

ACCURATE ASYMPTOTICAL TREATMENT OF ONE-ELECTRON PROCESSES IN SLOW COLLISIONS OF ATOMIC PARTICLES WITH SOLID SURFACES

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The energy eigenvalue problem for one electron in a field of an atomic core and a metal is solved. Obtained solutions are asymptotically correct. Two classes of eigenfunctions are derived: atomic function distorted in the metallic region and a metallic function distorted in the atomic region. These functions are used for the resonant ionization and neutralization probability calculations.

1. Introduction

When an atomic particle slowly collides with a solid surface, an electron exchange between metal and atom, as subsystems, occurs. The most probable are resonant processes: ionization of the atomic particle and positive ion neutralization in the vicinity of the metal surface, under the condition that they are energetically possible. Abreast with one-electron processes of these types, the Auger processes such as deexcitation of metastable atom or positive ion neutralisation also take place. These processes involve simultaneous transition of two electrons whereby one of them is ejected into the continuum. Due to their resonant nature, all one electron processes take place at relatively large distances between subsystems, and have high probabilities.

In the previous treatment of one-electron processes^{1,2)} the resonant ionization of an atomic particle was correctly described, and the probability for inverse process (resonant neutralization) was calculated by using the detailed balance principle. In this work, the latter probability is calculated in a direct manner.

When treating simultaneous two-electron transitions the most important is an equivalent description of the electron transition from the atom into the metal and the transition from the metal into the atom. The correct treatment of these two processes must include the distortion of the electronic motion in the sub-barrier region, where the electron interaction with both atomic core and image potentials is strong.

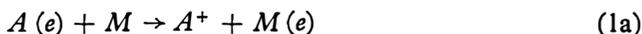
In the present paper we shall derive an asymptotically correct solution of the energy eigenvalue problem. Such a solution belongs to the continuous spectrum. Therefore it has to be normalized in terms of a Dirac δ -function. Two classes of the above mentioned solutions will be obtained. The first one appears in the description of the transition from the atom into the metal and coincides with solutions obtained earlier¹⁾, while the second class is responsible for the description of inverse processes and will be evaluated in the present paper for the first time.

In Section 2. we formulate the problem. In the Section 3., using the asymptotical JWKB metod³⁾, we find solutions for the above problem. In Section 4. we use these solutions to calculate the resonant ionization and neutralization probabilities. In Section 5. we give some concluding remarks.

Atomic units ($m_e = \hbar = e^2 = 1$) will be used throughout this paper.

2. Formulation of the problem

We consider the processes



of resonant ionization (1a) and neutralization (1b). A and A^+ designate the atomic particle and positive ion respectively, M signifies the metal and e is the electron taking part in the process. Let the origin of the coordinate system for our problem be located at the centre of the atomic core (with charge Z), and the position of the metallic surface be defined by $z = -R$, where R is the atom - surface distance. We describe the metal by Sommerfeld model (the potential well having depth U_0), and the atom — surface interaction by electrostatic image potentials. The Hamiltonian is, thus, given by

$$H = \begin{cases} -\frac{1}{2} \nabla^2 - U_0, & z < -R \\ -\frac{1}{2} \nabla^2 - \frac{Z}{r} + V_e(e) + V_e(C), & z > -R + \delta R \end{cases} \quad (2)$$

where δR is a linear dimension of the narrow region ($\delta R \sim 1-2$) around the surface in which the image potential approximation is not valid. $V_e(e)$ and $V_e(c)$ are the interactions of the electron e with its proper image, as well as with the atomic core image, given respectively by

$$V_e(e) = -\frac{1}{4}(z + R)^{-1} \tag{3a}$$

$$V_e(c) = Z[x^2 + y^2 + (z + 2R)^2]^{-1/2}. \tag{3b}$$

When solving the eigenvalue problem of the Hamiltonian (2), one should keep in mind that the dominant contribution to the electron transfer process gives a narrow cylindrical region around the z -axis¹¹. In this region (the barrier region B), as well as in the metallic region M and atomic region A (determined as a region where the Coulomb potential has dominant influence), the variables in the Schrodinger equation can be separated in the parabolic coordinates: $\xi = r + z$ ($0 < \xi < \infty$), ($\eta = r - z$ ($0 < \eta < \infty$), $\varphi = \arctan(y/x)$ ($0 < \varphi < 2\pi$).

