\dagger}} = -\partial_i \alpha^i \Pi_{\Phi^\dagger} - \mathrm{i} \varkappa \beta \Pi_{\Phi^\dagger}. \tag{11}$$

These equations can be also written in the form

$$\Pi_{\Phi^\dagger} = -\mathrm{i} K (\mathrm{i} \partial_\nu \gamma^\nu \Phi - \varkappa \Phi),$$

$$\Pi_\Phi = \mathrm{i} K (-\mathrm{i} \partial_\nu \Phi^\dagger \gamma^{\nu\dagger} - \varkappa \Phi^\dagger), \tag{12}$$

$$(-\mathrm{i} \partial_\mu \Pi_\Phi' \gamma^{\mu\dagger} - \varkappa \Pi_\Phi) = 0,$$

$$(\mathrm{i} \partial_\mu \gamma^\mu - \varkappa) \Pi_{\Phi^\dagger} = 0, \tag{13}$$

or using Ψ, $\Psi^\dagger$ from Eqs. (5) and (6)

$$\Psi = \mathrm{i} \partial_\nu \gamma^\nu \Phi - \varkappa \Phi$$

$$\overline{\Psi} = -\mathrm{i} \partial_\nu \overline{\Phi} \gamma^\nu - \varkappa \overline{\Phi}, \tag{14}$$

$$(\mathrm{i} \partial_\nu \gamma^\nu - \varkappa) \Psi = 0,$$

$$(-\mathrm{i} \partial_\mu \overline{\Psi} \gamma^\mu - \varkappa \overline{\Psi}) = 0. \tag{15}$$

By this the correct canonical as well as the Lagrange's formulation of the Dirac field is established.

We use the coordinates $x^\mu = (t, \vec{x})$ with the metric tensor

$$g_{00} = -g_{11} = -g_{22} = -g_{33} = 1, \qquad g_{\alpha\beta} \neq 0, \qquad \alpha \neq \beta$$

and units $c = \hbar = 1$.

3. *Solutions of the Lagrange's and canonical equations*

We seek solutions of the Lagrange's and the canonical equations in the form

$$\Phi^i(\vec{x}, t) = \frac{1}{L^{3/2}} \sum_{\vec{k}} a^i_{\vec{k}}(t)\, e^{i\vec{k}\vec{x}},$$

$$\Phi^{i\dagger}(\vec{x}, t) = \frac{1}{L^{3/2}} \sum_{\vec{k}} a^{i\dagger}_{\vec{k}}(t)\, e^{-i\vec{k}\vec{x}}, \tag{16}$$

$$\Psi^i(\vec{x}, t) = \frac{1}{L^{3/2}} \sum_{\vec{k}} A^i_{\vec{k}}(t)\, e^{i\vec{k}\vec{x}},$$

$$\Psi^{i\dagger}(\vec{x}, t) = \frac{1}{L^{3/2}} \sum_{\vec{k}} A^{i\dagger}_{\vec{k}}(t)\, e^{-i\vec{k}\vec{x}}, \qquad i = 1, 2, 3, 4. \tag{17}$$

Eq. (3) gives for the coefficients

$$a_{\vec{k}}(t) = \begin{bmatrix} a^1_{\vec{k}}(t) \\ a^2_{\vec{k}}(t) \\ a^3_{\vec{k}}(t) \\ a^4_{\vec{k}}(t) \end{bmatrix}, \tag{18}$$

$$\ddot{a}_{\vec{k}}(t) + 2\,\mathrm{i}\,\varkappa\,\gamma^0\,\dot{a}_{\vec{k}}(t) + (k^2 - \varkappa^2 - 2\,\varkappa\,k_l\,\gamma^l)\,a_{\vec{k}}(t) = 0. \tag{19}$$

Introducing the notation

$$\alpha_{\vec{k}}(t) = \begin{pmatrix} a^1_{\vec{k}}(t) \\ a^2_{\vec{k}}(t) \end{pmatrix}, \qquad \beta_{\vec{k}}(t) = \begin{pmatrix} a^3_{\vec{k}}(t) \\ a^4_{\vec{k}}(t) \end{pmatrix},$$

$$\xi_{\vec{k}}(t) = \begin{pmatrix} A^1_{\vec{k}}(t) \\ A^2_{\vec{k}}(t) \end{pmatrix}, \qquad \zeta_{\vec{k}}(t) = \begin{pmatrix} A^3_{\vec{k}}(t) \\ A^4_{\vec{k}}(t) \end{pmatrix}, \tag{20}$$

and the standard representation of γ^μ matrices

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

Eq. (19) can be written in the form

$$\ddot{a}_{\vec{k}}(t) + 2\,\mathrm{i}\,\varkappa\,\dot{a}_{\vec{k}}(t) + (k^2 - \varkappa^2)\,a_{\vec{k}}(t) - 2\varkappa\,(k_l\,\sigma^l)\,\beta_{\vec{k}}(t) = 0, \tag{21}$$

$$\ddot{\beta}_{\vec{k}}(t) - 2\,\mathrm{i}\,\varkappa\,\dot{\beta}_{\vec{k}}(t) + (k^2 - \varkappa^2)\,\beta_{\vec{k}}(t) + 2\varkappa\,(k_l\,\sigma^l)\,a_k = 0. \tag{22}$$

Let us notice that

$$(k_l\,\sigma^l)\,(k_l\,\sigma^l) = k^2. \tag{23}$$

From Eq. (21) using (23) it follows

$$\beta_k(t) = \frac{(k_l\,\sigma^l)}{2\,\varkappa\,k^2}\,[\ddot{a}_{\vec{k}} + 2\,\mathrm{i}\,\varkappa\,\dot{a}_{\vec{k}} + (k^2 - \varkappa^2)\,a_{\vec{k}}]. \tag{24}$$

