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THE ANALYSIS OF THE LOW ENERGY PART OF MUON SPECTRUM IN $K^+ \to \pi^0 \nu \mu^+$ DECAY

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Results are presented of an emulsion study of the low energy part of the μ^+ spectrum (5 MeV $< T_\mu <$ 16 MeV) resulting from 1331 $K^+ \to \pi^0 \nu \mu^+$ ($K_{\mu 3}$) decay of K^+ mesons at rest. The dependence of the matrix element of the energy of muon has been investigated. The value of formfactor ξ has been determined.

1. Introduction

The $K_{\mu 3}$, as semileptonic weak interaction, is a subject of continuous researches, from its discovery in 1957. This is a typical example of a high energy weak interaction which can be investigated experimentally in details, and at the same time a theoretical description of the process is relatively simple. The matrix element is a function of two formfactors f_+ and ξ.

Although there is a great number of experimental results, the errors of formfactors are still of the order of formfactors themselves, and even greater. Besides, the two methods used mostly the polarization[1,2,3] and the branching ratio $K_{\mu 3}/K_{e3}$[3,4,5,6,7,8] give the results which differ systematically. These results, except the work of G. Giacomelli[4] are obtained by the detectors which register high energy muons. The technique of the photonuclear emulsion used in this work, made the collection of the rich sample of low energy muons possible. From material which has been collected, the possibilities of using of emulsion technique and low energy part of spectrum to determine the formfactor ξ, have been investigated.

2. *Experimental procedure and results*

A part of stack of Ilford *K*5 emulsion, irradiated at Brookhaven National Laboratory in 1972, has been used as experimental material. The incident momentum of the K^+ mesons was parallel to the emulsion surface and was 330 MeV/c. After the K^+ mesons had been slowed down through the emulsion, they stopped and decayed at different depths. The development of the emulsion has been done at *CERN*.

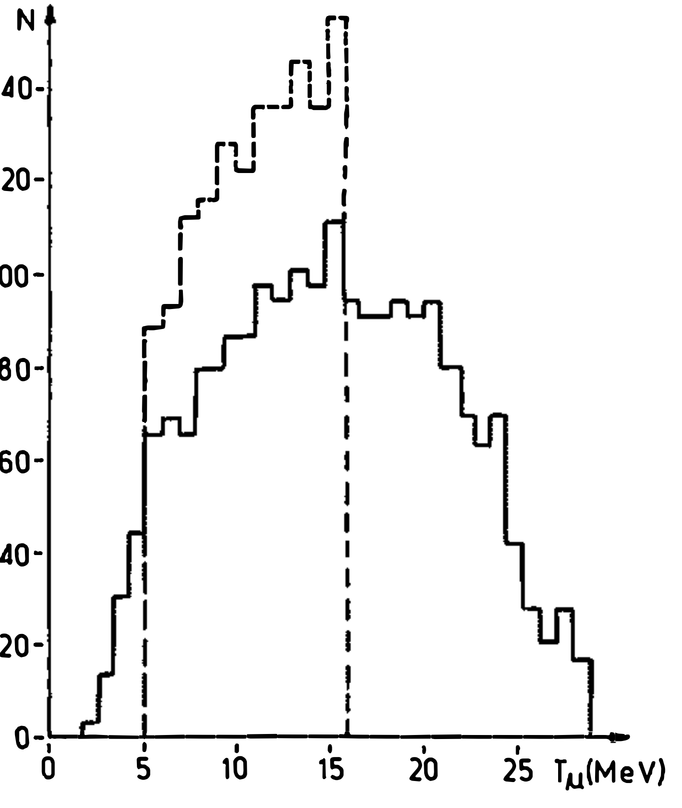

Fig. 1. Muons spectrum

In order to find decays of stopped K^+ mesons, the plates were scanned by microscope with magnification 15×20 using »the area scanning« method. The total area of 2000 cm^2 was scanned twice. The events with a grey secondary track with an ionisation at least greater than twice minimum, have been recorded. Then, the distance of these events from the emulsion surface, and the dip angle of the secondary track with respect to the emulsion surface (Θ), were measured.

When the dip was less than 60°, the track was followed till the end. The range of the track and the projected angle between the primary and secondary track were measured. Those events which secondary tracks ended by positron were identified as the $K_{\mu 3}$ decays.

The kinetic energy spectrum for all measured 2493 $K_{\mu 3}$ decays is given in Fig. 1 (full line). Related to Fig. 2, it can be seen that the experimental distribution does not follow the phase space distribution.

