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UNITARISED CAPTURE CROSS SECTIONS IN THE IMPACT PARAMETER REPRESENTATION

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The procedure of the unitarisation (normalisation) of the charge exchange capture cross sections was performed in the impact parameter representation using the relation between quantum-mechanical and impact parameter amplitudes. As one can expect, the normalisation effect is very essential for transitions with small resonance defect w in the region of intermediate and low relative velocities. The analytical expressions for 1s-electron capture amplitudes into nlm-states with $n < 3$ and the amplitudes, summed over all lm-states, were obtained in the Brinkman-Kramers approximation. The influence of the normalisation effect is illustrated by numerical calculations. The method of unitarisation, used in this paper, is rather effective in the region of intermediate velocities for the reactions, when the contribution from nonadiabatic transitions near quasiintersections of electronic levels of the 'ion + atom' system is negligible.

1. Introduction

In the previous paper[1] the relation between quantum-mechanical $f(q, v)$ and impact parameter $a(\varrho, v)$ amplitudes in charge exchange was obtained, using the Brinkman-Kramers (BK) approximation. In the present paper, with the help of this relation, the analytical formulae for $a(\varrho, v)$ were obtained for some $n'l'm' \to$ $\to nlm$ reactions and the procedure of multi-channel normalisation (unitarisation) for $a(\varrho, v)$ was performed to calculate partial and the total one-electron capture cross sections $\sigma(v)$. The aim of this work is to show that the region of validity of the BK approximation can be extended and spread into the intermediate relative velocities v.

Basic formulae for $a(\varrho, v)$ and $\sigma(v)$ are given in Section 2, in Section 3 numerical calculations of 1s-electron capture cross sections are compared with experimental data and the discussion is given in Section 4. Atomic units are used.

2. *Basic formulae*

According to Ref. 1 the one-electron capture amplitude for transition $n'l'm'$ — — nlm, i. e. for the reaction

$$B^{Z+} + A(n'l'm') \to B^{(Z-1)+}(nlm) + A^{+} \tag{1}$$

in the BK approximation can be written in the form:

$$|a(\varrho, v)|^2 \equiv W(\varrho, v) = \frac{4}{v^2}\left|\int_0^\infty \overline{K}d\overline{K}\, C_{l'm'}\, C_{lm}\, P_{l'}^{m'}(\Theta')\, P_l^m(\Theta)\, \mathcal{J}_{\Delta m}(\overline{K}\varrho) \times \right.$$

$$\left. \times\, Q_{n'l'}([\overline{K}^2 + (\overline{w}/v - v/2)^2]^{1/2})\, T_{nl}([\overline{K}^2 + (\overline{w}/v + v/2)^2]^{1/2})\right|^2 \tag{2}$$

$$C_{lm} = \left(\frac{(2l+1)(l-m)!}{(l+m)!}\right)^{1/2}; \ \Delta m = |m' - m|; \ \vec{\overline{K}} = \vec{q} - (\overline{w}/v + v/2)\,\vec{v}/v;$$

$$(\vec{\overline{K}}\vec{v}) = 0 \tag{3}$$

$$\Theta = \frac{\overline{w}/v + v/2}{[K^2 + (\overline{w}/v + v/2)^2]^{1/2}}; \ \Theta' = \frac{v/2 - \overline{w}/v}{[K^2 + (\overline{w}/v - v/2)^2]^{1/2}}$$

where w is the resonance (energy) defect, $\mathcal{J}(x)$ is the Bessel function, P_l^m is the associated Legendre polynomial. The product $Q_{n'l'}(\sqrt{x^2 - 2\overline{w}})\, T_{nl}(x)$ is the radial part of the charge transfer amplitude $f(q, v)$[2]:

$$Q_{nl}(x) = \int_0^\infty R_{nl}(r)\, j_l(xr)\, r^2\, \mathrm{d}r \tag{4}$$

$$T_{nl}(x) = \int_0^\infty R_{nl}(r)\, j_l(xr)\, V(r)\, r^2\, \mathrm{d}r \tag{5}$$

where $j(x)$ is the spherical Bessel function, $V(r)$ is the effective potential of the ion B^{Z+}; R_{nl} is the radial atomic wave function

