

GROUND STATE PROPERTIES OF SPIN-POLARIZED DILUTED ATOMIC HYDROGEN

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Employing the Ursell-Mayer procedure and the semi-free gas model analytic relations describing ground-state properties of spin-polarized hydrogen were derived. The obtained results are valid up to densities of 10^{26} atoms/m³. This region is somewhat larger for radial distribution function and for one-particle density matrix. It was found out that at the density of 2×10^{26} atoms/m³ the condensation of particles is 90% (at absolute zero).

1. Introduction

When studying the properties of helium which remains liquid, even at absolute zero, two facts have been emphasized: 1. the light atomic mass of helium and 2. the relatively small minimum of the potential well of interaction between two helium atoms. The fact that hydrogen, although it has a lighter mass, is not in the liquid state at the absolute zero, can be explained by a strong interaction between two hydrogen atoms with antiparallel electronic spins (Σ_g^+ -state). So before the occurrence of low temperatures the molecules of the hydrogen isotopes H₂, D₂ and T₂ are formed (hydrogen H, deuterium D and tritium T). Consequently, the second condition is not satisfied for hydrogen.

Kolos and Wolniewicz¹⁾ have also computed the interaction between two hydrogen atoms with parallel spins (Σ_u^+ -state), which are denoted by H \uparrow . The

presentation of this potential is given in Ref. 2. The depth of the potential well for $H \uparrow$ is twice smaller than for helium. Since the masses of hydrogen isotope atoms are considerably lighter than the mass of He atom, the quantum effects in $H \uparrow$, $D \uparrow$ and $T \uparrow$ should be more significant than in helium.

According to the assumption that the atoms of spin-polarized hydrogen $H \uparrow$ represent a set of bose particles³⁾ it was established^{2,4-10)} that such a system cannot form a liquid state, i. e. that such a system remains in a gaseous state even at absolute zero. Weak forces, low density and its gaseous state make this system an almost ideal theoretical model. The particles density of the ideal Bose-Einstein gas, ρ , and the temperature of the phase transition, T , at which the condensation begins in the momentum space, are expressed by the relation¹¹⁾

$$\rho = 2,612 \cdot (2\pi m k T)^{3/2} / h^3.$$

For the density of 4×10^{25} atoms/m³ it follows that the temperature of the phase transition in the system $H \uparrow$ is 0.21 K. The distance between the atoms for the above parameters is approximately 29×10^{-10} m. This appears to be a very diluted system if we compare it with superfluid helium whose density is 2.19×10^{28} atoms/m³ and the interatomic distance is cca 4×10^{-10} m.

Experimental studies of the spin-polarized hydrogen near absolute zero, where its quantum properties appear, is still in its initial phase. Because of the great instability with regard to the recombination $2H \rightarrow H_2 + Q$, where $Q = 7.209 \times 10^{-19}$ J (4.5 eV), it is very difficult to produce atomic hydrogen. Silvera and Walraven^{10,12)} produced atomic hydrogen at the temperature of 0.3 K in magnetic field up to 10 T. They found out there had not been any change in density in the time period of cca 9 minutes. The density they obtained was at least 10^{22} atoms/m³. W. Pescka and coworkers¹⁰⁾ state that it is now possible to stabilize 2 mg of atomic hydrogen in a half-litre, which corresponds to a density of 1.2×10^{27} atoms/m³.

The theories established so far in the study of physical properties of this system, are based on the numerical computations (Monte Carlo) and techniques previously employed in the study of boson systems. Because of the physical properties of this system (weak interaction potential, gaseous state at a relatively low density) it is possible to obtain simple analytical expressions of the physical quantities in the ground state. In this paper we defined, employing the Ursell-Mayer procedure¹³⁻¹⁶⁾, the energy of the ground state, pressure, compressibility, sound velocity, radial distribution function and one-particle density matrix as the density function. The expressions are valid for the densities a hundred times lower than the density of the liquid helium (Section 2). The results of the calculations and the discussion are presented in Section 3. It has also been shown that the ground state of tritium $T \uparrow$ is liquid.