After substitution of this expression in Eq. (22) one gets equation for $a_{\vec{k}}(t)$:

$$\ddot{a}_{\vec{k}} + 2\,(k^2 + \varkappa^2)\,\dot{a}_{\vec{k}} + (k^2 + \varkappa^2)^2\,a_{\vec{k}} = 0. \tag{25}$$

General solution of this equation is

$$a_{\vec{k}}(t) = a_{\vec{k}1}\,e^{-ik_0t} + a_{\vec{k}2}\,e^{ik_0t} + t\,(\gamma_{\vec{k}1}\,e^{-ik_0t} + \gamma_{\vec{k}2}\,e^{ik_0t}), \tag{26}$$

where $k_0 = \sqrt{k^2 + \varkappa^2}$ and

$$a_{\vec{k}1} = \begin{pmatrix} \overset{1}{a}_{\vec{k}1} \\ \overset{2}{a}_{\vec{k}1} \end{pmatrix}, \qquad a_{\vec{k}2} = \begin{pmatrix} \overset{1}{a}_{\vec{k}2} \\ \overset{2}{a}_{\vec{k}2} \end{pmatrix},$$
$$\gamma_{\vec{k}1} = \begin{pmatrix} \overset{1}{c}_{\vec{k}1} \\ \overset{2}{c}_{\vec{k}1} \end{pmatrix}, \qquad \gamma_{\vec{k}2} = \begin{pmatrix} \overset{1}{c}_{\vec{k}2} \\ \overset{2}{c}_{\vec{k}2} \end{pmatrix} \tag{27}$$

are arbitrary constant matrices.

The substitution $a_{\vec{k}}(t)$ from (26) into (24) gives $\beta_{\vec{k}}(t)$:

$$\beta_{\vec{k}}(t) = -(k_l\sigma^l)\left\{\frac{1}{k_0-\varkappa}\left(a_{\vec{k}2}-\frac{i}{\varkappa}\gamma_{\vec{k}2}\right)e^{ik_0t}-\frac{1}{k_0+\varkappa}\left(a_{\vec{k}1}-\right.\right.$$
$$\left.\left.-\frac{i}{\varkappa}\gamma_{\vec{k}1}\right)e^{-ik_0t}+t\left[\frac{1}{k_0-\varkappa}\gamma_{\vec{k}2}e^{ik_0t}-\frac{1}{k_0+\varkappa}\gamma_{\vec{k}1}e^{-ik_0t}\right]\right\}. \tag{28}$$

Eqs. (26) and (28) are general solution of the Lagrange's equation (3) in $\vec{k}$-space.

One can get the solutions of the canonical equations directly or it can be done by making use of the solution of the Lagrange's equations and the connection (14). We follow the last choice.

Eqs. (14) and (17) give

$$\xi_{\vec{k}}(t) = i\,\dot{a}_{\vec{k}}(t) - (k_l\sigma^l)\,\beta_{\vec{k}}(t) - \varkappa\,a_{\vec{k}}(t), \tag{29}$$

$$\zeta_{\vec{k}}(t) = -i\,\dot{\beta}_{\vec{k}}(t) + (k_l\sigma^l)\,a_{\vec{k}}(t) - \varkappa\,\beta_{\vec{k}}(t). \tag{30}$$

From here, taking into account (26) and (27), one gets

$$\xi_{\vec{k}}(t) = \mathrm{i}\,\frac{k_0}{\varkappa}\left(\gamma_{\vec{k}1}e^{-ik_0t}-\gamma_{\vec{k}2}e^{ik_0t}\right), \tag{31}$$

$$\zeta_{\vec{k}}(t) = i\,\frac{k_0}{\varkappa}(k_l\sigma^l)\left(\frac{1}{k_0+\varkappa}\gamma_{\vec{k}1}e^{-ik_0t}+\frac{1}{k_0-\varkappa}\gamma_{\vec{k}2}e^{ik_0t}\right). \tag{32}$$

Eqs. (26), (28), (31) and (32) give the general solution of the canonical equations (14) and (15) in $\vec{k}$-space.

In the next section it will be useful to have the inverse of this solution with respect to the constants $a_{\vec{k}i}$, $\gamma_{\vec{k}i}$, i. e. $a_{\vec{k}1}$, $a_{\vec{k}2}$, $\gamma_{\vec{k}1}$, $\gamma_{\vec{k}2}$ expressed by means of $a_{\vec{k}}(t)$, $\beta_{\vec{k}}(t)$, $\xi_{\vec{k}}(t)$ and $\zeta_{\vec{k}}(t)$.

From Eqs. (31) and (32), using (23), it follows

$$\gamma_{\vec{k}1} = \frac{\varkappa}{2\,\mathrm{i}\,k_0^2}\left[(k_l\sigma^l)\,\zeta_{\vec{k}}(t) + (k_0+\varkappa)\,\xi_{\vec{k}}(t)\right]e^{ik_0t}, \tag{33}$$

$$\gamma_{\vec{k}2} = \frac{\varkappa}{2\,\mathrm{i}\,k_0^2}\left[(k_l\sigma^l)\,\zeta_{\vec{k}}(t) - (k_0-\varkappa)\,\xi_{\vec{k}}(t)\right]e^{-ik_0t}. \tag{34}$$