$$\int_0^\infty R_{nl}^2(r)\, r^2\, \mathrm{d}r = 1. \tag{6}$$

If $f(q, v)$ values are known, one can calculate $W(\varrho, v)$ and perform the unitarisation procedure of the capture cross section in the form:

$$\sigma(n'l'm' - nlm) = \pi a_0^2\, 2 \int_0^\infty \varrho\, \mathrm{d}\varrho\, W(\varrho\, v) \tag{7}$$

$$W(\varrho, v) = \frac{N/3 \cdot W^{BK}_{n'l'm',nlm}(\varrho, v)}{1 + \frac{N}{3} \sum_{\widetilde{n}\widetilde{l}\widetilde{m}} W^{BK}_{n'l'm',\widetilde{n}\widetilde{l}\widetilde{m}}(\varrho, v)} \tag{8}$$

where W^{BK} is the BK capture impact parameter probability, defined in (2). The summation is made over all possible states $\widetilde{n}\widetilde{l}\widetilde{m}$ of the resulting ion $B^{(Z-1)+}$. The factor $N/3$ takes into account the number of equivalent electrons N in the target atomic shell and the asymptotic of the cross sections at high velocities[3]:

$$\sigma(v) \underset{v \gg \sqrt{\overline{\omega}}}{\approx} \sigma^{BK}(v)/3 = \pi a_0^2 \cdot \frac{2N}{3} \int_0^\infty W^{BK}_{n'l'm',nlm}(\varrho, v)\, \varrho\, \mathrm{d}\varrho. \tag{9}$$

Capture cross sections for transitions between terms $n'l' - nl$ and the total cross sections are defined respectively by

$$\sigma(n'l' - nl) = \sum_{mm'} \sigma(n'l'm' - nlm) \tag{10}$$

$$\sigma_t(v) = \sum_{\substack{nl\\ n'l'}} \sigma(n'l' - nl). \tag{11}$$

In some cases it is possible to obtain formulae for the functions $Q(x)$, $T(x)$ and $a(\varrho, v)$ in a close analytical form. For example, using hydrogen-like wave functions $R_{nl}(r) = Z^{3/2} R^H_{nl}(\mathrm{Z}r)$ with effective charge $Z = n\sqrt{2I}$ (I is the bound energy) and $V(r) = -Z/r$ we have:

$$Q_{1s} = \frac{2^2}{Z^{3/2}(x^2/Z^2 + 1)^2}; \qquad T_{1s} = \frac{2\sqrt{Z}}{x^2/Z^2 + 1} \tag{12}$$

$$Q_{2s} = \frac{4(x/Z)^2 - 1}{2\sqrt{2}Z^{3/2}(x^2/Z^2 + 1/4)^3}; \quad T_{2s} = \frac{\sqrt{Z}[4(x/Z)^2 - 1]}{4\sqrt{2}(x^2/Z^2 + 1/4)^2} \tag{13}$$

$$Q_{2p} = \sqrt{\frac{2}{3}}\frac{x/Z}{Z^{3/2}(x^2/Z^2 + 1/4)^3}; \quad T_{2p} = \sqrt{\frac{Z}{6}}\frac{x/Z}{(x^2/Z^2 + 1/4)^2} \tag{14}$$

$$Q_{3s} = \frac{4}{3\sqrt{3}}Z^{5/2}\frac{1}{(Z^2/9 + x^2)^4}\left(\frac{1}{81}Z^4 + x^4 - \frac{10}{27}Z^2x^2\right) \tag{15}$$

$$Q_{3p} = \frac{32}{27\sqrt{6}}\frac{Z^{7/2}x}{(Z^2/9 + x^2)^4}(x^2 - Z^2/9) \tag{16}$$

$$Q_{3d} = \frac{64}{81\sqrt{30}}Z^{9/2}x^2\frac{1}{(Z^2/9 + x^2)^4}. \tag{17}$$

For a given principal quantum number n the functions $Q(x)$ and $T(x)$ satisfy the known sum rules[4,2]:

$$\sum_{l=0}^{n-1}(2l + 1)Q_{nl}^2(x) = \frac{2^4}{n^3Z^3}\left(\frac{x^2}{Z^2} + \frac{1}{n^2}\right)^{-4} \tag{18}$$

$$\sum_{l=0}^{n-1}(2l + 1)T_{nl}^2(x) = \frac{2^2 Z}{n^3}\left(\frac{x^2}{Z^2} + \frac{1}{n^2}\right)^{-2}. \tag{19}$$