Because of the z -axial symmetry of the potential, there exists a solution of the eigenvalue problem

$$H(R) \Psi(R, \vec{r}) = -\frac{\gamma^2}{2} \Psi(R, \vec{r}) \tag{4}$$

with a defined value m of the angular momentum projection on the z -axis. Such a solution can be put in the form

$$\Psi = e^{im\varphi} \frac{1}{\sqrt{\xi\eta}} X(\xi)Y(\eta). \tag{5}$$

By separation of the variables in the eigenvalue problem (4) we obtain two one-dimensional problems:

$$\frac{d^2 X}{d\xi^2} + \left(\frac{1 - m^2}{4\xi^2} - \frac{\gamma^2}{4} - \frac{\mathcal{L}}{\xi} \right) X = 0 \tag{6}$$

and

$$\frac{d^2 Y}{d\eta^2} + \left(\frac{1 - m^2}{4\eta^2} - \frac{\gamma^2}{4} + \frac{\mathcal{L}}{\eta} + \right. \tag{7}$$

$$\left. \begin{cases} \frac{U_0}{2}, \eta > 2R \\ \frac{1}{4(2R - \eta)} - \frac{Z}{4R - \eta} + \frac{Z}{\eta}, 2(R - \delta R) < \eta < 2a \\ \frac{Z}{\eta}, \eta < 2a \end{cases} \right) Y = 0$$

where a is a linear dimension of the atomic region A . \mathcal{L} is the separation constant. The potential omitted in (7) can be treated as perturbation because it is negligible with respect to the remaining one. The corrected values of γ then will be dependent on R .

The boundary conditions for the eigenvalue problem (6) originate from the condition that the corresponding Hamiltonian must be of the Sturm — Liouville type. They are: $X(0) = X(\infty) = 0$. The application of these conditions gives

$$X = M_{-\frac{\mathcal{L}}{\gamma}, \frac{|m|}{2}}(\gamma \xi) \tag{8}$$

$$-\frac{\mathcal{L}}{\gamma} = n_1 + \frac{|m| + 1}{2}, n_1 = 0, 1, 2, \dots \tag{9}$$

The solutions of the equation (7) can be easily obtained for $\eta < 2a$ and $\eta > 2R$. For $\eta < 2a$, the solutions regular in origin are

$$Y = K M_{\frac{z+\mathcal{L}}{\gamma}, \frac{|m|}{2}}(\gamma \eta), \eta < 2a \tag{10a}$$

while in the metallic region, $\eta > 2R$

$$Y = K^{(1)} W_{\frac{z+\mathcal{L}}{\mathcal{H}}, \frac{|m|}{2}}(i \mathcal{H} \eta) + K^{(2)} W_{-\frac{z+\mathcal{L}}{i \mathcal{H}}, \frac{|m|}{2}}(-i \mathcal{H} \eta). \tag{10}$$

In the formulas (10 a, b) $M_{\lambda\mu}$ and $W_{\lambda\mu}$ are Whittaker functions, and $K, K^{(1)}$ and $K^{(2)}$ are arbitrary constants.

The notation

$$\mathcal{H}^2 = 2U_0 - \gamma^2 \tag{11}$$

was introduced.