In order to get corresponding equations for $a_{\vec{k}1}$ and $a_{\vec{k}2}$ we write Eqs. (26) and (28) in the form

$$a_{\vec{k}}(t) = a_{\vec{k}1} e^{-ik_0 t} + a_{\vec{k}2} e^{ik_0 t} + F_{\vec{k}}, \tag{35}$$

$$\beta_{\vec{k}}(t) = -(k_l \sigma^l)\left[-\frac{1}{k_0 + \varkappa} a_{\vec{k}1} e^{-ik_0 t} + \frac{1}{k_0 - \varkappa} a_{\vec{k}2} e^{ik_0 t} + G_{\vec{k}}\right], \tag{36}$$

where

$$F_{\vec{k}} = t\,\gamma_{\vec{k}1} e^{-ik_0 t} + t\,\gamma_{\vec{k}2} e^{ik_0 t}, \tag{37}$$

$$G_{\vec{k}} = \left(\frac{i}{\varkappa} - t\right)\left[\frac{1}{k_0 + \varkappa}\gamma_{\vec{k}1} e^{-ik_0 t} - \frac{1}{k_0 - \varkappa}\gamma_{\vec{k}2} e^{ik_0 t}\right] \tag{38}$$

with $\gamma_{\vec{k}1,2}$ given by (33), (34), or explicitely

$$F_{\vec{k}} = t\frac{\varkappa}{\mathrm{i}\, k_0^2}\left[\varkappa\, \xi_{\vec{k}}(t) + (k_l \sigma^l)\,\zeta_{\vec{k}}(t)\right], \tag{39}$$

$$G_{\vec{k}} = \left(\frac{i}{\varkappa} - t\right)\frac{\varkappa}{\mathrm{i}\, k_0^2}\left[\xi_{\vec{k}}(t) - \frac{\varkappa}{k^2}(k_l \sigma^l)\,\zeta_{\vec{k}}(t)\right]. \tag{40}$$

From Eqs. (35) and (36) it follows

$$a_{\vec{k}1} = \frac{1}{2k_0}\left[(k_l \sigma^l)\,\beta_{\vec{k}}(t) + k^2 G_{\vec{k}} + (k_0 + \varkappa)(a_{\vec{k}}(t) - F_{\vec{k}})\right] e^{ik_0 t}, \tag{41}$$

$$\beta_{\vec{k}2} = \frac{1}{2k_0}\left[-(k_l \sigma^l)\,\beta_{\vec{k}}(t) - k^2 G_{\vec{k}} + (k_0 - \varkappa)(a_{\vec{k}}(t) - F_{\vec{k}})\right] e^{-ik_0 t}. \tag{42}$$

4. Canonical quantization of the Dirac field

We assume the canonical quantization in the form[4)]

$$\{\ ,\ \}_{Poisson} \to \frac{1}{i}[\ ,\ ]_{Quantum}. \tag{43}$$

The Hamiltonian of the field then becomes an operator

$$H = \int \mathscr{H}\, d^3x = \int \left\{\frac{1}{K}\Pi_\Phi\, \beta\, \Pi_{\Phi^\dagger} - \Pi_\Phi(\partial_i \alpha^i \Phi + \mathrm{i}\,\varkappa\, \beta\, \Phi) + \right.$$
$$\left. + (-\partial_i \Phi^\dagger \alpha^i + \mathrm{i}\,\varkappa\, \Phi^\dagger \beta)\,\Pi_{\Phi^\dagger}\right\} d^3x \tag{44}$$

and the dynamical equation (9) goes over into

$$\frac{\mathrm{d}F}{\mathrm{d}t} = \frac{\partial F}{\partial t} + \frac{1}{\mathrm{i}}[F, H]. \tag{45}$$

The commutation relations are[4)]

$$\begin{aligned}
&[\Phi_a(\vec{x}, t), \Phi_b(\vec{y}, t)] = [\Phi_a(\vec{x}, t), \Phi_b^\dagger(\vec{y}, t)] = [\Phi_a^\dagger(\vec{x}, t), \Phi_b^\dagger(\vec{y}, t)] = 0,\\
&[\Pi_{\Phi_a}(\vec{x}, t), \Pi_{\Phi_b}(\vec{y}, t)] = [\Pi_{\Phi_a}(\vec{x}, t), \Pi_{\Phi_b^\dagger}(\vec{y}, t)] = [\Pi_{\Phi_a^\dagger}(\vec{x}, t), \Pi_{\Phi_b^\dagger}(\vec{y}, t)] = 0,\\
&[\Phi_a(\vec{x}, t), \Pi_{\Phi_b^\dagger}(\vec{y}, t)] = [\Phi_a^\dagger(\vec{x}, t), \Pi_{\Phi_b}(\vec{y}, t)] = 0,\\
&[\Phi_a(\vec{x}, t), \Pi_{\Phi_b}(\vec{y}, t)] = \mathrm{i}\,\delta_{ab}\,\delta(\vec{x} - \vec{y}),\\
&[\Phi_a^\dagger(\vec{x}, t), \Pi_{\Phi_b^\dagger}(\vec{y}, t)] = \mathrm{i}\,\delta_{ab}\,\delta(\vec{x} - \vec{y}), \qquad a, b = 1, 2, 3, 4.
\end{aligned} \tag{46}$$

Let us mention that Eqs. (46) must be the commutation relations, not anticommutation relations. The commutation relations only give correct canonical equations in accordance with (45).

From the Hamiltonian (44), the dynamical equation (45) and the commutation rules (46) follow the equations of motion for the operators Φ, $\Phi^\dagger$, Π_Φ, $\Pi_{\Phi^\dagger}$:

$$\begin{aligned}
\dot{\Phi} &= -\mathrm{i}\,[\Phi, H] = \frac{1}{K}\gamma^0\,\Pi_{\Phi^\dagger} - \partial_i\,\alpha^i\,\Phi,\\
\dot{\Phi}^\dagger &= -\mathrm{i}\,[\Phi^\dagger, H] = \frac{1}{K}\Pi_\Phi\,\gamma^0 - \partial_i\,\Phi^\dagger\,\alpha^i,\\
\dot{\Pi}_\Phi &= -\mathrm{i}\,[\Pi_\Phi, H] = -\,\partial_i\,\Pi_\Phi\,\alpha^i,\\
\dot{\Pi}_{\Phi^\dagger} &= -\mathrm{i}\,[\Pi_{\Phi^\dagger}, H] = -\,\partial_i\,\alpha^i\,\Pi_{\Phi^\dagger}.
\end{aligned} \tag{47}$$

These equations are in agreement with Eqs. (10) and (11).