In the case of the K-electron capture by fully stripped ions

$$Z + H_{Z'}(1s) \to H_Z(nlm) + Z'$$

we obtain from (2) and (12—17) for electron capture amplitudes:

$$|a_{1s,1s}| = \frac{2\varrho^2(ZZ')^{5/2}}{v\bar{k}_0^2}K_2(\varrho\bar{k}_0) \tag{20}$$

$$|a_{1s,2s}| = \frac{\varrho^2(ZZ')^{5/2}}{\sqrt{2}v\bar{k}_0^2}\left(K_2(\bar{k}_0\varrho) - \frac{\varrho Z^2 K_3(\bar{k}_0\varrho)}{12\bar{k}_0}\right) \tag{21}$$

$$|a_{1s,2p_0}| = \frac{\sqrt{2}\varrho^3(ZZ')^{5/2}Z(\overline{w}/v + v/2)}{12v\bar{k}_0^3}K_3(\bar{k}_0\varrho) \tag{22}$$

$$|a_{1s,2p\pm 1}| = \frac{\varrho^3(ZZ')^{5/2}Z}{12v\bar{k}_0^2}K_2(\bar{k}_0\varrho) \tag{23}$$

$$| a_{1s,3s} | = \frac{2}{3\sqrt{3}} \frac{(ZZ')^{5/2} \varrho^2}{v \bar{k}_0^2} \left| K_2 (\bar{k}_0 \varrho) - \frac{8 Z^2}{3^4 \bar{k}_0} \varrho K_3 (\bar{k}_0 \varrho) + \right.$$

$$\left. + \frac{Z^4 \varrho^2}{3^6 \bar{k}_0^2} K_4 (\bar{k}_0 \varrho) \right| \tag{24}$$

$$| a_{1s,3p_0} | = \frac{2^3}{3^4 \sqrt{2}} \frac{(ZZ')^{5/2} Z (\bar{w}/v + v/2) \varrho^3}{v \bar{k}_0^3} | K_3 (\bar{k}_0 \varrho) - \frac{Z^2 \varrho}{36 \bar{k}_0} K_4 (\bar{k}_0 \varrho) | \tag{25}$$

$$| a_{1s,3p\pm 1} | = \frac{2^2}{3^4} \frac{(ZZ')^{5/2} Z \varrho^3}{v \bar{k}_0^2} \left| K_2 (\bar{k}_0 \varrho) - \frac{Z^2 \varrho}{36 \bar{k}_0} K_3 (\bar{k}_0 \varrho) \right| \tag{26}$$

$$| a_{1s,3d_0} | = \frac{2^3 (ZZ')^{5/2} Z^2 \varrho^3}{3^5 \sqrt{6}\, v \bar{k}_0^3} \left| \left[2 \left(\frac{\bar{w}}{v} + \frac{v}{2} \right)^2 + \bar{k}_0^2 \right] \frac{\varrho}{2^3 \bar{k}_0} K_4 (\bar{k}_0 \varrho) - \right.$$

$$\left. - K_3 (\bar{k}_0 \varrho) \right| \tag{27}$$

$$| a_{1s,3d\pm 1} | = \frac{(ZZ')^{5/2} Z^2 \varrho^4}{3^5 v \bar{k}_0^3} \left(\frac{\bar{w}}{v} + \frac{v}{2} \right) K_3 (\bar{k}_0 \varrho) \tag{28}$$

$$| a_{1s,3d\pm 2} | = \frac{(ZZ')^{5/2} Z^2 \varrho^4}{2 \cdot 3^5 v \bar{k}_0^2} K_2 (\bar{k}_0 \varrho) \tag{29}$$

$$\bar{k}_0^2 = \left(\frac{\bar{w}}{v} + \frac{v}{2} \right)^2 + \frac{Z^2}{n^2}, \quad \bar{w} = \frac{(Z')^2}{2(n')^2} - \frac{Z^2}{2n^2} = I' - I \tag{30}$$

where $K_n (x)$ is the MacDonald function; in Eqs. (20—29) $n' = 1$, $I' = Z^2/2$.

The analogous formulae may be obtained for the capture from L —, M — etc. target shells to the different states nlm.