3. JWKB solutions of the differential equation in η

As it was mentioned above, the transition processes take place at the distance R between metal and core, such that $R \gg 1$. Under the condition $\gamma R \gg 1$, the solutions (10a) and (10b) can be connected through the barrier region by the use of the asymptotical JWKB method. The equation (7) can be put in the form

$$\frac{d^2 Y}{d\eta^2} + p^2(\eta) Y = 0. \tag{12}$$

By definition, the turning point η_p is a zero of the function $p^2(\eta)$. It is the point of the energy level $\gamma^2/2$ intersection with a corresponding η -potential: $-p^2(\eta)/2 + \gamma^2/2$. Close to the metal η_p is

$$\eta_p = 2R \left[1 - \frac{1}{2\gamma^2 R} + 0 \left(\frac{1}{R^2} \right) \right]. \tag{13}$$

The function $p^2(\eta)$ monotonically increases in the interval $\eta \in [2a, 2R - 2\delta R] + [2R, \eta^*]$. η^* is a point inside the metal which is sufficiently far from η_p . If we suppose the same monotonic nature of the function $p^2(\eta)$ in the vicinity of the metallic surface: $\eta \in (2R - 2\delta R, 2R]$, we can obtain the relations between the constants K and $K^{(1)}, K^{(2)}$ in (10 a, b) by using the JWKB connection formulae. So, we get the solutions of the eigenvalue problem in the region of physical interest.

The connection formulae are³⁾

$$Y(2a) = C |p|^{-1/2} \exp |w| + D |p|^{-1/2} \exp [-|w|] \tag{14}$$

$$\text{(if } |C \exp |w| \lesssim D \exp [-|w|])$$

hence

$$Y(\eta^*) = 2D |p|^{-1/2} \cos \left(|w| - \frac{\pi}{4} \right)$$

and

$$Y(\eta^*) = a_1 |p|^{-1/2} \exp \left[i |w| + \frac{i\pi}{4} \right] + a_2 |p|^{-1/2} \exp \left[-i |w| - \frac{i\pi}{4} \right] \tag{15}$$

$$\left(\text{if } \frac{a_1 + a_2}{|a_1| + |a_2|} \approx 0 \right)$$

hence

$$Y(\eta_a) = (a_1 + a_2) |p|^{-1/2} \exp |w|$$

where

$$w(\eta) = \int_{\eta_p}^{\eta} p(\eta) d\eta. \tag{16}$$

The integral $w(\eta)$, (16), can be calculated approximately. It is

$$\exp |w(\eta)| = \eta^{\frac{Z+\mathcal{L}}{\gamma}} \exp \left(-\frac{1}{2} \gamma \eta \right) \Phi(R), \quad \eta \sim \eta_a \tag{17}$$

where

$$\Phi(R) = \gamma^{-\frac{1}{2\gamma}} (2e)^{-\frac{1}{4\gamma}} 2^{-\frac{1}{2\gamma}} \frac{\mathcal{L}}{\gamma} R^{-\frac{Z-\mathcal{L}}{\gamma}} - \frac{1}{4\gamma} e^{\gamma R} \tag{18}$$

and

$$\exp(i|w(\eta)|) = e^{i\Delta} (\mathcal{H}\eta)^{\frac{i\mathcal{L}}{\mathcal{H}}} \exp\left(\frac{i\mathcal{H}\eta}{2}\right), \quad \eta \sim \eta^* \tag{19}$$

where Δ is a real value which depends on the potential in the narrow region around the metal surface.

From the formulae (17) and (19) the expressions for $Y(2a)$ and $Y(\eta^*)$ in (14) can be easily obtained. These solutions should be matched with the asymptotical forms of the solutions (10a) and (10b). Using the connecting formulae (14) and (15), the needed relations between the constants K and $K^{(1)}, K^{(2)}$ can be obtained. The asymptotical form of the solutions (10a) essentially depend on the value $(Z + \mathcal{L})/\gamma$. For the $(Z + \mathcal{L})/\gamma = n_2 + (|m| + 1)/2$, $n_2 = 0, 1, \dots$, what is, taking into account (9), the case of atomic levels

$$\gamma = \frac{Z}{n}, \quad n = 1, 2, \dots; \quad n_1 = 0, 1, \dots, n-1; \quad |m| = 0, 1, \dots, n - n_1 - 1, \tag{20}$$

the solution Y exponentially decreases with increasing η , while in the remaining cases it increases exponentially.