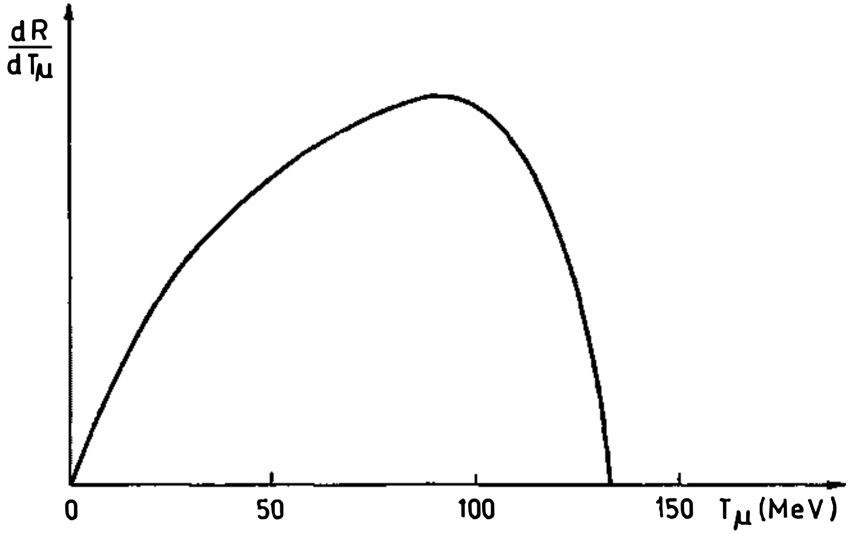

Fig. 2. The phase space distribution for muons

This does not agree either with earlier experiments or with theoretical predictions. Because of that, all events with $T_\mu > 16$ MeV were rejected before further analysis.

The value $T \;= 5$ MeV is chosen to be the lower limit of the spectrum. (Below 3 MeV, there are great fluctations in scanning efficiency. The contamination by $K^+ \to \pi^+ \pi^0 \pi^0$ decays with very short range of pion, and $\pi^+ \to \mu^+$ decays from beem direction has been avoided by rejecting muons with kinetic energy of about 4 MeV.)

In the interval 5 MeV $\leq T_\mu <$ 16 MeV the spectrum is characterized by high and constant scanning efficiency, that can be seen in Fig. 3.

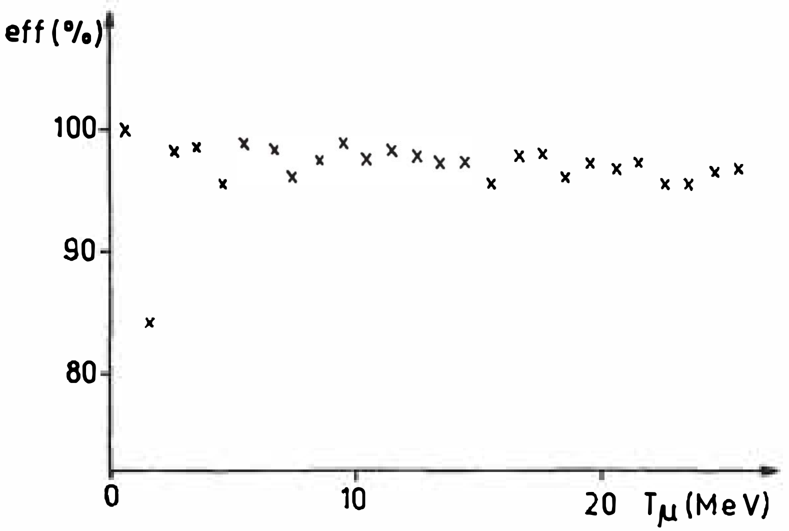

Fig. 3. The dependence of scanning efficiency on energy

The distributions of events were analysed with respect to the depth of emulsion, to the sine of a dip angle and to the azimuth, in order to establish geometrical losses (because of location and configuration of events). To make the kinetic energy spectrum corrected to each event dependently on its depth and angle Θ, a corresponding weight was attributed.

The correction connected with scanning efficiency was made by multiplication of the number of events in each energy interval by the value of eff^{-1} where eff is scanning efficiency in a given energy interval.

The corrected spectrum is illustrated in Fig. 1 (dotted line). It includes 1331 events.