The BK cross sections, calculated in the impact parameter approximation (2), (9), give the same results as those in quantum-mechanical treatment[5] in the whole range of velocity v with asymptotic:

$$\sigma (v) \sim (nn')^{-3} Z^{5+2l} (Z')^{5+2l'} v^{-2(6+l+l')}, \quad v \to \infty. \tag{31}$$

For different applications it is useful also to know analytical expressions for the capture probabilities and cross sections, summed over all quantum numbers lm, i. e. for the reaction

$$Z + H_{Z'} (n') \to Z' + H_Z (n).$$

Calculation of the sum $\sum_{l'm'\,lm} |a^{BK}_{n'l'm',nlm}(\varrho, v)|^2$ directly from (3) seems to be not possible. However, using Eqs. (2, 4, 5, 18, 19) one can obtain:

$$f_{n'n}(q, v) = \frac{(4\pi)^2\, 8\, (ZZ')^{5/2}}{(n'n)^{3/2}\left[q^2 + (\bar{w}/v + v/2)^2 + \frac{Z^2}{n^2}\right]^3} \tag{32}$$

$$W_{n'n} = |\, a_{n'n}(\varrho, v)\,|^2 =$$

$$= \frac{4\,(ZZ')^5\, \varrho^4}{(n')^2\,(nn')^3\left[(\bar{w}/v + v/2)^2 + \frac{Z^2}{n^2}\right]^2} K_2^2\left(\varrho\sqrt{(\bar{w}/v + v/2)^2 + \frac{Z^2}{n^2}}\right) \tag{33}$$

where $\bar{w}$ is defined in (30).

Cross sections, calculated with (32) or (33), give the same result as was obtained in Ref. 2:

$$\sigma(n' - n) = \pi a_0^2 \cdot \frac{2^8\, N\,(ZZ')^5}{15 v^2\, n^3\,(n')^5}\left[\left(\frac{\bar{w}}{v} + \frac{v}{2}\right)^2 + \frac{Z^2}{n^2}\right]^{-5}. \tag{34}$$

It is interesting to note that although

$$\sum_{l'm'lm} |a_{n'l'm',\,nlm}(\varrho, v)|^2 \neq |a_{n'n}(\varrho, v)|^2$$

(except $n' = 1 \to n = 1$ transition) corresponding cross sections are equal, i.e.

$$\frac{2N}{3}\sum_{l'm'lm}\int_0^\infty \varrho\, \mathrm{d}\varrho\, |a_{n'l'm',nlm}(\varrho, v)|^2 = \frac{2N}{3}\int_0^\infty \varrho\, \mathrm{d}\varrho\, |a_{n'n}(\varrho, v)|^2 = \sigma(n' - n) \tag{35}$$

where $a_{n'l'm'\,nlm}$, $a_{n'n}$ and $\sigma(n' - n)$ are given in (2), (33) and (34), respectively.

3. *Numerical calculations*

Normalised (7, 8) and BK (9) cross sections were computed using analytical formulae (20—29, 33) for charge exchange between H^+, He^+, He^{2+}, Li^{2+} ions and H, He atoms and for 1*s*-electron capture by multiply charged ions in the velocity region $v = 0.01$—10 a. u. Calculated partial and the total cross sections are given in Figs. 1—10 together with other theoretical results and experimental data available. In the present calculations we put in Eq. (3) $\tilde{n}_{max} = 10$ with expression $W_{n',n}(\varrho, v)$ given in (33) for $\tilde{n} > 3$ and effective charge $Z = n\sqrt{2I}$, where I is the bound energy.

The total capture cross sections for $H^+ + H$ collisions are given in Fig. 1. It is seen the surprisingly good agreement of the calculated cross section with other theoretical results and experiment in the whole energy range. At $v > 2$ a. u. the normalised cross section σ^N is equal to BK cross section (9): $\sigma^{BK}/\sigma^N = 1$; at $v = 0.01$ the discrepancy is much larger $\sigma^{BK}/\sigma^N \approx 10^4$.