In the case $\gamma = Z/n$, the function Y in the atomic region coincides with an atomic function. In the case when solution is an »outgoing wave« in the metal ($K^{(1)} = 0$), the relation between the constants $K^{(2)}$ and K can be obtained. If the constant $K^{(1)}$ and $K^{(2)}$ are both nonzero, the JWKB method yields only one relation between three constants: $K, K^{(1)}$ and $K^{(2)}$, and the corresponding solution remains undetermined. In the case $\gamma \neq Z/n$, Y function in the metallic region is a sum of incoming and outgoing solution. In this case the connection formula (14) gives the relation between $K^{(2)}$ and K , as well as a relation between $K^{(1)}$ and $K^{(2)}$.

The general form of the eigenvalue solution (15) is

$$\Psi_{\gamma n, m} = \frac{e^{im\varphi}}{\sqrt{\xi}\eta} M_{n_1 + \frac{|m|+1}{2}, \frac{|m|}{2}, |m|}(\gamma\xi) \times \tag{21}$$

$$\times \begin{cases} K M_{\lambda, \frac{|m|}{2}}(\gamma\eta), & A \\ K^{(1)} W_{\frac{\mathcal{L}}{i\mathcal{H}}, \frac{|m|}{2}}(i\mathcal{H}\eta) + K^{(2)} W_{-\frac{\mathcal{L}}{i\mathcal{H}}, \frac{|m|}{2}}(-i\mathcal{H}\eta), & M \end{cases}$$

where $\lambda = (Z + \mathcal{L})/\gamma$. Because the relations between the constants K and $K^{(1)}, K^{(2)}$ are different in the case $\gamma = Z/n$ from the case $\gamma \neq Z/n$, the two classes of functions Φ are obtained: Ψ^I and Ψ^{II} . These functions belong to the rigged Hilbert space, and, as they depend on indices γ, n_1, m , they can be normalized, uniquely, by the condition

$$\langle \Psi_{\gamma n, m} | \Psi_{\gamma' n', m'} \rangle = \delta(\gamma - \gamma') \delta_{n_1, n'_1} \delta_{m, m'}. \tag{22}$$

The normalization condition (22) is equivalent to the normalization condition for the corresponding eigen differential in γ (normalized to unity). So, under the condition $R \gg 1$, the normalization condition for the function Ψ^I is approximately

$$1 = d\gamma \left\| K \frac{e^{im\varphi}}{\sqrt{\xi \eta}} M_{n_1 + \frac{|m|+1}{2}, \frac{|m|}{2}}(\gamma \xi) M_{\lambda, \frac{|m|}{2}}(\gamma \eta) \right\|^2, \quad (23)$$

while for the function Ψ^{II} the following approximate expression holds

$$1 = \left\| \frac{e^{im\varphi}}{\sqrt{\xi \eta}} M_{n_1 + \frac{|m|+1}{2}, \frac{|m|}{2}}(\gamma \xi) \times \right. \\ \left. \times \frac{1}{\sqrt{d}\gamma} \int_{\gamma}^{\gamma+d\gamma} [K^{(1)} W_{\frac{\mathcal{L}}{i\mathcal{L}}, \frac{|m|}{2}}(i\mathcal{L}\eta) + K^{(2)} W_{-\frac{\mathcal{L}}{i\mathcal{L}}, \frac{|m|}{2}}(-i\mathcal{L}\eta)] d\gamma \right\|_M^2. \quad (24)$$

The integration in norm (24) is carried out over the metallic region M.