We perform further analysis in the $\vec{k}$-space. By making use of (16) and (17), i. e.

$$\Phi^i(\vec{x}, t) = \frac{1}{L^{3/2}}\sum_{\vec{k}} a^i_{\vec{k}}(t)\,e^{i\vec{k}\vec{x}},$$

$$\Phi^{i\dagger}(\vec{x}, t) = \frac{1}{L^{3/2}}\sum_{\vec{k}} a^{i\dagger}_{\vec{k}}(t)\,e^{-i\vec{k}\vec{x}},$$

$$\Pi_{\Phi i} = K\,\Psi'^{i\dagger}(\vec{x}, t) = K\,\frac{1}{L^{3/2}}\sum_{\vec{k}} A^{i\dagger}_{\vec{k}}(t)\,e^{-i\vec{k}\vec{x}},$$
$$\Pi_{\Phi i\dagger} = K\,\Psi^{i}(\vec{x}, t) = K\,\frac{1}{L^{3/2}}\sum_{\vec{k}} A^{i\dagger}_{\vec{k}}(t)\,e^{i\vec{k}\vec{x}} \tag{48}$$

the quantum rules (46) in the $\vec{k}$-space are

$$\begin{aligned}
&[a^{i}_{\vec{k}}(t), a^{j}_{\vec{k}'}(t)] = [a^{i}_{\vec{k}}(t), a^{j\dagger}_{\vec{k}'}(t)] = [a^{i\dagger}_{\vec{k}}(t), a^{j\dagger}_{\vec{k}'}(t)] = 0,\\
&[A^{i}_{\vec{k}}(t), A^{j}_{\vec{k}'}(t)] = [A^{i}_{\vec{k}}(t), A^{j\dagger}_{\vec{k}'}(t)] = [A^{i\dagger}_{\vec{k}}(t), A^{j\dagger}_{\vec{k}'}(t)] = 0,\\
&[a^{i}_{\vec{k}}(t), A^{j}_{\vec{k}'}(t)] = [a^{i\dagger}_{\vec{k}'}(t), A^{j\dagger}_{\vec{k}'}(t)] = 0,\\
&[a^{i}_{\vec{k}}(t), A^{j\dagger}_{\vec{k}'}(t)] = \frac{\mathrm{i}}{K}\,\delta_{ij}\,\delta_{\vec{k}\vec{k}'},\\
&[a^{i\dagger}_{\vec{k}}(t), A^{j}_{\vec{k}'}(t)] = \frac{\mathrm{i}}{K}\,\delta_{ij}\,\delta_{\vec{k}\vec{k}'}, \qquad i, j = 1, 2, 3, 4.
\end{aligned} \tag{49}$$

Having these commutation rules we now evaluate the commutation rules for the time independent coefficients ($a^{i}_{\vec{k}n}, a^{i\dagger}_{\vec{k}m}, c^{i}_{\vec{k}n}, c^{i\dagger}_{\vec{k}m}$):

$$\begin{aligned}
&[a^{i}_{\vec{k}1}, a^{j\dagger}_{\vec{k}'1}] = \frac{1}{K}\,\delta^{ij}\,\delta_{\vec{k}\vec{k}'}\,\frac{k^2}{2\,k_0^3},\\
&[a^{i}_{\vec{k}2}, a^{j\dagger}_{\vec{k}'2}] = -\frac{1}{K}\,\delta^{ij}\,\delta_{\vec{k}\vec{k}'}\,\frac{k^2}{2\,k_0^3},\\
&[a^{i}_{\vec{k}1}, c^{j\dagger}_{\vec{k}'1}] = \frac{i}{K}\,\delta^{ij}\,\delta_{\vec{k}\vec{k}'}\,\frac{\varkappa\,(k_0 + \varkappa)}{2\,k_0^2},\\
&[a^{i}_{\vec{k}2}, c^{j\dagger}_{\vec{k}'2}] = -\frac{\mathrm{i}}{K}\,\delta^{ij}\,\delta_{\vec{k}\vec{k}'}\,\frac{\varkappa\,(k_0 - \varkappa)}{2\,k_0^2}, \qquad i, j = 1, 2,
\end{aligned} \tag{50}$$

and all other commutators are zero.

Introducing the new coefficients according to

$$a^j_{\vec{k}i} = \sqrt{\frac{1}{K}\frac{k^2}{2\,k_0^3}}\, a^j_{\vec{k}Ni},$$

$$c^i_{\vec{k}1} = \frac{1}{\mathrm{i}}\frac{\varkappa\, k_0}{k_0 - \varkappa}\sqrt{\frac{1}{K}\frac{k^2}{2\,k_0^3}}\, c^i_{\vec{k}1N}, \tag{51}$$

$$c^i_{\vec{k}2} = \frac{1}{\mathrm{i}}\frac{\varkappa\, k_0}{k_0 + \varkappa}\sqrt{\frac{1}{K}\frac{k^2}{2\,k}}\, c^i_{\vec{k}2N}, \qquad i, j = 1, 2,$$

the commutation relations (50) become

$$[a^i_{\vec{k}1N}, a^{j\dagger}_{\vec{k}'1N}] = \delta^{ij}\,\delta_{\vec{k}\vec{k}'},$$

$$[a^i_{\vec{k}2N}, a^{j\dagger}_{\vec{k}'2N}] = -\,\delta^{ij}\,\delta_{\vec{k}\vec{k}'}, \tag{52}$$

$$[a^i_{\vec{k}1N}, c^{j\dagger}_{\vec{k}'1N}] = \delta^{ij}\,\delta_{\vec{k}\vec{k}'},$$

$$[a^i_{\vec{k}N2}, c^{j\dagger}_{\vec{k}'N2}] = -\,\delta^{ij}\,\delta_{\vec{k}\vec{k}'}, \qquad i, j = 1, 2,$$

and all other commutators are zero.

