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A METHOD FOR MEASURING THE ELASTIC RATIO K_{11}/K_{33} OF A MESOGENE IN NEMATIC PHASE[(−)]

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A capacitance measurement method is presented, able to determine the elastic ratio K_{11}/K_{33} of a nematic liquid crystal using a hybrid aligned cell. Moreover, the perturbation method results a suitable calculation procedure for describing the director distorition.

The hybrid aligned Nematic Liquid Crystal (NLC) cells have been studied from an optical point of view by S. Matsumoto et al.[1], and appear particularly suitable for realizing coulored displays[2].

The rheological properties of a NLC depend of course on the elastic constants: thereby the importance of the methods involving the absolute or relative measurements of these parameters.

In the present paper the planar elasticity of a hybrid aligned NLC cell will be discussed by means of a perturbation method, with the aim of describing a capacitance measurement able to desume the elastic ratio K_{11}/K_{33}.

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The authors purpose is to describe an experimental method based on dielectric measurements with the following characteristics: a) extreme simplicity, b) absence of external fields inducing distortion, and to point out that the perturbation method is a suitable calculation procedure for describing the director lines pattern in the cases a) $K_{11} \sim K_{33}$, b) $K_{11} \ll K_{33}$, c) $K_{11} \gg K_{33}$.

In the elastic continuum theory the distortion free energy density of a NLC is given by[3]

$$F(\vec{r}) = \frac{1}{2} K_{11} [\nabla \cdot \vec{n}(\vec{r})]^2 + \frac{1}{2} K_{22} [\vec{n}(\vec{r}) \cdot \nabla \times \vec{n}(\vec{r})]^2 +$$

$$+ \frac{1}{2} K_{33} [\vec{n}(\vec{r}) \times \nabla \times \vec{n}(\vec{r})]^2 \tag{1}$$

where the three terms represent the splay, twist and bend contribution, respectively. Taking into account that a hybrid cell has no twist, the Euler-Lagrange Eq., minimizing the total free energy of the cell, can be written[4]

$$(\vec{n} \times \nabla)^2 \varphi + \left\{ \nabla \times \vec{n} \cdot \vec{k} \cdot (\nabla \cdot \vec{n}) - \frac{\nabla^2 \varphi}{A} \right\} = 0 \tag{2}$$

where $\vec{k}$ is the unit vector normal to the distortion(:) plane, φ is the tilt angle between the director $\vec{n}$ and the normal $\vec{j}$ to the cell plates, and $A = 1 - K_{11}/K_{33}$. From (2) it follows that the general solution $\varphi(x, y)$ must be harmonic in the one constant hypothesis. Furthermore, the symmetric condition of the problem allows us to write in this case(=)

$$\varphi = \varphi(\eta). \tag{3}$$

Consequently, Eq. (2) becomes

$$(1 - A \sin^2 \varphi) \frac{d^2 \varphi}{d \eta^2} - A \sin \varphi \cos \varphi \left(\frac{d \varphi}{d \eta} \right)^2 = 0 \tag{4}$$

and, by integrating after trivial calculation[5,6], if $K^{33} \neq 0$:

$$(1 - A \sin^2 \varphi) \left(\frac{d \varphi}{d \eta} \right)^2 = B^2 \tag{5}$$

where B^2 is to be determined by the conditions

$$\varphi(0) = 0 \tag{6}$$
$$\varphi(1) = \pi/2$$

in the strong anchoring hypothesis.

(:) The operator $(\vec{n} \times \nabla)^2$ represents $\sin^2 \varphi\, \partial^2_{yy} - \sin 2\varphi\, \partial_x \cdot \partial_y + \cos^2 \varphi\, \partial^2_{xx}$.

(=) The reduced coordinates are defined as $\xi \equiv (x - x_0)/h$, $\eta = y/h$.

In the one constant approximation, it results $A = 0$; then we get

$$B_0 = \pi/2 \tag{7}$$

and Eq. (5) gives

$$\varphi_0(\eta) = B_0\,\eta$$

$$\eta = \frac{1}{B_0}\,\mathrm{arc\,cos\,exp}\,(-B_0\,\xi). \tag{8}$$

On the other hand, if $K_{11} \ll K_{33}$, Eq. (5) gives

$$\varphi(\eta) = \mathrm{arc\,sin}\,\eta \tag{9}$$

$$\eta = [\xi(2 \cdot \xi)]^{1/2}.$$

On the contrary, if $K_{11} \gg K_{33}$, Eq. (4) gives directly

$$\varphi(\eta) = \mathrm{arc\,cos}\,(1 - \eta) \tag{10}$$

$$\xi = \frac{1}{2}\ln\left[\frac{1 + \sqrt{\eta(2-\eta)}}{1 - \sqrt{\eta(2-\eta)}}\right] - \sqrt{\eta(2-\eta)}.$$

As a consequence, the tilt angle is sensitive on the particular value of K_{11}/K_{33} (see Fig. 1a), 1b)): this fact can suggest a capacitance measurement in order to determine the elastic ratio, using a hybrid aligned NLC cell.