2. Physical quantities of the ground state

For the boson system of N identical particles confined to a volume Ω , ($\Omega \rightarrow \infty$, $N \rightarrow \infty$) with Hamiltonian

$$H = -\frac{\hbar^2}{2m} \sum_i \Delta_i + \frac{1}{2} \sum_{i,j} V_{ij} \quad (1)$$

the wave function in a two particle approximation is

$$\Psi(1,2,\dots,N) = \prod_{i<j}^N f_{ij}, \quad (2)$$

where f_{ij} is the distance function between two particles

$$f_{ij} = e^{\frac{1}{2} u_{ij}} \quad (3)$$

$$u_{ij} = - \left(\frac{\alpha}{r_{ij}} \right)^\beta \quad (4)$$

for the Lennard-Jones potential 12—6; α and β are variational parameters. Otherwise this form of the wave function is a generalized solution of the two-particle problem at short inter-particle distances¹⁰⁾. Interaction between H \uparrow atoms in the region of short interatomic distances can be analytically expressed by the Lennard-Jones potential 12—6⁶⁾

$$V(r) = 4 \varepsilon \left[\left(\frac{\sigma}{r} \right)^{12} - \left(\frac{\sigma}{r} \right)^6 \right], \text{ for } r \leq 6.87 \times 10^{-10} \text{ m}, \quad (5)$$

whereas for greater distances it is shown that the function

$$V(r) = - (C_6 r^{-6} + C_8 r^{-8} + C_{10} r^{-10}) \quad (6)$$

approximates the real interaction well. Parameters in the potential have the following values: $\varepsilon = 8.92 \times 10^{-23}$ J, $\sigma = 3.69 \times 10^{-10}$ m, $C_6 = 6.22071 \times 10^{-60}$ Jm⁶, $C_8 = 3.334 \times 10^{-74}$ Jm⁸ and $C_{10} = 2.46564 \times 10^{-90}$ Jm¹⁰. There are other forms of interaction potential between two H \uparrow atoms^{3,4)}.

In this paper the initial variational parameters α and β are defined according to the Ljolie relation¹⁶⁾

$$\left(\frac{du}{dr} \right)^2 = 16 \cdot \varepsilon \cdot \frac{m}{\hbar^2} \cdot \left(\frac{\sigma}{r} \right)^{12} \quad (7)$$

and there are

$$\alpha = (4 \sqrt{\varepsilon \cdot m \sigma^6 / 5 \hbar})^{1/5}, \quad \beta = 5. \quad (8)$$

Ground state energy of the boson system for the wave function (2) is

$$\begin{aligned} \frac{E}{N} &= \varrho \Omega^{-1} \int e^{u_{1,2}} F_2(1,2) G_2(1,2) d\vec{r}_1 d\vec{r}_2 + \\ &\varrho^2 \Omega^{-1} \int e^{u_{1,2} + u_{1,3} + u_{2,3}} F_3(1,2,3) G_3(1,2,3) d\vec{r}_1 d\vec{r}_2 d\vec{r}_3, \end{aligned} \quad (9)$$

where $\varrho = N/\Omega$,

$$F_2(1, 2) = -\frac{\hbar^2}{4m} \nabla_1 u_{12} - \frac{\hbar^2}{8m} \nabla_1 u_{12} \nabla_1 u_{12}, \quad (10)$$

$$F_3(1, 2, 3) = -\frac{\hbar^2}{8m} \nabla_1 u_{12} \cdot \nabla_1 u_{13}, \quad (11)$$

$$G_2(1, 2) = \Omega^2 e^{-u_{11}} \int \Psi^2(1, 2, \dots, N) d\vec{r}_3 \dots d\vec{r}_N / I_N, \quad (12)$$

$$G_3(1, 2, 3) = \Omega^2 e^{-u_{11} - u_{12} - u_{13}} \int \Psi^2(1, 2, \dots, N) d\vec{r}_4 \dots d\vec{r}_N / I_N. \quad (13)$$

$$I_N = \int \Psi^2(1, 2, \dots, N) d\vec{r}_1 \dots d\vec{r}_N. \quad (14)$$

Radial distribution function¹⁷⁾

$$g(1, 2) = \varrho^{-2} \cdot N(N-1) \int \Psi^*(1, 2, \dots, N) \Psi(1, 2, \dots, N) d\vec{r}_3 \dots d\vec{r}_N / I_N \quad (15)$$

is connected with the function $G_2(1, 2)$ by the relation

$$g(1, 2) = e^{u_{11}} G_2(1, 2). \quad (16)$$

One-particle density matrix

$$\varrho_1(1, 1') = N \int \Psi^*(1', 2, \dots, N) \cdot \Psi(1, 2, \dots, N) d\vec{r}_2 d\vec{r}_3 \dots d\vec{r}_N \quad (17)$$