It is important to notice the essential difference between these quantum rules and those for the massless field[4]. The difference comes from the structure of the solution (26) and (28) which has its origin in the mass term of the Dirac equation.

One finds useful the following transformation:

$$a^i_{\vec{k}1N} = \frac{1}{2\sqrt{2}}\,(3\, b^i_{\vec{k}1} - d^{i\dagger}_{\vec{k}1}),$$

$$a^i_{\vec{k}2N} = \frac{1}{2\sqrt{2}}\,(3\, b^{i\dagger}_{\vec{k}2} - d^i_{\vec{k}2}), \tag{53}$$

$$c^i_{\vec{k}1N} = \frac{1}{2\sqrt{2}}\,(b^i_{\vec{k}1} + d^{i\dagger}_{\vec{k}1}),$$

$$c^i_{\vec{k}2N} = \frac{1}{2\sqrt{2}}\,(b^{i\dagger}_{\vec{k}2} + d^i_{\vec{k}2}), \qquad i = 1, 2.$$

The new operators $b^{i}_{\vec{k}j}$ and $d^{i}_{\vec{k}j}$ satisfy the commutation rules

$$
\begin{aligned}
&[b^{i}_{\vec{k}n}, b^{j\dagger}_{\vec{k}'m}] = \delta^{ij}\,\delta_{nm}\,\delta_{\vec{k}\vec{k}'},\\
&[d^{i}_{\vec{k}n}, d^{j\dagger}_{\vec{k}'m}] = \delta^{ij}\,\delta_{nm}\,\delta_{\vec{k}\vec{k}'},\\
&[b^{i}_{\vec{k}n}, d^{j\dagger}_{\vec{k}'m}] = 0,\\
&[b^{i}_{\vec{k}n}, d^{j\dagger}_{\vec{k}'m}] = 0, \qquad i, j, n, m = 1, 2,
\end{aligned} \tag{54}
$$

and other commutators are zero.

The advantage of these operators is evident.

5. *Constants of motion*

The scalar constant of motion, which follows from

$$
j^{\mu} = \mathrm{i}\left(\Phi^{\dagger}\frac{\partial \mathscr{L}}{\partial(\partial_{\mu}\Phi^{\dagger})} - \frac{\partial \mathscr{L}}{\partial(\partial_{\mu}\Phi)}\,\Phi\right), \qquad \partial_{\mu} j^{\mu} = 0, \tag{55}
$$

is given by

$$
Q = \mathrm{const} \int j^{0}\, d^{3}x = \mathrm{const} \int (\Phi^{\dagger}\Psi + \Psi^{\dagger}\Phi)\, d^{3}x. \tag{56}
$$

Using the expressions (16), (17), (18), (20), (26), (27) and (28) it becomes

$$
\begin{aligned}
Q = \mathrm{const} \sum_{\vec{k}, i=1,2} &\{(a^{i\dagger}_{\vec{k}1N}\, c^{i\dagger}_{\vec{k}1N} + c^{i\dagger}_{\vec{k}1N}\, a^{i}_{\vec{k}1N}) - (a^{i\dagger}_{\vec{k}2N}\, c^{i}_{\vec{k}2N} + \\
&+ c^{i\dagger}_{\vec{k}2N}\, a^{i}_{\vec{k}2N}) - c^{i\dagger}_{\vec{k}1N}\, c^{i}_{\vec{k}1N} + c^{i\dagger}_{\vec{k}2N}\, c^{i}_{\vec{k}2N}\}
\end{aligned} \tag{57}
$$

or written by means of the operators $b^{i}_{\vec{k}j}$ and $d^{i}_{\vec{k}j}$ (Eqs. (53))

$$
Q = \mathrm{const} \sum_{\vec{k}, i=1,2} \{b^{i\dagger}_{\vec{k}1}\, b^{i}_{\vec{k}1} - d^{i}_{\vec{k}1}\, d^{i\dagger}_{\vec{k}1} - b^{i}_{\vec{k}2}\, b^{i\dagger}_{\vec{k}2} + d^{i\dagger}_{\vec{k}2}\, d^{i}_{\vec{k}2}). \tag{58}
$$

We write also Q in the explicite particle number operator form:

$$Q = \text{const} \sum_{\vec{k}} \{ b^{1\dagger}_{\vec{k}1} b^{1}_{\vec{k}1} + b^{2\dagger}_{\vec{k}1} b^{2}_{\vec{k}1} - (d^{1\dagger}_{\vec{k}1} d^{1}_{\vec{k}1} + d^{2\dagger}_{\vec{k}1} d^{2}_{\vec{k}1}) - \tag{59}$$

$$- (b^{1\dagger}_{\vec{k}2} b^{1}_{\vec{k}2} + b^{2\dagger}_{\vec{k}2} b^{2}_{\vec{k}2}) + d^{1\dagger}_{\vec{k}2} d^{1}_{\vec{k}2} + d^{2\dagger}_{\vec{k}2} d^{2}_{\vec{k}2} - 4 \}.$$

The energy-momentum constant of motion follows from

$$T_\alpha{}^\beta = \partial_\alpha \Phi^\dagger \frac{\partial \mathscr{L}}{\partial (\partial_\beta \Phi^\dagger)} + \frac{\partial \mathscr{L}}{\partial (\partial_\beta \Phi)} \partial_\alpha \Phi - \delta_\alpha{}^\beta \mathscr{L}, \qquad \partial_\beta T_\alpha{}^\beta = 0 \tag{60}$$

and for the Lagrangian (1) reads

$$P_0 = \text{const} \int (\mathrm{i}\, \Psi^\dagger \dot{\Phi} - \mathrm{i}\, \dot{\Phi}^\dagger \Psi' - \Psi' \Psi') \, d^3 x, \tag{61}$$

$$P_j = \text{const} \int \mathrm{i}\, (\Psi^\dagger \partial_j \Phi - \partial_j \Phi^\dagger \Psi') \, d^3 x.$$