3. *Analysis and discussion of results*

The form of matrix element for $K_{\mu 3}$ decay is:

$$M = \frac{g}{\sqrt{2}} f_+ \left[P_K - 1/2\,(1 - \xi)\,(P_\mu + P_\nu) \right] \overline{\nu}\,(\vec{P}_\nu)\, \gamma_L\,(1 + \gamma_5)\, \mu\,(\vec{P}_\mu) \qquad (1)$$

where g is coupling constant of weak interactions, $\vec{P}_\mu$ and $\vec{P}_\nu$ are momenta of muon and neutrino, P_μ and P_ν are their four momentum, and $\nu\,(\vec{P}_\nu)$ and $\mu\,(\vec{P}_\mu)$ are leptons wave functions.

The dependence of the phase space distribution of muons kinetic energy is given in the Fig. 2.

From formula (1) and the expression for three particle phase space, a differential transition probability is:

$$dW = \frac{g^2}{16\pi^3} |f_+|^2 [A + B \,\mathrm{Re}\xi + C|\xi|^2] \, dT_\pi \, dT_\mu \tag{2}$$

where A, B and C are functions of T_π, T_ν and P_μ.

The theoretical probability of emission of muon in a given energy interval, can be obtained by integration of formula (2) over all values of pions energy. In order to make the integration, it is necessary to suppose that formfactors are real and independent on T_π. Up to now, experimental results[5,10] show that this supposition is not rough, and hence it is used to calculate the muons spectrum. Using that way, the theoretical distribution is obtained as:

$$\frac{dN}{dE_\mu} = \frac{f_+^2 g^2}{(2\pi^3) \, 2m_K} [A(E_\mu) + B(E_\mu)\xi + C(E_\mu)\xi^2] \tag{3}$$

where E_μ is a total energy of muon and functions $A(E_\mu)$, $B(E_\mu)$ and $C(E_\mu)$ are known.

From the formula (3) it follows:

$$\frac{dN}{dE_\mu} = K \cdot F(E_\mu, \xi). \tag{4}$$

The coefficient K includes besides coupling constants, the value of formfactor f_+ and mass of K^+ meson too.

In order to determine the square of the matrix element, it is supposed that it is constant in energy interval of 1 MeV, and then, the corrected number of events in each interval was divided by the value of phase space. The result which is obtained is represented in Fig. 4. Units are arbitrary and dependent on statistics. It can be seen that in this energy interval, the square of the matrix element is practically independent of kinetic energy.

To determine the formfactor ξ, the theoretical energy spectrum of muons should be compared with the corrected experimental distribution.

If it is supposed that in the interval of 1 MeV f_+ does not depend on E_μ, for two succesive intervals, according to the formula (4), the relation:

$$\frac{\left(\frac{dN}{dE_\mu}\right)_i}{\left(\frac{dN}{dE_\mu}\right)_{i+1}} = \frac{F(E_\mu, \xi)}{F(E_\mu + 1, \xi)} \tag{5}$$

will be held. The left side represents the ratio of corrected number of events in two succesive intervals, while the right side is a theoretical value of that ratio for a given value ξ.