is also an important quantity because it is connected with the particle density in the condensate. Penrose and Onsager¹⁸⁾ showed that in limes $r \rightarrow \infty$ the function $\varrho_1(r) \rightarrow \varrho_0$, where ϱ_0 is the number of particles with the momentum equal to zero. Functions G_2 and G_3 , as showed in Refs. 14, 15 can be written as a series with respect to the density

$$G_2(1, 2) = 1 + \varrho [k(1, 2) - \gamma] + \varrho^2 \left[\gamma^2 - \frac{1}{2} \delta - \gamma \cdot k(1, 2) + \frac{1}{2} h(1, 2) \right] + \dots, \quad (18)$$

$$G_3(1, 2, 3) = 1 + \varrho [p(1, 2, 3) - \lambda] + \dots \quad (19)$$

where

$$k(1, 2) = \int (e^{u_{11} + u_{12}} - 1) d\vec{r}_3, \quad (20)$$

$$h(1, 2) = \int (e^{u_{1s} + u_{2s}} - 1)(e^{u_{1s} + u_{2s}} - 1) d\vec{r}_3 d\vec{r}_4, \quad (21)$$

$$p(1, 2, 3) = \int (e^{u_{1s} + u_{2s} + u_{3s}} - 1) d\vec{r}_4, \quad (22)$$

$$\gamma = \frac{1}{\Omega^2} \int e^{u_{1s}} (e^{u_{1s} + u_{2s}} - 1) d\vec{r}_1 d\vec{r}_2 d\vec{r}_3, \quad (23)$$

$$\delta = \frac{1}{\Omega^2} \int e^{u_{1s} + u_{2s}} (e^{u_{1s} + u_{2s}} - 1)(e^{u_{1s} + u_{2s}} - 1) d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 d\vec{r}_4, \quad (24)$$

$$\lambda = \frac{1}{\Omega^3} \int e^{u_{1s} + u_{2s} + u_{3s}} (e^{u_{1s} + u_{2s} + u_{3s}} - 1) d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 d\vec{r}_4. \quad (25)$$

Integrating the relations (23—25) we obtain

$$\gamma = 8\pi \int [e^{u(r)} - 1] r^2 dr + O\left(\frac{1}{\Omega}\right), \quad (26)$$

$$\delta = \gamma^2 + O\left(\frac{1}{\Omega}\right), \quad (27)$$

$$\lambda = \frac{3}{2}\gamma + O\left(\frac{1}{N}\right), \quad (28)$$

where the terms denoted by $O\left(\frac{1}{\Omega}\right)$ are proportional to $\frac{1}{\Omega^n}$ ($n = 1, 2, \dots$), and which in limes $\Omega \rightarrow \infty$ can be omitted.

Employing the same procedure as when calculating the function G_2 for one-particle density matrix we can write

$$\begin{aligned} \varrho_1(1, 1') &= \varrho \cdot \Omega^{-1} \cdot \left\{ \int d\vec{r}^2 e^{\frac{1}{2}u_{1s} + \frac{1}{2}u_{1's}} [1 + \varrho \int d\vec{r}^3 (e^{\frac{1}{2}u_{1s} + \frac{1}{2}u_{1's} + \frac{1}{2}u_{2s}} - 1)] \right\} \cdot \\ &\quad \cdot (1 + \varrho\gamma + \dots). \end{aligned} \quad (29)$$

Hence in limes we find

$$\lim_{r \rightarrow 0} \varrho_1(r) = \varrho, \quad (30)$$

$$\lim_{r \rightarrow \infty} \varrho_1(r) = \varrho \left(1 + \frac{1}{2}\varrho\gamma + \dots \right). \quad (31)$$

The expression for energy (9) as the density function then becomes

$$\frac{E}{N} = \varrho I_1 + \varrho^2 I_2 + \varrho^3 I_3 + \dots, \quad (32)$$

where

$$I_1 = \frac{1}{\Omega} \int e^{u_{11}} F_2(1, 2) d\vec{r}_1 d\vec{r}_2, \quad (33)$$

$$I_2 = I_{21} + I_{22} + I_{23}, \quad (34)$$

$$I_3 = I_{31} + I_{32} + I_{33} + I_{34} + I_{35}, \quad (35)$$

$$I_{21} = \frac{1}{\Omega} \int e^{u_{11}} F_2(1, 2) (e^{u_{11} + u_{11}} - 1) d\vec{r}_1 d\vec{r}_2 d\vec{r}_3, \quad (36)$$