Let us consider now for example a NLC having $K_{11} \sim K_{33}$, that is $A \ll 1$: by applying the perturbation method in order to solve Eq. (5), and by putting

$$\varphi(\eta) = \sum_{k=0}^{\infty} \varphi_k(\eta) \cdot A^k$$

$$B^2 = \sum_{k=0}^{\infty} B_k^2 \cdot A^k \tag{11}$$

we obtain

$$\varphi_1(\eta) = -(1/8)\sin 2\,B_0\,\eta \tag{12}$$

$$B_1^2 = -(1/2)\,B_0^2$$

$$\varphi_2(\eta) = -(1/16)\sin 2B_0\,\eta + (5/256)\sin 4B_0\,\eta \tag{13}$$

$$B_2^2 = -(1/32)\,B_0^2.$$

These terms are enough since the power series converges rapidly; (+) obviously Eqs. (7) and (8) give the zero order term.

(+) In fact, by putting $\varphi_{(l)} = \varphi_{(l-1)} + A^l\,\varphi_l$ and $\varphi_{(0)} = \varphi_0$ the maximum values of both $(\varphi_{(2)} - \varphi_{(1)})/\varphi_{(2)}$ and $(B_{(2)}^2 - B_{(1)}^2)/B_{(2)}^2$ results 0.3%, if $A = 0.25$ (MBBA, see Ref. 7)).

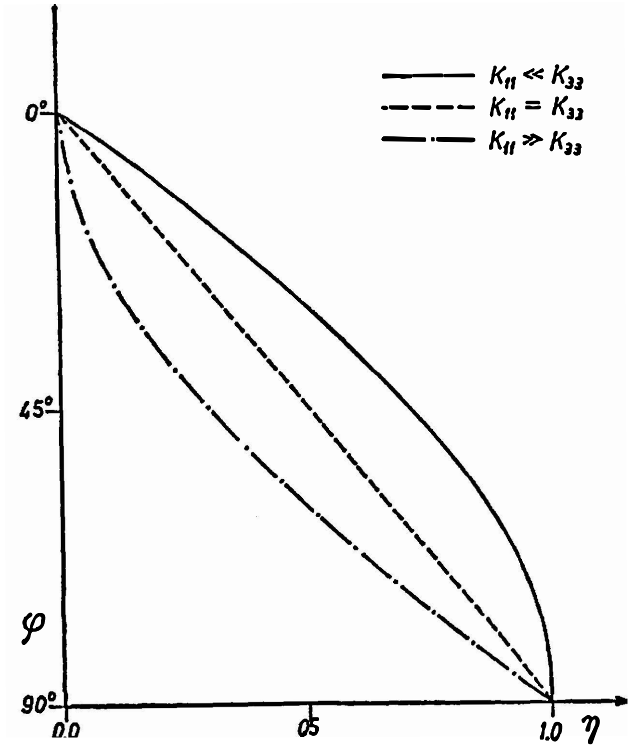

Fig. 1a: Tilt angle φ vs. the reduced thickness η of the NLC slab for different values of K_{11}/K_{33}.

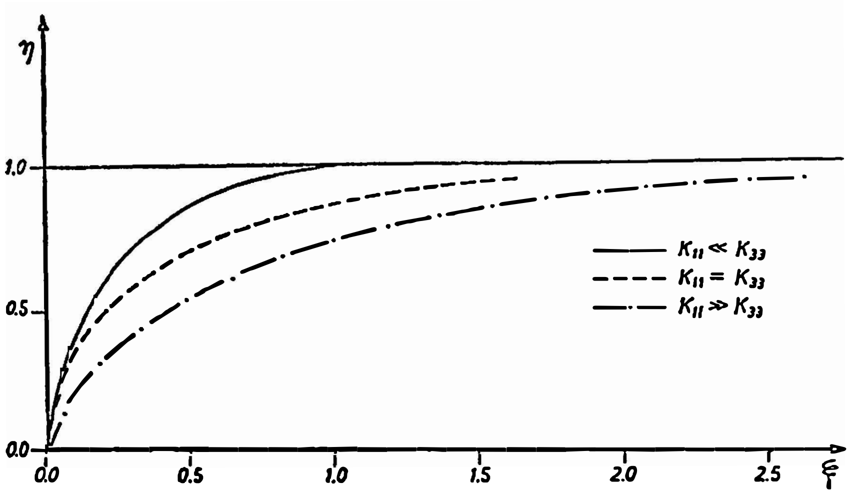

Fig. 1b: Director lines of the NLC slab for different values of K_{11}/K_{33}.