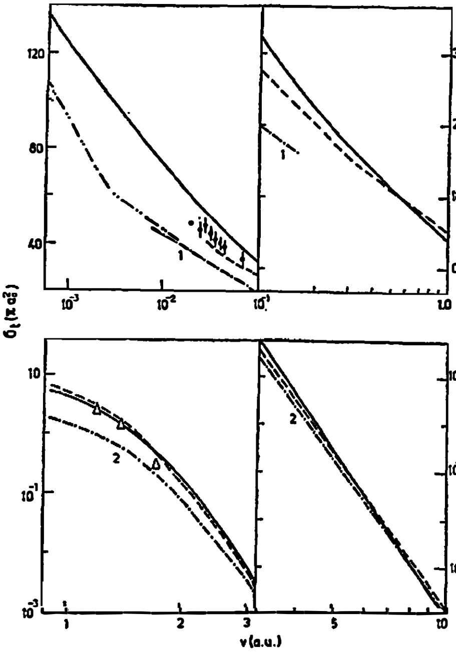

Fig. 1 Total charge transfer cross sections in $H^+ + H$ collisions. Calculated: 1-[12], 2-the eikonal approximation[13]; — ·· — the perturbed stationary states method[14]; △-a three-dimensional Monte Carlo approach[15]; present work: —— normalised cross sections (11, 8). Observed: ...[16], 000 for $D^+ + D$ reaction[16], — — — average of experimental results[17].

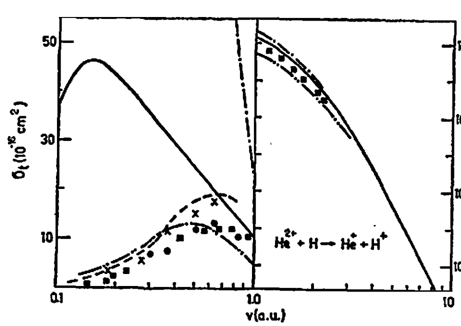

Fig. 2. Total one-electron capture cross sections in $He^{2+} + H$ collisions. Calculated: — — — molecular treatment[6]; present work: — · — corrected BK approximation (9), —— normalised cross section (11, 8). Observed: × × ×[18]; ...[19]; ■ [20].

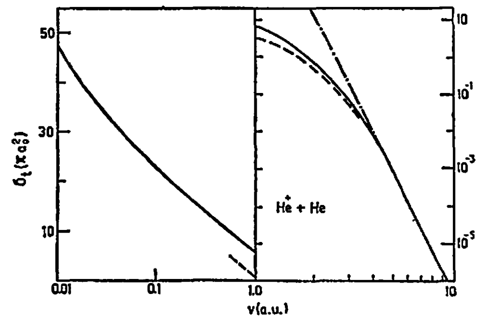

Fig. 3. Total one-electron capture cross sections in $He^+ + He$ collisions. Observed: — — — average of experimental results[17]; Calculated: present work: —·—·— BK approximation (9), —— normalised cross section (11, 8); in the scale of the figure experimental results are in good agreement with calculations by modified Bates approximation[21] in the region $v = 0.7$—3 a. u.

The total cross sections for $He^{2+} + H$ collisions are given in Fig. 2. In this case the quasi-resonant reaction $He^{2+} + H \rightarrow He\,(n = 2) + H^+$ with small $w \approx$ ≈ 0.05 a. u. plays an important role, therefore it is necessary to make the normalisation of the capture cross sections. At $v < 0.5$ a. u. the agreement of experimental data with results, performed by more accurate, but complicated methods (for example, by the molecular treatment[6]) is better. The influence of the normalisation effect is illustrated in Figs. 3—5, where the total cross sections for $He^+ + H$, $H^+ + He$, and $He^{2+} + He$ reactions are given.

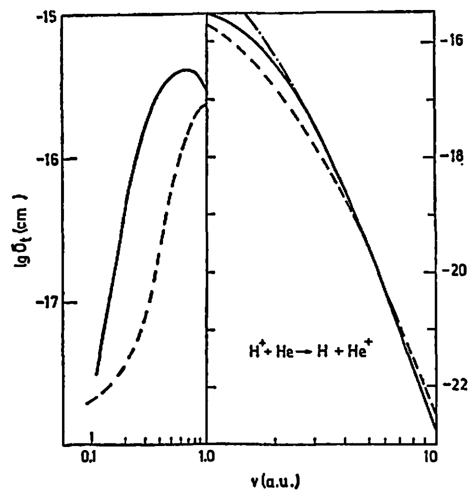

Fig. 4. Total one-electron capture cross sections in $H^+ + He$ collisions. Observed: — — — average of experimental results[17,22]. Calculated: —·—·— BK approximation (9), —— normalised cross section (11, 8) -present work; in the scale of the figure experimental results are in good agreement with calculations by Born approximation[23] in the region $v = 1 - 3$ a. u.