On the base of the normalization conditions (23) and (24), in the case $\gamma = Z/n, b_1 + n_2 + |m| + 1 = n$, one obtains the following expressions for the constants $K, K^{(1)}$ and $K^{(2)}$:

$$K_{n_1, n_2, m} = \frac{1}{\sqrt{d}\gamma} \frac{\gamma}{Z\pi} \left[\frac{(|m| + n_1)! (|m| + n_2)!}{(|m|!)^4 n_1! n_2!} \right]^{1/2} \quad (22a)$$

$$K^{(1)} = 0 \quad (22b)$$

$$K_{n_1, n_2, m}^{(2)} = \frac{1}{\sqrt{d}\gamma} e^{-\frac{\pi \mathcal{L}}{2\mathcal{L}}} e^{i\left(\lambda - 1 + \frac{\pi}{4} - \frac{|m|+1}{2}\right)} \times \\ \times \sqrt{\frac{\gamma}{\pi} \frac{\gamma}{\pi Z}} \left[\frac{(|m| + n_1)!}{(|m| + n_2)! n_1! n_2! (|m|!)^2} \right]^{1/2} \times \\ \times \gamma^{\lambda + \frac{1}{2\gamma}} (2e)^{\frac{1}{4\gamma} 2^{\frac{1}{2\gamma} - n_1 - \frac{|m|+1}{2}}} R^{\lambda + \frac{1}{4\gamma}} e^{-\gamma R}. \quad (22c)$$

In the case $\gamma \neq Z/n$, the corresponding constants are

$$K_{\gamma, n_1, m} = e^{i\lambda - \frac{i\pi}{4}} \frac{1}{\pi} \left[\frac{(n_1 + |m|)!}{2 n_1!} \right]^{1/2} \frac{\Gamma\left(-\lambda + \frac{|m|+1}{2}\right)}{(|m|!)^2} \times \\ \gamma^{\lambda + \frac{1}{2\gamma}} (2e)^{\frac{1}{4\gamma} 2^{\frac{1}{2\gamma} - n_1 - \frac{|m|+1}{2}}} R^{\lambda + \frac{1}{4\gamma}} e^{-\gamma R} \quad (23a)$$

$$K_{\gamma n_1 m}^{(1)} = \sqrt{\frac{\gamma}{\pi}} \frac{1}{\pi} \left[\frac{(n_1 + |m|)!}{2 n_1 (|m|!)^2} \right]^{1/2} e^{-\frac{\pi \mathcal{E}}{2 \mathcal{H}}} \quad (23b)$$

$$K_{\gamma n_1 m}^{(2)} = e^{2i \Delta - \frac{i\pi}{2}} K_{\gamma n_1 m}^{(1)}. \quad (23c)$$

We obtain the solutions Ψ^I and Ψ^{II} by substituting (22), that is (23) into (21).

Dependence on R (factor $R^{\lambda + \frac{1}{4\gamma}} \exp(-\gamma R)$) appears in the distorted tail of the atomic function Ψ^I in the metal, as well as in the distorted tail of the metallic function Ψ^{II} in the atom region.

4. Resonant ionization and neutralization probability per unit time

The resonant neutralization (1a) at fixed distance between metal and atom is electron transition from the metal into the region where the Coulomb field has dominant influence; while the resonant ionization process (1b) is the inverse one. If the atomic motion is sufficiently slow, the state of the system varies adiabatically and, therefore, the probability per unit time of the above mentioned process is given by

$$W = -d \gamma \int_{S_{FIN}} \vec{j} \cdot d\vec{S} \quad (24)$$

where

$$\vec{j} = \text{Im}(\Psi^* \nabla \Psi) \quad (25)$$

and S_{FIN} is a boundary surface of the final region for the given process. It is the metallic surface in the case of ionization, (1a), and the boundary surface of the atomic region ($z = -a$) in the resonant ionization, (1b), case. The functions Ψ^I and Ψ^{II} stand for $\Psi_{\gamma n_1 m}$ in (25) in cases of ionization and neutralization, respectively. Initial state at fixed distance R belongs to the continuous spectrum, in both cases, and corresponds to the energy interval $(\gamma, \gamma + d\gamma)$.

