

COSMIC RAY MUON CHARGE RATIO DERIVED FROM THE NEW SCALING VARIABLE MODEL

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The charge ratio of sea level muons has been estimated from the new scaling variable model and the CERN Intersecting Storage Ring data of Capiluppi et al. for $pp \rightarrow \pi^\pm X$ and $pp \rightarrow K^\pm X$ inclusive reactions. The estimated muon charge ratio is found to be 1.21 and the result has been compared with the experimental data of Parker et al., Burnett et al., Ashley et al. and Muraki et al.

1. Introduction

The study of the charge ratio of the sea level muons is of phenomenological importance in the cosmic ray physics and also in the study of high energy nuclear interactions.

Theoretical calculations of the charge ratio of sea level cosmic muons are based on the assumptions on the high energy accelerator data and such calculations have been done by different authors¹⁻⁷⁾. The calculated muon charge ratios range from 1.26—1.54. The experimental data on the other hand changes from 1.26 to 1.33 at mean muon energy changes from a few GeV to about 3 TeV.

In the present investigation we have fitted the $pp \rightarrow K^\pm X$ inclusive reaction ISR data of Capiluppi et al.⁸⁾ by means of the new scaling variable model⁹⁾ in a similar process as done in a recent work¹⁰⁾ for $pp \rightarrow \pi^\pm X$ inclusive reactions. The fitting parameters have been used to derive the charge ratio of sea level muons and the calculated result has been compared with the measured data of Parker et al.¹¹⁾, Burnett et al.¹²⁾, Muraki et al.¹³⁾ and Ashley II et al.¹⁴⁾.

2. Theoretical aspects

The CERN Intersecting Storage Ring data of Capiluppi et al.⁸⁾ on the inclusive reactions $p p \rightarrow K^+ X$ and $p p \rightarrow K^- X$, respectively and the results have been plotted as functions of the scaling variable

$$\eta = 2 p_T^2 x / [c' \ln (s/s_0)] \quad (1)$$

where $x = 2 p_L / \sqrt{s}$, p_L is the centre of mass longitudinal momentum of the detected particle and \sqrt{s} is the total centre of mass energy; p_T is the centre of mass trans-