$$I_{22} = \frac{1}{\Omega} \int e^{u_{11} + u_{11} + u_{11}} F_3(1, 2, 3) d\vec{r}_1 d\vec{r}_2 d\vec{r}_3, \quad (37)$$

$$I_{23} = -\gamma I_1, \quad (38)$$

$$I_{31} = \frac{1}{2\Omega} \int e^{u_{11} + u_{11}} F_2(1, 2) (e^{u_{11} + u_{11}} - 1) \cdot (e^{u_{11} + u_{11}} - 1) d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 d\vec{r}_4, \quad (39)$$

$$I_{32} = \frac{1}{\Omega} \int e^{u_{11} + u_{11} + u_{11}} F_3(1, 2, 3) (e^{u_{11} + u_{11} + u_{11}} - 1) d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 d\vec{r}_4, \quad (40)$$

$$I_{33} = -\gamma I_{21}, \quad (41)$$

$$I_{34} = \left(\gamma^2 - \frac{1}{2} \cdot \delta \right) \cdot I_1, \quad (42)$$

$$I_{35} = -\lambda I_{22}. \quad (43)$$

The functions F_2 and F_3 in the actual selection of function u_{ij} from the relation (4) have the following form

$$\begin{aligned} F_2(i, j) &= -\frac{\hbar^2}{4m} \left(\frac{d^2 u_{ij}}{dr_{ij}^2} + \frac{2}{r_{ij}} \frac{du_{ij}}{dr_{ij}} \right) - \frac{\hbar^2}{8m} \left(\frac{du_{ij}}{dr_{ij}} \right)^2 + \frac{1}{2} V_{ij} = \\ &= -\frac{\hbar^2}{8m\alpha^2} \left[\beta^2 \left(\frac{\alpha}{r_{ij}} \right)^{12} - 2\beta(\beta - 1) \left(\frac{\alpha}{r_{ij}} \right)^7 \right], \end{aligned} \quad (44)$$

$$F_3(i, j, k) = -\frac{\hbar^2}{8m\alpha^2} \beta^2 \left(\frac{\alpha}{r_{ij}} \right)^6 \left(\frac{\alpha}{r_{ik}} \right)^6 \frac{\vec{r}_{ij} \vec{r}_{ik}}{r_{ij} r_{ik}}. \quad (45)$$

Using the term for energy we can calculate the pressure P , compressibility κ and sound velocity s according to the relations

$$P = - \frac{\partial E}{\partial \Omega} = e^2 \frac{\partial E/N}{\partial \rho} = e^2 [I_1 + 2\rho I_2 + 3\rho^2 I_3 + \dots], \quad (46)$$

$$\frac{1}{\kappa} = - \Omega \frac{\partial P}{\partial \Omega} = e^2 [2 I_1 + 4\rho I_2 + 9\rho^2 I_3 + \dots], \quad (47)$$

$$s = \left(\frac{1}{m} \frac{\partial P}{\partial \rho} \right)^{1/2} = \left[\frac{2\rho}{m} (I_1 + 3\rho I_2 + 6\rho^2 I_3 + \dots) \right]^{1/2}. \quad (48)$$

3. Results and discussion

Variational parameter α determined by relation (8) for the hydrogen H mass amounts to 3.748×10^{-10} m. We considered two additional values of the variational parameter α . Calculations were carried out for two potentials. Quantities having index LJ determine values defined by the Lennard-Jones potential (5) taken for all r ; other quantities containing the potential, but not having index LJ , refer to the computation which for the distances greater than 6.78×10^{-10} m includes as the potential the form (6).

Integrals I_1 and I_2 were computed numerically by the Simpson and Gauss QG 24 formula with a relative error from 0.1% to 0.4%. Integrals I_{31} and I_{32} were computed numerically using the program VEGAS¹⁹⁾ after they had been reduced to nine-folds. Standard deviations of the integrals I_{31} and I_{32} are 4–6% and 7–10%. The results of the integral computations are shown in Table 1.