By making use of (16), (17), (18), (20), (26), (27) and (28) it becomes

$$P_0 = \text{const} \sum_{\vec{k},\, i=1,2} k_0 \Big\{ (a^{i\dagger}_{\vec{k}1N} c^{i}_{\vec{k}1N} + c^{i\dagger}_{\vec{k}1N} a^{i}_{\vec{k}1N}) + (a^{i\dagger}_{\vec{k}2N} c^{i}_{\vec{k}2N} + c^{i\dagger}_{\vec{k}2N} a^{i}_{\vec{k}2N}) - \tag{62}$$

$$- \left(1 - \frac{\varkappa}{k_0 - \varkappa}\right) c^{i\dagger}_{\vec{k}1N} c^{i}_{\vec{k}1N} - \left(1 + \frac{\varkappa}{k_0 + \varkappa}\right) c^{i\dagger}_{\vec{k}2N} c^{i}_{\vec{k}2N} \Big\},$$

$$P_j = \text{const} \sum_{\vec{k},\, i=1,2} k_j \{ (a^{i\dagger}_{\vec{k}1N} c^{i}_{\vec{k}1N} + c^{i\dagger}_{\vec{k}1N} a^{i}_{\vec{k}1N}) - (a^{i\dagger}_{\vec{k}2N} c^{i}_{\vec{k}2N} + c^{i\dagger}_{\vec{k}2N} a^{i}_{\vec{k}2N}) - \tag{63}$$

$$- c^{i\dagger}_{\vec{k}1N} c^{i}_{\vec{k}1N} + c^{i\dagger}_{\vec{k}2N} c^{i}_{\vec{k}2N} \},$$

or using the operators $b_{\vec{k}i}$ and $d_{\vec{k}i}$

$$P_0 = \text{const} \sum_{\vec{k},\, i=1,2} k_0 \Big\{ b^{i\dagger}_{\vec{k}1} b^{i}_{\vec{k}1} - d^{i}_{\vec{k}1} d^{i\dagger}_{\vec{k}1} + b^{i}_{\vec{k}2} b^{i\dagger}_{\vec{k}2} - d^{i\dagger}_{\vec{k}2} d^{i}_{\vec{k}2} + \tag{64}$$

$$+ \frac{1}{2} \frac{\varkappa}{k_0 - \varkappa} (b^{i\dagger}_{\vec{k}1} + d^{i}_{\vec{k}1}) (b^{i}_{\vec{k}1} + d^{i\dagger}_{\vec{k}1}) - \frac{1}{2} \frac{\varkappa}{k_0 + \varkappa} (b^{i}_{\vec{k}2} + d^{i\dagger}_{\vec{k}2}) (b^{i\dagger}_{\vec{k}2} + d^{i}_{\vec{k}2}) \Big\},$$

$$P_j = \text{const} \sum_{\vec{k},\, i=1,2} k_j \{ b^{i\dagger}_{\vec{k}1} b^{i}_{\vec{k}1} - d^{i}_{\vec{k}1} d^{i\dagger}_{\vec{k}1} - b^{i}_{\vec{k}2} b^{i\dagger}_{\vec{k}2} + d^{i\dagger}_{\vec{k}2} d^{i}_{\vec{k}2} \}. \tag{65}$$

The angular momentum density tensor

$$M^{\alpha;\beta\gamma} = (x^\gamma T^{\beta\alpha} - x^\beta T^{\gamma\alpha}) - \frac{\partial \mathscr{L}}{\partial(\partial_\alpha \Phi)}\left(\frac{\mathrm{i}}{2}\sigma^{\beta\gamma}\right)\Phi - \Phi^\dagger\left(-\frac{\mathrm{i}}{2}\sigma^{\beta\gamma\dagger}\right)\frac{\partial \mathscr{L}}{\partial(\partial_\alpha \Phi^\dagger)} \quad (66)$$

for the Lagrangian (1) becomes

$$M^{\alpha,\beta\gamma} = (x^\gamma T^{\beta\alpha} - x^\beta T^{\gamma\alpha}) + \frac{K}{2}\overline{\Psi}\,\gamma^\alpha \sigma^{\beta\gamma}\Phi + \frac{K}{2}\overline{\Phi}\,\sigma^{\beta\gamma}\gamma^\alpha \Psi. \quad (67)$$

From here the constant of motion is

$$M^{\beta\gamma} = \text{const} \int M^{0,\beta\gamma} d^3x =$$
$$= \text{const} \int \left\{x^\gamma T^{\beta 0} - x^\beta T^{\gamma 0} + \frac{K}{2}(\Psi^\dagger \sigma^{\beta\gamma}\Phi + \overline{\Phi}\,\sigma^{\beta\gamma}\gamma^0 \Psi)\right\} d^3x. \quad (68)$$

The spin part of this constant, using Eqs. (16) and (17), reads

$$S^{ij} = \text{const}\frac{K}{2}\sum_{\vec{k}} [A^{m}_{\vec{k}}{}^{\dagger}(\sigma^{ij})_{ml}\, a^{l}_{\vec{k}} + a^{m}_{\vec{k}}{}^{\dagger}(\sigma^{ij})_{ml}\, A^{l}_{\vec{k}}]. \quad (69)$$

The spin pseudovector is then

$$S_r = \frac{1}{2}\varepsilon_{ijr}\, S^{ij}$$

and using the expressions (20)

$$S_r = \text{const}\frac{K}{2}\sum_{\vec{k}} \{\xi^\dagger_{\vec{k}}\sigma_r\, a_{\vec{k}} + \zeta^\dagger_{\vec{k}}\sigma_r\, \beta_{\vec{k}} + a^\dagger_{\vec{k}}\sigma_r\, \xi_{\vec{k}} + \beta^\dagger_{\vec{k}}\sigma_r\, \zeta_{\vec{k}}\}. \quad (70)$$