Partial cross sections are given in Figs. 6—8. It is seen that the pure BK approximation does not describe in principle the region of the maximum of the cross sections, while normalisation procedure leads to the better agreement with experi-

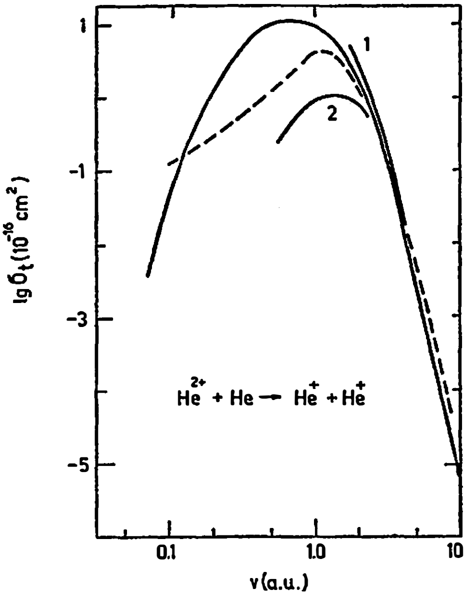

Fig. 5. Total one-electron capture cross sections in He^{2+} + He collisions. Calculated: the continuum distorted wave method[24]; 2-modified Bates approximation[21]; present work: —— normalised cross section (11, 8). Observed: — — — average of experimental results[17,21].

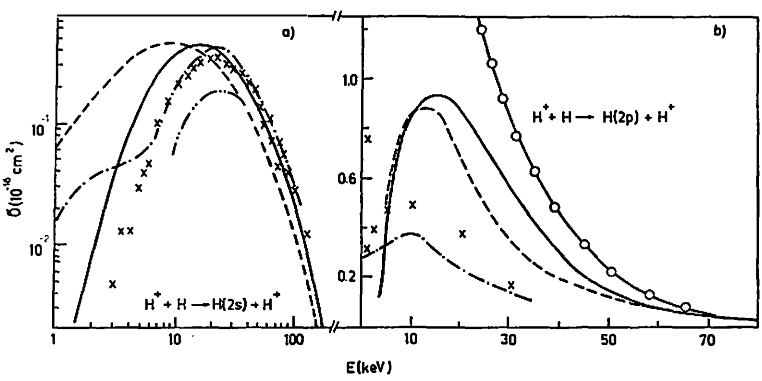

Fig. 6. Partial cross sections for reactions $H^+ + H(1s) \to H(2s) + H^+$ (*a*) and $H^+ + H(1s) \to$ $\to H(2p) + H^+$ (*b*).

a. Calculated: — — — Born approximation[26]; — · — pseudo-state expansion method[27]; the eikonal approximation[13]; present work: —— normalised cross section (10, 8). Observed: xxx from Ref. 17.

b. Calculated: — — — Born approximation[28]; — · — impulse approximation[29]; present work: —o— BK approximation (9), — — — normalised cross section (11, 8). Observed: x x x[30].

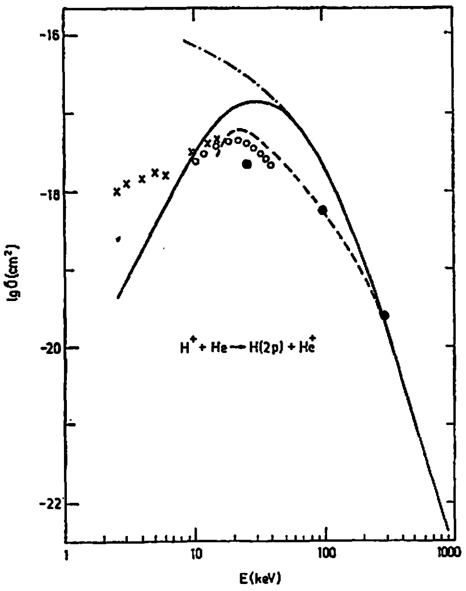

Fig. 7. Capture cross sections for reaction $H^{+} + He \rightarrow H(2p) + He^{+}$.
Calculated: ... 11-state close coupling[30]; present work: —·—·— BK approximation (9), —— normalised cross section (10, 8). Observed: × × ×[31], 0 0 0[32], — — — from Ref. 17.