TABLE 1

α	I_1	$I_{1(LJ)}$	I_{21}	$I_{21(LJ)}$	I_{22}	I_{23}	$I_{23(LJ)}$
3.700	6.6088	5.1844	-4.9916	-3.7072	0.6075	6.2069	4.8692
3.748	5.8584	4.4358	-4.6619	-3.3268	0.6400	5.7189	4.3302
3.800	5.2813	3.8594	-4.4123	-3.0192	0.7650	5.3736	3.9269
$I_{31(LJ)}$	I_{32}	I_{33}	$I_{33(LJ)}$	I_{34}	$I_{34(LJ)}$	I_{35}	
0.303	-0.0844	-4.6881	-3.4818	2.9148	2.2865	0.8559	-0.9392
0.68	-0.1316	-4.5509	-3.2476	2.7914	2.1136	0.9372	-0.9764
0.64	-0.615	-4.4894	-3.0272	2.7339	1.9978	1.0302	-1.0175

The values of integrals calculated for three parameters α . Integrals $I_1, I_{1(LJ)}$ are given in 10^{-51} J m^3 , $I_{21}, I_{21(LJ)}, I_{22}, I_{23}, I_{23(LJ)}$, in 10^{-78} J m^6 and $I_{31(LJ)}, I_{32}, I_{33}, I_{33(LJ)}, I_{34}, I_{34(LJ)}, I_{35}$ in 10^{-105} J m^9 ; α is in 10^{-10} m and γ in 10^{27} m^3 .

Expressions for energy, pressure, compressibility and sound velocity can be written in form of

$$\begin{aligned} \frac{E}{N} &= \{\rho' I'_1 + 0.1 \rho'^2 I'_2 + 0.01 \rho'^3 I'_3\} \times 10^{-23} \text{ J}, \\ P &= \rho'^2 \{I'_1 + 0.2 \rho'^2 I'_2 + 0.03 \rho'^2 I'_3\} \times 10^{-1} \text{ N/m}^2, \\ \frac{1}{\kappa} &= \rho'^2 \{2 I'_1 + 0.4 \rho' I'_2 + 0.09 \rho'^2 I'_3\} \times 10^{-1} \text{ N/m}^2, \\ s &= \frac{\rho'}{m_0} \left\{ 2 I'_1 + 0.6 \rho' I'_2 + 0.12 \rho'^2 I'_3 \right\}^{1/2} \times 10 \text{ m/s}, \end{aligned}$$

where we introduced

$$\begin{aligned} \rho &= \rho' \times 10^{26} \text{ atoms/m}^3, I_1 = I'_1 \times 10^{-51} \text{ Jm}^3, I_2 = I'_2 \times 10^{-78} \text{ Jm}^6, \\ I_3 &= I'_3 \times 10^{-105} \text{ Jm}^9, m = m_0 \times 10^{-27} \text{ kg}. \end{aligned}$$

Since the values of the quantities I'_1 , I'_2 and I'_3 for parameter $\alpha = 3.748 \times 10^{-10}$ m and the exact L-J potential are equal to 4.436, 1.643 and 0.351, it follows that the obtained expression for energy is valid for densities up to 10^{26} atoms/m³. In that case the third-order term is 1260 times smaller than the first-order term, and about 47 times smaller than the second one. If we compare the values of our energies for these densities with the corresponding Monte Carlo calculations we can see that our energies are greater. It is our opinion that this is due to the fact that our expression has not been minimized with respect to α , which will be done in our next calculations. Energy, pressure, compressibility and sound velocity for $\alpha = 3.748 \times 10^{-10}$ m are shown in Figure 1.

The radial distribution function up to the second order in density according to relations (16), (18), (20), (21), (26) and (27) has the form:

$$g(r_{12}) = e^{u_{12}} \left(1 + \rho l_{12} + \frac{1}{2} \rho^2 l_{12}^2 \right), \quad (49)$$

where the function l_{12} is

$$l_{12} = \int (e^{u_{13}} - 1)(e^{u_{23}} - 1) d\vec{r}_3. \quad (50)$$

Values of the function $l_{1,2}$ are computed numerically and displayed in Table 2. Radial distribution function for several densities is plotted in Figure 2. Graphs (c) and (d) refer to densities $\rho = 10^{27}$ atoms/m³ and $\rho = 7.5 \times 10^{27}$ atoms/m³ for which other expressions are not valid. Because of the form of the function