In the resonant ionization case, \vec{j} has to be evaluated at the metallic surface by using function (21) with conditions (22b) and (22c). In this case, using the asymptotical form of the above mentioned function,

$$\vec{j} \cdot d\vec{S} = -\frac{2\eta}{\xi + \eta} \cdot \frac{\mathcal{H}}{2} |K_{n_1 n_2 m}^{(2)} e^{\frac{\pi \mathcal{E}}{2 \mathcal{H}}}|^2 \frac{1}{\xi \eta} |M_{n_1 + \frac{|m|}{2} + 1, \frac{|m|}{2}}(\gamma \xi)|^2. \quad (26)$$

The final result can be obtained by the proper integration of (24). The resonant ionization probability per unit time thus becomes

$$W_{n_1 n_2 m}^I = \frac{\gamma^3 2^{-2n_1 - |m| - 1 + \frac{1}{\gamma}}}{Z (n_2 + |m|)! n_2!} \gamma^{2\lambda + \frac{1}{\gamma}} (2e)^{\frac{1}{2\gamma}} R^{2\lambda + \frac{1}{2\gamma}} e^{-2\gamma R} \quad (27)$$

where

$$\lambda = \frac{Z}{\gamma} - n_1 - \frac{|m| + 1}{2} = n_2 + \frac{|m| + 1}{2}, \gamma = \frac{Z}{n}. \tag{27a}$$

The expression (27) can be corrected using the fact that the energy $-\gamma^2/2$ is perturbed because of the image force influence in the atomic region. The perturbed γ can be written as

$$\gamma = \frac{Z}{n} - \frac{2Z - 1}{4ZR} n. \tag{28}$$

This correction is significant only in the exponent in (27). The corrected W^I value coincide with the expression of Grozdanov, Janev²⁾ up to the factor 1/2.

In the resonant neutralization case, \vec{j} at the surface $z = -a$, again can be found from the asymptotical form of $\Psi_{\gamma n, m}$, (21), with K given by expression (23a). Thus we obtain

$$\begin{aligned} \vec{j} \cdot \frac{d\vec{S}}{dS} = & -\frac{2\eta}{\xi + \eta} \frac{\gamma}{2} |K_{\gamma n, m}|^2 \frac{1}{\xi\eta} |M_{n_1 + \frac{|m| + 1}{2}, \frac{|m|}{2}}(\gamma\xi)|^2 \times \\ & \times \frac{2(|m|!)^2 \sin\left(\pi\lambda - \pi\frac{|m| + 1}{2}\right)}{\Gamma\left(-\lambda + \frac{|m| + 1}{2}\right) \Gamma\left(\lambda + \frac{|m| + 1}{2}\right)}. \end{aligned} \tag{29}$$

Carrying out the integration (24), we obtain the final result for the resonant neutralization probability:

$$\begin{aligned} W_{\gamma n, m}^N = & \gamma d\gamma \frac{2^{-2n_1 - |m| - 1 + \frac{1}{\gamma}}}{\Gamma\left(\frac{Z}{\gamma} - n_1\right) \Gamma\left(\frac{Z}{\gamma} - n_1 - |m|\right)} \gamma^{2\lambda + \frac{1}{\gamma}} (2e)^{\frac{1}{2\gamma}} \times \\ & \times R^{2\lambda + \frac{1}{2\gamma}} e^{-2\gamma R}. \end{aligned} \tag{30}$$

In the case $\gamma \rightarrow Z/n$, the expression in denominator tends to $(n_2 + m)! n_2!$, so that the expression (30) tends to the resonant ionization probability (27), multiplied by $2d\gamma/\gamma^2$.