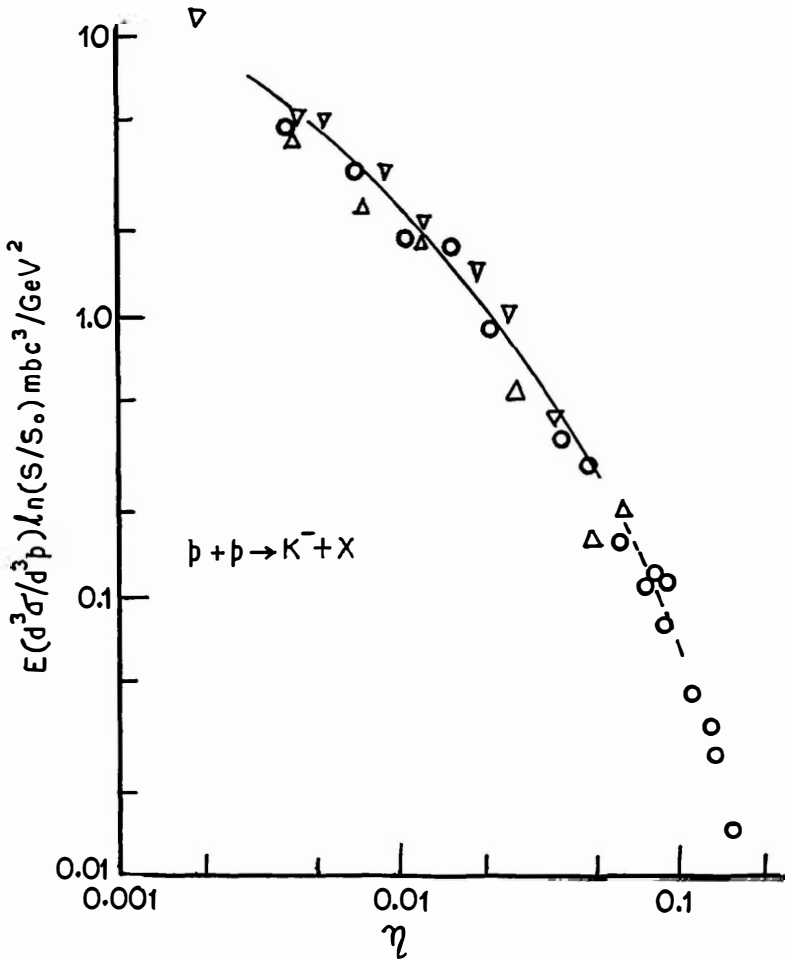


Fig. 1. The invariant cross section data of Capiluppi et al.⁸⁾ for $pp \rightarrow K^+ X$ inclusive reaction, $E(d^3 \sigma / d^3 p) \ln(s/s_0)$ vs η have been plotted. Full curve is the fit to the data from the relation (2) for $A = 24$, $a = 157.2$ and $b = 1800$. ISR data: \blacktriangle - $\sqrt{s} = 30.6$ GeV; \bullet - $\sqrt{s} = 44.6$ GeV and \blacktriangledown - $\sqrt{s} = 53$ GeV.

verse momentum and $s_0 = 1 \text{ (GeV)}^2$, $c' = 1 \text{ (GeV/c)}^2$ for $0.075 < x < 0.3$ and $0.2 < p_T < 1.5 \text{ GeV/c}$. The parametrization of the invariant cross section used in the present work is of the form

$$E(d^3\sigma/d^3p) \ln(s/s_0) = A \exp(-a\eta + b\eta^2). \quad (2)$$

The fit in Fig. 1 for positive kaons corresponds to $A = 24$, $a = 157.2$, $b = 1800$ and the fit to Fig. 2 corresponds to $A = 10.3$, $a = 150$ and $b = 1500$.

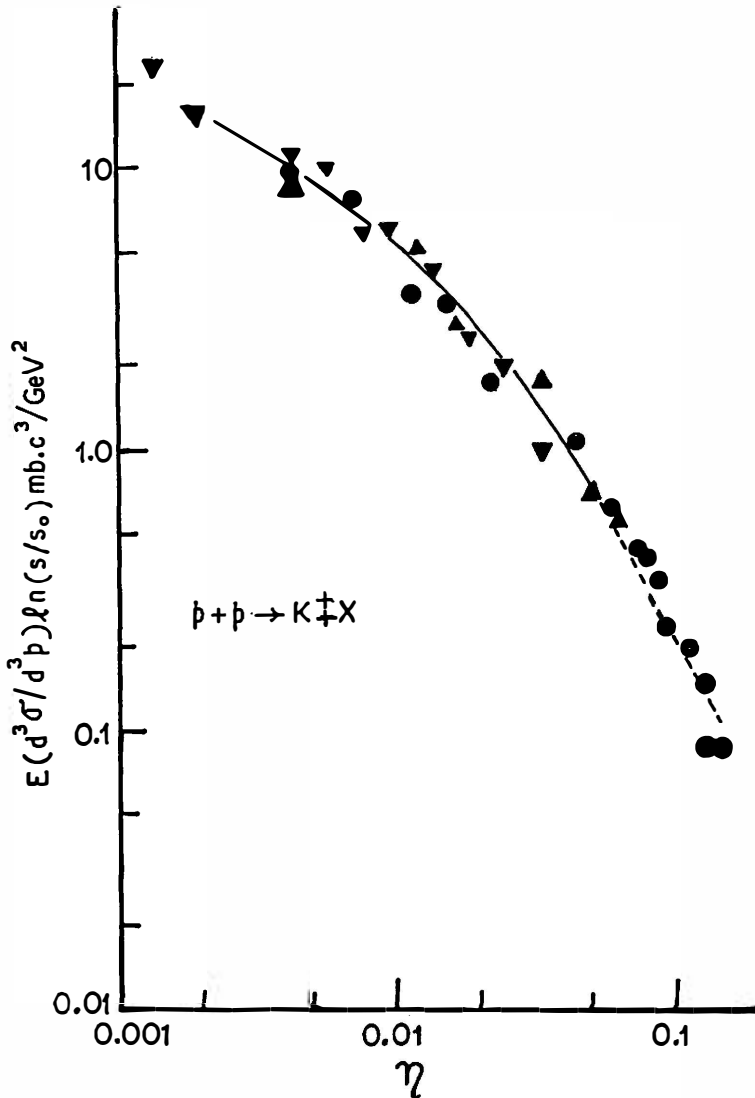


Fig. 2. The invariant cross section data of Capiluppi et al.⁸⁾ for $pp \rightarrow K^- X$ inclusive reactions. $E(d^3\sigma/d^3p) \ln(s/s_0)$ vs η have been plotted. Full curve is the fit from the relation (2) for $A = 10.3$, $a = 150$ and $b = 1500$. ISR data: $\Delta - \sqrt{s} = 30.6 \text{ GeV}$; $\circ - \sqrt{s} = 44.6 \text{ GeV}$ and $\triangle - \sqrt{s} = 53 \text{ GeV}$.