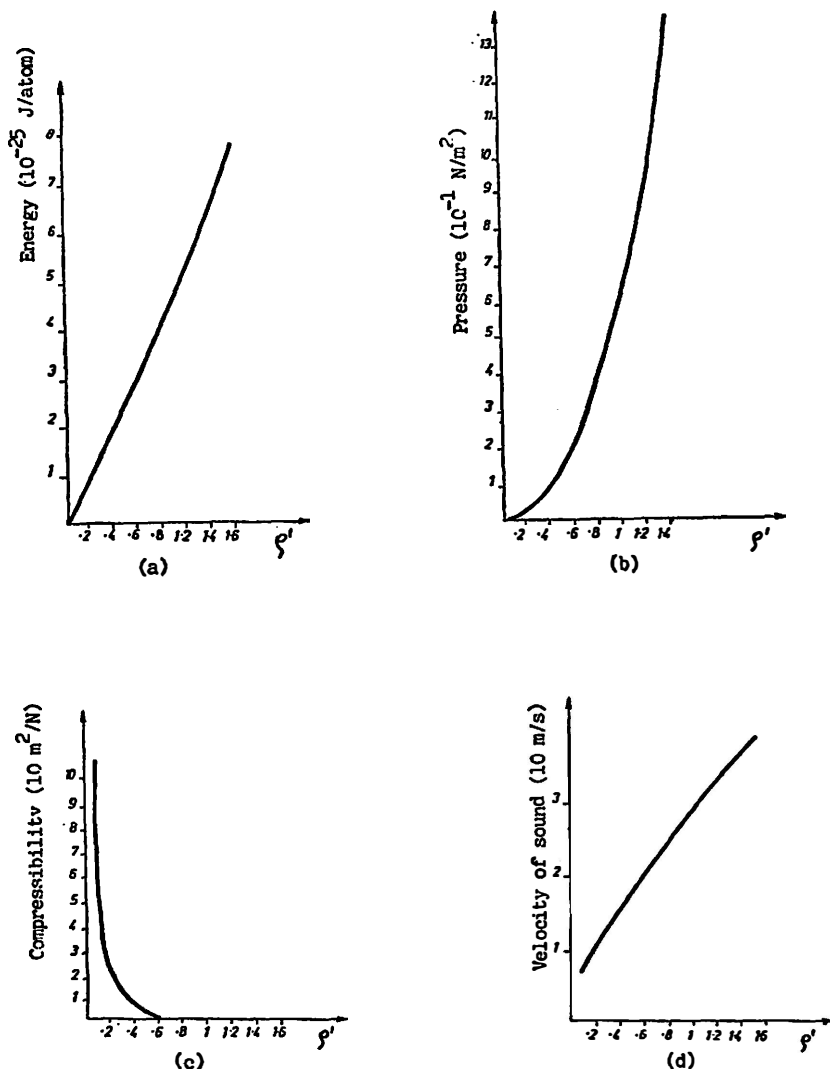


Fig. 1. Energy, pressure, compressibility and sound velocity as functions of density $\rho = \rho' \times 10^{26}$ atoms/m³.

$l(r)$ in point $r/a = 0.1$ the values of the product ρl amount to 0.2356 and 1.767 for densities $\rho = 10^{27}$ atoms/m³ and $\rho = 7.5 \times 10^{27}$ atoms/m³. Thus, expression (49) can be considered correct also for the densities up to 10^{27} atoms/m³. The third-order term in (49) is smaller than the second also at densities of 7.5×10^{27} atoms/m³. In Ref. 5 employing the Monte Carlo technique the form of the function $g(r)$ at density $\rho = 7.5 \times 10^{27}$ atoms/m³ was obtained. The agreement between our function $g(r)$ and this one is very good. Consequently, it can be concluded that additional terms in series for radial distribution function become decisive for densities greater than 10^{28} atoms/m³.

TABLE 2

r/α	$l(r)$ (10^{-28} m^3)						
0.1	2.356	0.8	1.670	2.0	0.318	3.0	0.051
0.2	2.312	1.0	1.390	2.2	0.218	3.5	0.024
0.3	2.244	1.2	1.122	2.4	0.149	4.0	0.012
0.4	2.155	1.4	0.868	2.4	0.149	4.5	0.007
0.5	2.050	1.6	0.644	2.6	0.104	5.0	0.004
0.6	1.930	1.8	0.459	2.8	0.073	5.5	0.002

The function $l(r)$ for parameter $\alpha = 3.748 \times 10^{-10} \text{ m}$.