For one particle state with the momentum $\vec{k}$, the projection $\frac{\vec{k}\vec{S}}{k}$, which is also a constant of motion, is

$$\frac{\vec{k}\vec{S}}{k} = \text{const}\frac{K}{2}\left\{-\mathrm{i}\frac{2k_0}{k_0+\varkappa}[\gamma^\dagger_{\vec{k}1}(\vec{k}\vec{\sigma})\, a_{\vec{k}1} - a^\dagger_{\vec{k}1}(\vec{k}\vec{\sigma})\,\gamma_{\vec{k}1}] +\right.$$
$$+\mathrm{i}\frac{2k}{\cancel{k_0+\varkappa}}[\gamma^\dagger_{\vec{k}2}(\vec{k}\vec{\sigma})\, a_{\vec{k}2} - a^\dagger_{\vec{k}2}(\vec{k}\vec{\sigma})\,\gamma_{\vec{k}2}] - \quad (71)$$
$$\left. -\frac{2}{\varkappa}\frac{k_0-\varkappa}{k_0+\varkappa}\gamma_{\vec{k}1}(\vec{k}\vec{\sigma})\,\gamma^\dagger_{\vec{k}1} + \frac{2}{\varkappa}\frac{k_0+\varkappa}{k_0-\varkappa}\gamma^\dagger_{\vec{k}2}(\vec{k}\vec{\sigma})\,\gamma_{\vec{k}2}\right\},$$

where we have used Eqs. (26) and (28).

Using the new operators given by (51) and (53) and taking the polar axis in the direction of the vector $\vec{k}$, it becomes

$$\frac{\vec{k}\vec{S}}{k} = \text{const}\,\frac{1}{2}\,\{(b^{1\dagger}_{\vec{k}1}\,b^{1}_{\vec{k}1} - b^{2\dagger}_{\vec{k}1}\,b^{2}_{\vec{k}1}) - (d^{1}_{\vec{k}1}\,d^{1\dagger}_{\vec{k}1} - d^{2}_{\vec{k}1}\,d^{2\dagger}_{\vec{k}1}) - - (b^{1}_{\vec{k}2}\,b^{1\dagger}_{\vec{k}2} - b^{2}_{\vec{k}2}\,b^{2\dagger}_{\vec{k}2}) + (d^{1\dagger}_{\vec{k}2}\,d^{1}_{\vec{k}2} - d^{2\dagger}_{\vec{k}2}\,d^{2}_{\vec{k}2})\}. \tag{72}$$

6. *Diagonalization of the energy operator*

The energy operator is not diagonalized. The term with the same indices has the form

$$H = \lambda\, A^\dagger A + \eta\, B^\dagger B + \omega\,(A^\dagger B^\dagger + A\, B) \tag{73}$$

where λ, η, ω are constants and A, $A^\dagger$, B, $B^\dagger$ operators satisfying the commutation rules

$$[A, A^\dagger] = [B, B^\dagger] = 1, \qquad [A^\dagger, B] = [A, B] = 0. \tag{74}$$

The Hamiltonian H can be diagonalized by the following canonical transformation[5)]

$$\mu_1 = A\,\text{ch}\,\Theta + B^\dagger\,\text{sh}\,\Theta, \qquad \mu_2 = A^\dagger\,\text{sh}\,\Theta + B\,\text{ch}\,\Theta \tag{75}$$

and c. c. The new operators satisfy the commutation rules

$$[\mu_i, \mu_j^\dagger] = \delta_{ij}, \qquad [\mu_i, \mu_j] = 0, \qquad i, j = 1, 2. \tag{76}$$

The condition that the operator H is diagonalized after the transformation (75),

$$H = E_0 + \sum_{i=1,2} E_i\, \mu_i^\dagger\, \mu_i, \tag{77}$$

gives

$$\begin{aligned} \text{th}\,\Theta &= \frac{1}{2\,\omega}\{(\eta - \lambda) + \sqrt{(\lambda + \eta)^2 - 4\,\omega^2}\} \equiv D, \\ E_1 &= \frac{1}{2}\{(\lambda - \eta) + \sqrt{(\lambda + \eta)^2 - 4\,\omega^2}\}, \\ E_2 &= \frac{1}{2}\{(\eta - \lambda) + \sqrt{(\lambda + \eta)^2 - 4\,\omega^2}\}, \\ E_0 &= \frac{D}{D^2 - 1}\sqrt{(\lambda + \eta)^2 - 4\,\omega^2}. \end{aligned} \tag{78}$$

We get for the energy operator terms given by (64)

$$\mathrm{th}\,\Theta \to 1, \qquad E_1 = -E_2 \to k_0, \qquad E_0 = 0. \tag{79}$$

Consequently,

$$P_0 = \mathrm{const} \sum_{\substack{\vec{k} \\ i=1,2}} k_0 \left(\overset{i\dagger}{\mu}_{\vec{k}1} \overset{i}{\mu}_{\vec{k}1} - \overset{i}{\nu}_{\vec{k}1} \overset{i\dagger}{\nu}_{\vec{k}1} + \overset{i}{\mu}_{\vec{k}2} \overset{i\dagger}{\mu}_{\vec{k}2} - \overset{i\dagger}{\nu}_{\vec{k}2} \overset{i}{\nu}_{\vec{k}2}\right\},$$

where $\mu_1 \to \mu$ and $\mu_2 \to \nu$, i. e.