The pion-nucleon or kaon-nucleon flux ratio at the top of the atmosphere can be estimated by the following relation after Ref. 10

$$\frac{i(E_i) dE_i}{N(E_i) dE_i} = \frac{\pi A I(\beta)}{2(\gamma - 2)\sigma_{in}} \quad (3)$$

where

$$I(\beta) = \frac{\exp(-a^2/4b)}{b^{\frac{1}{2}}} [\exp(a^2/4b) D(a/2b^{\frac{1}{2}}) + \exp\{b(\beta - a/2b)^2\} D\{b(\beta - a/2b)\}].$$

$D(x)$ is the Dawson's integral; $\sigma_{in} = 35$ mb; $N(E) dE$ is the differential flux of the primary proton spectrum taken after Ryan et al.¹⁵⁾ where the spectral amplitude is $N_0 = 2$ and the spectral index is $\gamma + 1 = 2.75$; $i = \pi$ or k .

3. Results and discussions

Using the Goddard primary proton spectrum¹⁵⁾ of the following form

$$N(E_p) dE_p = 2 E_p^{-2.75} dE_p \quad (4)$$

where E_p is the proton energy expressed in GeV and $N(E_p) dE_p$ is the intensity expressed in $(\text{cm}^2 \text{ sec sr GeV})^{-1}$, the kaon-nucleon flux ratios have been estimated and the results have been presented in the Table 1 along with the pion-nucleon flux ratios found from our previous work¹⁰⁾.

TABLE 1

Inclusive reactions	Fractional moments	Calculated values	Reference
$p + p \rightarrow \pi^+ + X$		0.083	Bhattacharyya et al. ¹⁰⁾
$p + p \rightarrow \pi^- + X$		0.055	
$p + p \rightarrow K^+ + X$		0.0128	Present work
$p + p \rightarrow K^- + X$		0.0096	

Fractional energy moments for $pp \rightarrow \pi^\pm X$ and $pp \rightarrow K^\pm X$ inclusive reactions

On assuming that pions and kaons retain the same direction as the parent particles and taking into account pion fractional loss and production through nucleon-nucleon and nucleon-pion interaction as well as pion decay, the usual diffusion equation for kaon and atmospheric cases⁷⁾ at high energies, the muon charge ratio can be estimated by the following expression

$$\frac{\mu^+}{\mu^-} = \frac{\left[\frac{N_0}{\Delta_0} (Z_{p\pi^+} + Z_{p\pi^-}) L_\pi + (Z_{p\pi^+} - Z_{p\pi^-}) L'_\pi + \frac{b_{k\mu} B_k}{B_\pi} \left\{ \frac{1 - (m_\mu/m_k)^2}{1 - (m_\mu/m_\pi)^2} \right\}^{\gamma+1} \right.}{\left[\frac{N_0}{\Delta_0} (Z_{pK^+} + Z_{pK^-}) L_k + (Z_{pK^+} - Z_{pK^-}) L'_k \right]} \cdot \frac{\left[\frac{N_0}{\Delta_0} (Z_{p\pi^+} + Z_{p\pi^-}) L_\pi - (Z_{p\pi^+} - Z_{p\pi^-}) L'_\pi + \frac{b_{k\mu} B_k}{B_\pi} \left\{ \frac{1 - (m_\mu/m_k)^2}{1 - (m_\mu/m_\pi)^2} \right\}^{\gamma+1} \right.}{\left[\frac{N_0}{\Delta_0} (Z_{pK^+} - Z_{pK^-}) L_k - (Z_{pK^+} - Z_{pK^-}) L'_k \right]}$$

where $N_0/\Delta_0 = 1.245$, $L_\pi = 162.3 \text{ g} \cdot \text{cm}^{-2}$; $L'_\pi = 124.4 \text{ g} \cdot \text{cm}^{-2}$;
 $L_k = 165.7 \text{ g} \cdot \text{cm}^{-2}$; $L'_k = 127.1 \text{ g} \cdot \text{cm}^{-2}$;
 $b_{k\mu} = 0.64$; $B_\pi = 115 \text{ GeV}$; $B_k = 854 \text{ GeV}$;

m_k , m_π and m_μ are the masses of kaon, pion and muon, respectively. The muon charge ratio μ^+/μ^- has been calculated by the expression (5) and the result is presented in the Table 2 along with the theoretically calculated results of different authors.