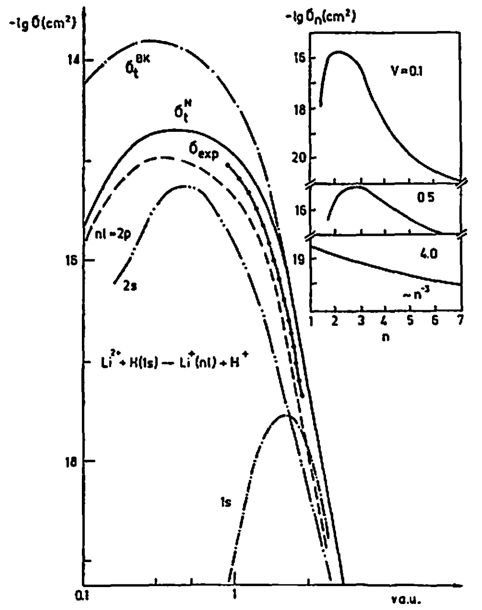

Fig. 8. Partial and the total capture cross sections in $Li^{3+} + H$ collisions. Observed total: — · —[33]
Calculated: present work: —·—·— BK approximation (9), —— normalised cross section (11, 8). On the right the distribution on principal quantum number n is also shown at different velocities v (Eqs. (8, 10, 33)).

ment. In Fig. 8 it is also shown the distribution of the cross sections on the principal quantum numbers n in reaction $Li^{2+} + H \rightarrow Li^{+}(n) + H^{+}$ at different relative velocities $v = 0.1$, 0.5 and 4.0 a. u.

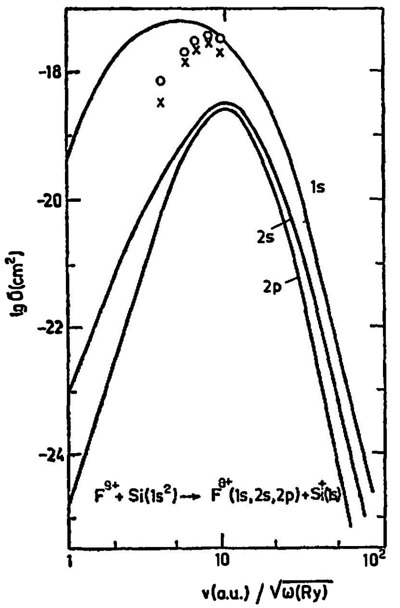

Fig. 9. $1s$-electron capture cross sections for reaction $F^{9+} + Si(1s^2) \rightarrow F^{8+}(1s, 2s, 2p) + Si^{+}(1s)$. Calculated: 0 0 0 $K - K$ capture cross section, modified Bates approximation[34]; present work: —— normalised cross section (11, 8). Observed $K - K$ reaction: × × ×[35] (1 Ry = 13.6 eV).

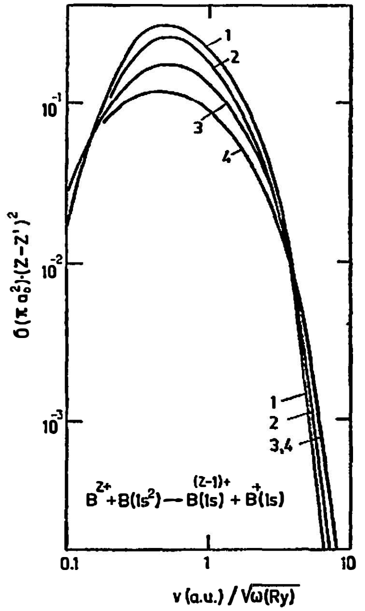

Fig. 10. Scaled $K - K$ capture cross sections in collisions of fully-stripped ions with identical atoms, present work Eqs. (10, 8): 1-$Ne^{10+} + Ne$, 2 — $Ar^{18+} + Ar$, 3 — $Kr^{36+} + Kr$, 4 — $Xe^{54+} + Xe$ (1 Ry = = 13.6 eV).