In order to calculate the total probability per unit time, we integrate (3), with statistical weight ϱ_M , over all possible values of γ from the filled metallic band. Due to the resonant nature of the process, we obtain merely expression (30) multiplied by ϱ_M . ($d\gamma$ automatically disappearing). The value γ figuring in (30) is the resonant value Z/n .

5. Discussion and concluding remarks

As it is well known, the formula (27) gives, in comparison with the corresponding perturbational expression⁴⁾, the correct preexponential factor. The nonperturbational approach to the problem was given by Chaplik⁵⁾, for the case $Z = 1$ and $m = 0$, as well as by Janev¹⁾ in general case. In these papers the state function has not been matched with the metallic one (10b). In the present paper it is shown that, the same result for the probability W^I can be obtained with the corrected function, because of the asymptotical nature of the calculations. As it was mentioned before, the probability W^I can be compared in the most direct manner with the paper of Grozdanov, Janev²⁾. In this paper the imaginary part of the complex energy has been found by the etalon equation method. Let us note that, for $Z = 0$, the same R dependence of the ionization probability was obtained⁶⁾. The transition probability W^N is calculated in direct manner for the first time here.

The obtained probabilities depend on parabolic quantum numbers n_1 , n_2 and m (i. e. γ , n_1 and m). The commonly used dependence on γ , l and m can be obtained by the proper summation of the expression (27) or (30). Assuming the passage of the atomic particle near the surface being adiabatic the probability (per passage) P is calculated using the expression

$$P = 1 - \exp \left[- \frac{2}{v_0} \int_{R_c}^{\infty} W(R) dR \right]$$

where v_0 is the velocity component of the particle, perpendicular to the surface, and R_c is some critical distance such that for $R < R_c$ the process in question becomes ineffective¹⁾.

Finally, we would like to note that two classes of functions (Ψ^I and Ψ^{II}), obtained in the present paper, inspite they enable us to calculate W^I and W^N directly, they are the basic element in the description of simultaneous two-electron transitions. So they can be used in the cases of: resonant deexcitation, Auger deexcitation and neutralization, e. t. c.

The main purpose of the present work is calculation of one electron ionization and neutralization probabilities during the slow collisions of atomic particles with solid surfaces. Therefore we did not discuss at all the connection of the results of this work with the problems of adsorption or desorption of atomic particles on surfaces. These problems are discussed in details elsewhere^{7), 8)}.

References

- 1) R. K. Janev, J. Phys. B: Atom. Mol. Phys. **7** (1974) 1650;
- 2) T. P. Grozdanov and R. K. Janev, J. Phys. B: Atom. Mol. Phys. **10** (1977) 1385;
- 3) N. Fröman and P. O. Fröman, *WKB Approximation*, North Hol and P. C., Amsterdam 1965;
- 4) J. W. Gadzuk, Surface Sci. **6** (1967) 133;
- 5) A. V. Chaplik, Zh. Eksp. Teor. Fiz. **54** (1968) 332 (Sov. Phys. — JETP **27** 178);
- 6) M. E. Kishinevskii, Zh. Tech. Fiz. **48** (1978) 773;
- 7) B. Lundqvist et al., Phys. Rev. Lett. **37** (1976) 292;
- 8) B. Gumhalter and V. Zlatić, J. Phys. C **13** (1980) 1679.

TAČAN ASIMPTOTSKI TRETMAN JEDNOELEKTRONSKIH PROCESA
PRI SPORIM SUDARIMA ATOMSKIH ČESTICA SA POVRŠINOM ČVRSTOG
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Rešen je svojstveni problem energije jednog elektrona u polju atomskog ostatka i metala. Dobijena rešenja su asimptotski korektna. Nađene su dve klase svojstvenih funkcija: atomska funkcija perturbovana u oblasti metala i funkcija metala perturbovana u atomskoj oblasti. Ove funkcije su iskorišćene za izračunavanje verovatnoća rezonantne jonizacije i neutralizacije.