$$\overset{i}{b}_{\vec{k}j} = \overset{i}{\mu}_{\vec{k}j}\,\mathrm{ch}\,\overset{i}{\Theta}_{\vec{k}j} - \overset{i\dagger}{\nu}_{\vec{k}j}\,\mathrm{sh}\,\overset{i}{\Theta}_{\vec{k}j},$$

$$\overset{i}{d}_{\vec{k}j} = \overset{i}{\nu}_{\vec{k}j}\,\mathrm{ch}\,\overset{i}{\Theta}_{\vec{k}j} - \overset{i\dagger}{\mu}_{\vec{k}j}\,\mathrm{sh}\,\overset{i}{\Theta}_{\vec{k}j}, \quad \text{and c. c.,} \tag{80}$$

and

$$[\overset{i}{\mu}_{\vec{k}n}, \overset{j\dagger}{\mu}_{\vec{k}'m}] = \delta^{ij}\,\delta_{nm}\,\delta_{\vec{k}\vec{k}'},$$

$$[\overset{i}{\nu}_{\vec{k}n}, \overset{j\dagger}{\nu}_{\vec{k}'m}] = \delta^{ij}\,\delta_{nm}\,\delta_{\vec{k}\vec{k}'}, \tag{81}$$

$$[\overset{i}{\nu}_{\vec{k}n}, \overset{j}{\mu}_{\vec{k}'m}] = [\overset{i}{\nu}_{\vec{k}n}, \overset{j\dagger}{\mu}_{\vec{k}'m}] = 0, \qquad i, j, n, m = 1, 2.$$

The transformation (80) for other constants of motion gives

$$P_j = \mathrm{const} \sum_{\substack{\vec{k} \\ i=1,2}} k_j \left\{\overset{i\dagger}{\mu}_{\vec{k}1} \overset{i}{\mu}_{\vec{k}1} - \overset{i}{\nu}_{\vec{k}1} \overset{i\dagger}{\nu}_{\vec{k}1} - \overset{i}{\mu}_{\vec{k}2} \overset{i\dagger}{\mu}_{\vec{k}2} + \overset{i\dagger}{\nu}_{\vec{k}2} \overset{i}{\nu}_{\vec{k}2}\right\}, \tag{82}$$

$$\underline{Q} = \mathrm{const} \sum_{\substack{\vec{k} \\ i=1,2}} \left(\overset{i\dagger}{\mu}_{\vec{k}1} \overset{i}{\mu}_{\vec{k}1} - \overset{i}{\nu}_{\vec{k}1} \overset{i\dagger}{\nu}_{\vec{k}1} - \overset{i}{\mu}_{\vec{k}2} \overset{i\dagger}{\mu}_{\vec{k}2} + \overset{i\dagger}{\nu}_{\vec{k}2} \overset{i}{\nu}_{\vec{k}2}\right), \tag{83}$$

$$\frac{\vec{k}\vec{S}}{k} = \mathrm{const}\,\frac{1}{2}\,\{(\overset{1\dagger}{\mu}_{\vec{k}1} \overset{1}{\mu}_{\vec{k}1} - \overset{2\dagger}{\mu}_{\vec{k}1} \overset{2}{\mu}_{\vec{k}1}) - (\overset{1}{\nu}_{\vec{k}1} \overset{1\dagger}{\nu}_{\vec{k}1} - \overset{2}{\nu}_{\vec{k}1} \overset{2\dagger}{\nu}_{\vec{k}1}) -$$

$$- (\overset{1}{\mu}_{\vec{k}2} \overset{1\dagger}{\mu}_{\vec{k}2} - \overset{2}{\mu}_{\vec{k}2} \overset{2\dagger}{\mu}_{\vec{k}2}) + (\overset{1\dagger}{\nu}_{\vec{k}2} \overset{1}{\nu}_{\vec{k}2} - \overset{2\dagger}{\nu}_{\vec{k}2} \overset{2}{\nu}_{\vec{k}2})\}. \tag{84}$$

Therefore, all constants of motion are diagonalized and have clear particle interpretation. However, th $\Theta \to 1$ leads to ch $\Theta \to \infty$, sh $\Theta \to \infty$. Although this does not affect the quantum rules and the constants of motion it indicates that the Dirac field with a mass term has to be treated differently (not differently in canonical sense).

7. *Conclusion*

Consistent and correct canonical quantization of the Dirac field with a mass term leads rather to the commutation and not anticommutation rules for the canonical pairs. The constants of motion of the field exhibit particle description. The spin of the particles is 1/2.

The diagonalization of the energy operator involves infinities. These infinities do not affect the quantum rules neither the constants of motion but indicate that the Dirac field with a mass term should be treated differently, not differently in canonical sense.

References

1) J. Brana and K. Ljolje, FIZIKA **10** (1978) 85;
2) J. Brana, Dissertation, (1977);
3) K. Ljolje and S. Vobornik, FIZIKA **11** (1979) 171;
4) J. Brana, K. Ljolje and S. Vobornik, FIZIKA, **12** (1980) 195;
) A. S. Davidov, *Kvantovaja mehanika,* Moskva, (1973), p. 230.

KVANTIZIRANJE DIRACOVOG POLJA U NOVOJ KANONSKOJ FORMULACIJI

J. BRANA i K. LJOLJE

Prirodno-matematički fakultet, Odsjek za fiziku,

Univerzitet u Sarajevu, Sarajevo

UDK 530.19

Originalni znanstveni rad

Na osnovu nove kanonske formulacije Diracovog polja sa masom mirovanja u radu je izvršeno konzistentno kanonsko kvantiziranje ovog polja. Kao i u slučaju Diracovog polja bez mase mirovanja, nađeno je da je kvantiziranje moguće provesti samo sa komutacionim, a ne i antikomutacionim pravilima. Izračunate konstante kretanja pokazuju čestičnu interpretaciju sa spinom 1/2. Međutim, dijagonaliziranje ovih konstanata sadrži izvjesne singularnosti. Ove singularnosti ukazuju na drugačiji pristup Diracovom polju sa masom mirovanja i drugačije njegovo fizikalno razumijevanje.