Furthermore, if the dissipation factor of the hybrid aligned NLC cell can be neglected, the cell capacitance is given by[8,9]:

$$C = C_{\parallel} \{1 + e \langle \sin^2 \varphi \rangle + e^2 \langle \sin^4 \varphi \rangle\}^{-1} \tag{14}$$

where $C_{\parallel}$ is the capacitance of an ideal identical homeotropic NLC cell, and $e \equiv \varepsilon_a/\varepsilon_{\parallel}$ is the ratio between the anisotropy of the dielectric constant and the parallel dielectric constant.

Then, it is necessary to calculate $\langle \sin^2 \varphi \rangle$, $\langle \sin^4 \varphi \rangle$ vs. A, by using the system (11) stopped to the second order term, with the aim of desuming A by means of a capacitance measurement.

It results [++]:

$$\langle \sin^2 \varphi \rangle = \frac{1}{2} - M(A)$$

$$\langle \sin^4 \varphi \rangle = \frac{3}{8} - M(A) - N(A) \tag{15}$$

where

$$M(A) \equiv \frac{A}{16}\left(1 + \frac{9}{16} A\right)$$

$$N(A) \equiv \frac{7}{1024} A^2. \tag{16}$$

[++] In order to obtain Eqs. (15) (16), we have rewritten the expansion (11) stopped to the second order in A:

$$\varphi = \frac{\pi}{2}\eta + \frac{1}{8} A \left\{- \left(1 + \frac{A}{2}\right) \sin(\pi \eta) + \frac{5}{16} A \sin(2\pi \eta)\right\}. \tag{11'}$$

Then $\sin\varphi$ has been calculated by Taylor expansion around $\frac{\pi}{2}\eta$, by maintaining the first three expansion terms and neglecting the terms greater than A^2. It results

$$\sin\varphi = v_0 - A v_1 - A^2 v_2 \tag{12'}$$

where

$$\begin{cases} v_0 \equiv \sin(\pi/2)\eta \\ v_1 \equiv (1/4)[\sin(\pi/2)\eta - \sin^3(\pi/2)\eta] \\ v_2 \equiv (1/32)[\sin(\pi/2)\eta - 13\sin^3(\pi/2)\eta + 2\sin^5(\pi/2)\eta]. \end{cases}$$

By computing $\sin^2\varphi$, $\sin^4\varphi$ with the same approximation, we obtain the mean values (15), (16).

By substituting the system (15) in Eq. (14), we get:

$$A^2 + R_1 A + R_0 = 0 \quad (17)$$

where

$$R_1 \equiv (16/9)(1 + e)[1 - (43/36)e]^{-1}$$

$$R_2 \equiv [(C_{\parallel}/C) - 1 - (e/2)(1 + (3/4)e)][(9/256)e(1 - (43/36)e)]^{-1}.$$

As an example of the method application, a measurement has been performed on a hybrid aligned NLC cell ($h = 10.5\ \mu$m, $S = 24.01$ cm^2) filled with anhydrous MBBA by CEN-LETI (Grenoble).

The measurement, at room temperature $T = (22.9 \pm 0.1)$°C, implies the use of a bridge method discussed in a previous paper[9] ; the bridge driving voltage is chosen as $V = 7.8$ mV at frequency $f = 1$kHz, in order to avoid a detectable change in the director flow lines pattern, due to ECB effect[(*) 10].

The experimental result is:

$$C = (10.18 \pm 0.01)\ \text{nF}. \quad (18)$$

By computing M from Eq. (17) with the data $\varepsilon_{\parallel} = 4.70$, $\varepsilon_{\perp} = 5.40$[11], the relation (17) gives $A = 0.02$ and finally[(§)]:

$$\frac{K_{11}}{K_{33}} = 0.98 \pm 0.07. \quad (19)$$

(*) The ECB distortion in this case is given by $\varphi_{ECB} \cong \frac{1}{2}\left(\frac{V}{V_{Th}}\right)^2 \sin \pi \eta$ where $V_{Th} \cong 3V$ is the ECB threshold for MBBA. Then the maximum distortion results $\varphi_{ECB} \sim 0.07°$ and can be neglected.