TABLE 2

Authors	Integral primary spectral index (γ)	Calculated muon charge ratio (μ^+/μ^-)
Garaffo et al. ²⁾	1.70	1.38
Yekutieli and Rotter ³⁾	1.70	1.26
Morrison and Elbert ¹⁶⁾	1.70	1.26
Hume et al. ¹⁷⁾	1.60	1.30
Erlykin, Ng, Wolfendale ¹⁸⁾	1.62	1.36
Present work	1.75	1.21

Integral primary spectral index and muon charge ratio derived by different authors

The calculated charge ratio of sea muons has been presented in Fig. 3 along with the experimental data of Parker et al.¹¹⁾, Burnett et al.¹²⁾, Muraki et al.¹³⁾ and Ashley II et al.¹⁴⁾. It is evident from Fig. 3 that the calculated muon charge ratio is in accord with the experimental data of Muraki et al.¹³⁾ and Ashley II et al.¹⁴⁾.

It is also found from the Table 2 that the present result is in accord with the theoretical results of Yekutieli and Rotter³⁾ and Morrison and Elbert¹⁶⁾.

More recently Allkofer et al.¹⁹⁾ have reported in Kyoto Cosmic Ray Conference (1979) the behaviour of the muon charge ratio similar to that of Muraki et al.¹³⁾. Earlier Hayakawa²⁰⁾ has pointed out that when the meson multiplicity in-

increases with energy the positive to negative ratio has to decrease. So the rise of meson charge ratio (μ^+/μ^-) at high energy is not supported by the data of Muraki et al.¹³⁾ and Ashley II et al.¹⁴⁾.

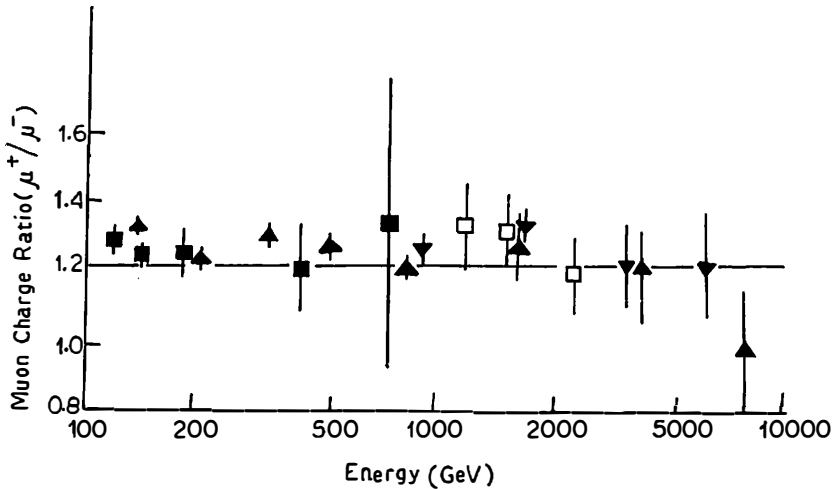


Fig. 3. The charge ratio of muons at different energies: Full line is the derived result from the present model. Experimental data: \triangle — Parker et al.¹¹⁾, \blacksquare — Burnett et al.¹²⁾, \blacktriangle — Muraki et al.¹³⁾ and \blacksquare — Ashley II et al.¹⁴⁾.

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ODNOS NABOJA ZA MIONE IZ KOZMIČKIH ZRAKA IZVEDEN IZ
MODELA NOVE OMJERBENE PROMJENLJIVE

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Procijenjen je odnos naboja miona na morskom nivou iz modela nove omjerbene promjenljive i podataka sa CERN-ovog prosječnog prstena Capilupija i dr. za uključne procese $p\bar{p} \rightarrow \pi^\pm X$ i $p\bar{p} \rightarrow K^\pm X$. Za procijenjeni odnos mionskog naboja dobivena je vrijednost 1.21 i rezultat je uspoređen sa eksperimentalnim podacima Parkera i dr., Burnetta i dr., Ashleya i dr. i Murakija i dr.