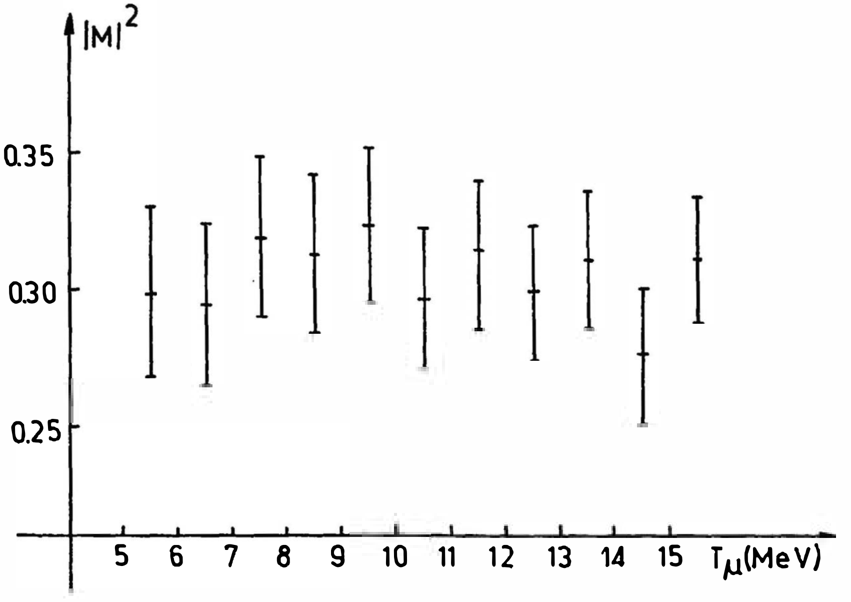

Fig. 4. Matrix element

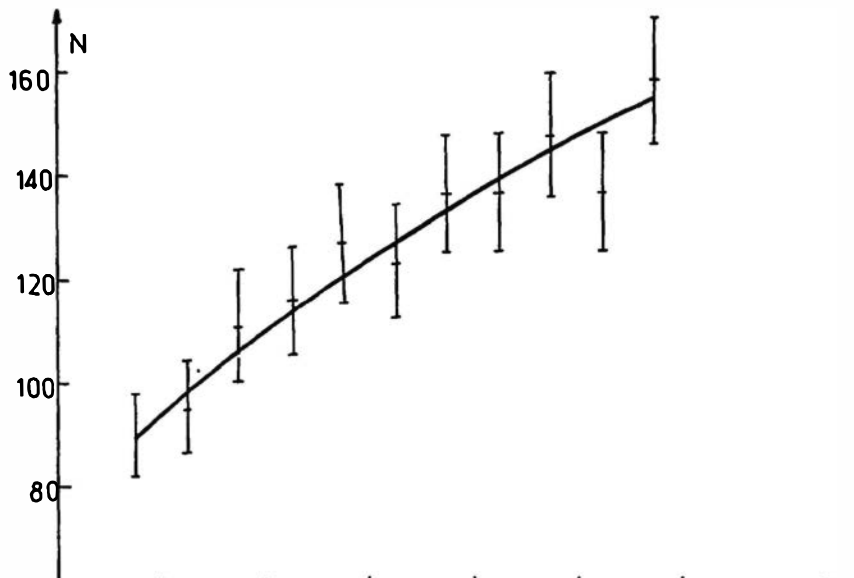

Fig. 5. The theoretical and experimental distributions of muons.

After the function F had been calculated for a series of values ξ in the interval $(-10 \leq \xi \leq +10)$, the best value ξ was determined using the χ^2 test. It is found that when $\xi = 0.2$, χ^2 is minimum, and that its increase is faster for values on the left of minimum.

The normalised theoretical curve for $\xi = 0.2 \pm 0.4$ compared to the experimental data has been represented in Fig. 5. The normalisation has been done so that the area under the curve would be equal to the area under dotted line in the Fig. 1 in the interval of kinetic energy betweeen 5 and 16 MeV. It can be seen that the theoretical curve fits very well the experimental distribution.

The results of the recent experimental works and of the only experiment using the emulsion technique up to now, have been presented in Table 1.

TABLE 1

reference	detector	method	T_μ (MeV)	number of events	ξ
Giacomelli[4]	emuls.	μ spect.	28	87	0.7
X2 coll.[8]	freon	polariz. Dalitz	$46 < T < 93$ $i > 105$	10900	-1.0 ± 0.3 0.6 ± 1.2 0.6
Chiang[3]	wire chamb.	$K_{\mu 3}/K_{e3}$	all	3695	-0.21 ± 0.35
Braun[9]	freon	$K_{\mu 3}/K_{e3}$	> 11	1800	-1.3 ± 0.7
this work	emuls.	μ spect.	$5 \leq T < 16$	1331	0.2 ± 0.4

To compare our value of formfactor with the other results given in Table 1, it should be mentioned that our result is obtained using very narrow energy interval. But, the average number of events in a unit interval is high and nearly equal to the average number of events in the experiment of X2 Collaboration in what the other part of the spectrum was studied. However, in our energy interval the phase space is small compared with the total available.

From Table 1 it can be seen that the recent values of formfactor group around zero, and our result is also close to zero. The drawback of the emulsion in the sense of impossibility of detection of π^0 mesons, is so compensated by possibility of investigating with a good statistics, the part of phase space in which a small number of events is expected.

4. *Conclusion*

Using the emulsion technique the matrix element and the formfactor ξ in the $K_{\mu 3}$ decay were investigated. It was found that in the interval 5 MeV $\leq T_\mu < 16$ MeV the matrix element does not depend on muon energy. From the χ^2 test the best value of ξ vas determined to be $\xi = 0.2 \pm 0.4$.

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ANALIZA NISKOENERGETSKOG DELA SPEKTRA MIONA U $K^+ \rightarrow \pi^0 \nu \mu^+$ RASPADU

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Prikazani su rezultati ispitivanja niskoenergetskog dela spektra miona u $K_{\mu3}$ raspadu. Analizirana je zavisnost matričnog elementa od kinetičke energije miona i određena vrednost formfaktora ξ.