(§) We can differentiate Eq. (17) by considering both e and $C_{\parallel}/C$ affected by uncertainties. In the hypothesis of errors normal distribution, we get for the probable errors the expression:

$$|\Delta A| = \left\{E_1^2\left[\left(\frac{\Delta \varepsilon_{\perp}}{\varepsilon_{\perp}}\right)^2 + \left(\frac{\Delta \varepsilon_{\parallel}}{\varepsilon_{\parallel}}\right)^2\right] + E_2^2\left[\left(\frac{\Delta \varepsilon_{\parallel}}{\varepsilon_{\parallel}}\right)^2 + \left(\frac{\Delta \langle \varepsilon \rangle}{\langle \varepsilon \rangle}\right)^2\right]\right\}^{1/2}$$

where

$$E_1 \equiv \{1 + (3/2)e - (1/8)(1 + 2e)A - (9/128)[1 - (43/18)eA^2\}(1 - e) \cdot$$
$$\cdot \{(9/128)e[1 - (43/36)e]A + (1/8)e(1 + e)\}^{-1}$$

and

$$E_2 \equiv (C_{\parallel}/C)\{(9/256)e[1 - (43/36)e]A + (1/16)e(1 + e)\}^{-1}$$

by consequence, uncertainties of 1% on ε_i (where $i = \parallel, \perp, \langle\,\rangle$) give a probable error $\Delta(K_{11}/K_{33})/(K_{11}/K_{33}) \sim 13\%$, while $|\Delta \varepsilon_i/\varepsilon_i| \sim 5 \cdot 10^{-4}$ (easily to obtain with good instrumentation) should give only $\Delta(K_{11}/K_{33})/(K_{11}/K_{33}) \sim 7\%$.

This value is consistent with the measurement of the elastic ratio performed by R. Malvano and the authors on a homeotropic NLC cell with the same capacitance bridge technique in presence of both an electric and a magnetic field[12].

In conclusion, this method is limited by: 1) the strong anchoring hypothesis; 2) the knowledge of $\varepsilon_{\parallel}$, $\varepsilon_{\perp}$ with very small errors; 3) the small conductance condition.

This last hypothesis can be omitted if $\sigma_{\parallel}$, $\sigma_{\perp}$ are known with great accuracy because Eq. (14) can be easily generalized[13].

Moreover, the advantages of this method are: 1) the great simplicity; 2) the absence of external field producing the NLC distortion; 3) the measurement rapidity; 4) the accuracy.

Furthermore, the perturbation method is very useful for describing the distortion of a NLC having the splay and bend elastic constants quite near or quite different: in the first case, the perturbation parameter is A and the zero order term is given by Eq. (8); in the latter one, if $K_{11} \ll K_{33}$ (or $K_{11} \gg K_{33}$) the perturbation parameter is 1—A (or 1/A) and the zero order term is given by Eq. (9) (or by Eq. (9) or by Eq. (10), respectively).

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References

1) S. Matsumoto, M. Kawamoto, K. Mizunaya, J. Appl. Phys., **47** (1976) 3842;

2) R. Riva, Fisica e Tecnologia, **2** (1979 114;

3) See for example S. Chandrasekar, *Liquid Crystals*, Cambridge University Press (1977), p. 98;

4) G. Barbero, A. Strigazzi, Rev. Roumaine Phys., **26** (1981), in press;

5) P. Pieranski, F. Brochard, E. Guyon, J. de Physique, **33** (1972) 681;

6) H. Gruler, T. J. Sheffer, G. Meier, Z. Naturf. **27a** (1972) 966;

7) E. B. Priestly, P. J. Wojtowicz, Ping Sheng, *Introduction to Liquid Crystals*, Plenum Press, New York (1975) p. 352;

8) T. J. Sheffer, J. Nehrign, J. Appl. Phys., **48** (1977) 1783;

9) R. Malvano, A. Strigazzi, Atti Accad. Sci. Torino, **112** (1978) 193;

10) D. Meyerhofer, Phys. Lett., **51A** (1975) 407;

11) H. Kelker, R. Hatz, *Handbook of Liquid Crystals*, Verlag Chemie (1980), p. 178;

12) G. Barbero, R. Malvano, A. Strigazzi, Nuovo Cimento B (1980), in press;

13) H. Gruler, L. Cheung, J. Appl. Phys. **46** (1975) 5097.

METODA MJERENJA ODNOSA ELASTIČNIH KONSTANTI K_{11}/K_{33} TEKUĆIH KRISTALA U NEMATIČKOJ FAZI

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Prikazana je metoda određivanja odnosa elastičnih konstanti K_{11}/K_{33} nematičkih tekućih kristala mjerenjem kapaciteta ćelije s različito orijentirajućim stijenkama. Pokazano je da je perturbaciona metoda podesna za izračunavanje ovisnosti smjera jediničnog vektora o položaju među rubnim plohama.