One-particle density matrix for $r \rightarrow 0$ yields the particle density according to relation (30). In limit $r \rightarrow \infty$ relation (31) is valid; assuming that $|\rho\gamma| \sim 0.5$ for $\alpha = 3.748 \times 10^{-10} \text{ m}$, it follows that this relation is valid in the region of densities smaller than $2 \times 10^{26} \text{ atoms/m}^3$. For example, it is computed that the number of particles in the condensate (at absolute zero) for density $\rho = 2 \times 10^{26} \text{ atoms/m}^3$ amounts to 90%. For densities $9.95 \times 10^{26} \text{ atoms/m}^3$ Ristig²⁰⁾ states the result of 88.4% in a »uniform limit« and 79.1% in a »hypernetted chain« approximation. The comparison with these results confirms the quality of the obtained analytical expressions.

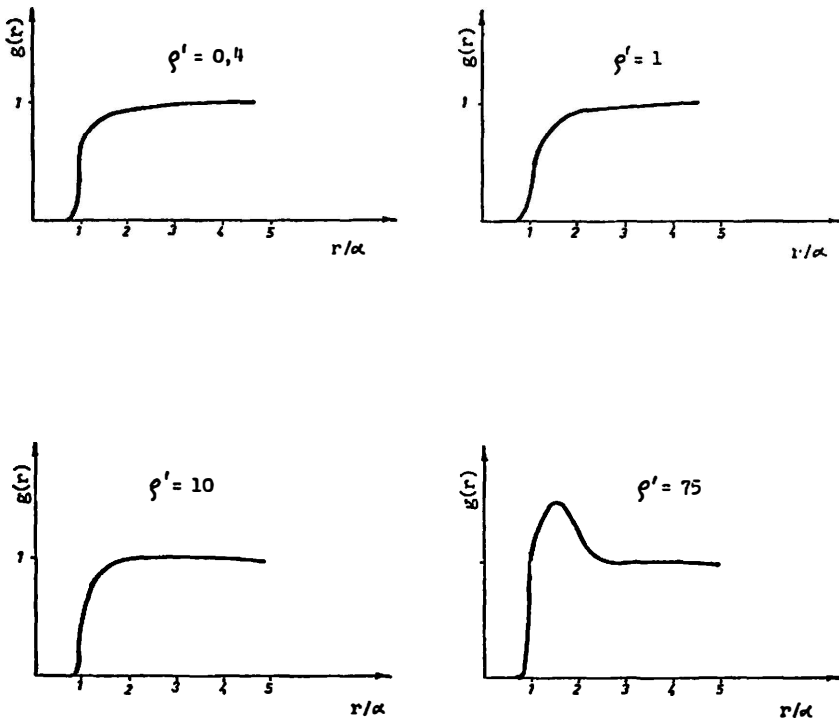


Fig. 2. Radial distribution function $g(r)$ at four densities $\rho = \rho' \times 10^{26} \text{ atoms/m}^3$ and $\alpha = 3.748 \times 10^{-10} \text{ m}$.

Let us consider how, according to the mentioned results, we can conclude that tritium $T \uparrow$ is a liquid and not a gas at absolute zero. Kinetic energy contributions should be multiplied by the hydrogen mass and divided by the tritium mass. Thus, we obtain for example $I_1 = -7.7 \times 10^{-51} \text{ Jm}^3$. Energy is therefore negative, which means the system is in a bound state. Taking into consideration the form of the force between $T \uparrow$ atoms and the fact that their mass is lighter than the mass of He^4 atom, which is a liquid, we should expect the state of tritium $T \uparrow$ to be liquid. Naturally our conclusion should be confirmed by a detailed analysis; however, we do not think the expected conclusion will be altered.

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FIZIČKE OSOBINE SPIN-POLARIZIRANOG ATOMARNOG VODIKA U OSNOVNOM STANJU

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Koristeći Ursell-Mayerov razvoj i Ljoljin model poluslobodnog plina izvedene su relacije koje opisuju osnovno stanje spin-polariziranog vodika $H \uparrow$. Dobijeni analitički izrazi vrijede za gustoće do 10^{26} atoma/ m^3 . Područje primjenljivosti za radialnu funkciju raspodjele i jednočestičnu matricu gustoće je nešto veće. Pokazano je da je ovaj bozonski sistem, za navedeno područje gustoća, na apsolutnoj nuli u plinskom stanju. Nadalje je utvrđeno da pri gustoći od $2 \cdot 10^{26}$ atoma/ m^3 kondenzacija čestica u impulsnom prostoru iznosi 90%.