In Figs. 9, 10 the K-electron capture cross sections by multiply charged ions are given for reactions with relatively small energy defects $w = (Z')^2/2 - Z^2/2 = I' - I \ll I', I$. In the region of maximum, the BK cross sections are 3—5 times higher than calculated by the present method. Calculations showed that $K - K$ capture cross sections reach their maximum σ_{max} at the velocity v_{max}, defined by

$$v_{max} \approx \sqrt{w} \tag{36}$$

and

$$\sigma_{max} = \begin{cases} 0.1\,(Z' - Z)^2\,\pi a_0^2, & \text{if } Z \approx Z' \\ 32\,Z^5/(Z')^7\,\pi a_0^2, & \text{if } Z' > Z. \end{cases} \tag{37}$$

Formulae (36, 37), obtained from analysis of the normalised cross sections, give the same results as obtained in Refs. 2, 6, 7 for K-electron capture by protons.

These formulae aslo may be considered as a scaling law for $K - K$ electron capture. For the total cross sections σ_t the scaling coordinates are[8)]

$$\sigma_t/\sqrt{I}, \quad v^2/\sqrt{Z}, \quad Z = n\sqrt{2I}$$

and correspond to the asymptotic

$$\sigma_t \approx 1.88 \cdot 10^{-4} \frac{N\,(I'/\mathrm{Ry})^{5/2}\,(I/\mathrm{Ry})\,\mathrm{cm}^2}{(E^H\,(\mathrm{keV})/\sqrt{Z})^6}, \quad E\,(\mathrm{Kev}) > 25\,I/\mathrm{Ry} \tag{38}$$

where 1 Ry = 13.6 eV.

4. *Conclusion*

Normalisation procedure, used in the present paper, is very important for transitions with a small energy defect w in the region of velocities, defined by semiclassical limits:

$$\sqrt{2\bar{w}/M} < v < \sqrt{2\bar{w}},$$

where M is the reduced mass of colliding particles. The smaller $\bar{w}$, the more levels of the resulting ion $B^{(Z-1)+}$ one must take into account to obtain normalised capture probability (8). At $v \gg \sqrt{2\bar{w}}$ the procedure of normalisation gives the same results as corrected BK approximation (9).

As was shown above, the normalisation of the capture cross sections reduces the BK cross sections in the region of maximum, $v \sim (0.3–3)\sqrt{\bar{w}}$, for all reactions considered. However, in a large number of transitions, for instance, involving multiply charged ions, the charge transfer cross sections may exceed the corresponding BK cross sections at low energies, if the ionisation potential of the resulting ion $B^{(Z-1)+}$ is much higher than that of the target atom A. For such processes it is necessary to take into account the contribution from nonadiabatic transitions near quasi-intersections of electronic levels of the 'ion + atom' system (see, for example Refs. 10, 11). It is possible, in principle, to include the contribution from such semicrossings in the framework of the normalisation method, considered here, but this question will be discussed in another paper.

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References

1) V. P. Shevelko, J. Phys. **B 13** (1980) L319;
2) V. P. Shevelko, Zs. Phys. **A 287** (1978) 19;
3) R. M. Drisko, Thesis, pp 26—28, Carnegie Inst. Tech. 1955;
4) R. M. May, Phys. Rev. **136 A** (1964) 669;
5) D. R. Bates and R. McCarroll, Adv. Phys. **11** (1962), 39;
6) R. D. Piacentini and A. Salin, J. Phys. **B 7** (1974) 1666; ibid. **10** (1977) 1515;
7) C. L. Cocke, R. K. Gardner, B. Curnutte, T. Bratton and T. K. Saylor, Phys. Rev. **A 16** (1977) 2248;
8) H. Tawara, *Atomic Collision Research in Japan*. Progr. Rep. No5, (1979) 79;
9) A. Müller and V. P. Shevelko, Sov. Phys. — JTP, **50** (1980) 985; GSI-tr-80/13 (1980);
10) L. P. Presnyakov and A. D. Ulantzev, Sov. J. Quant. Electronics, **4** (1975) 1320;
11) H. Ryufuku and T. Watanabe, Phys. Rev. **A 18** (1978) 2005; ibid. **20** (1979) 1828;
12) F. J. Smith, Proc. Phys. Soc. **92** (1967) 866;
13) D. P. Dewangen, J. Phys. **B 10** (1977) 1083;
14) A. V. Matveenko and L. I. Ponomarev, Sov. Phys. — JETP, **57** (1969) 2084;
15) R. E. Olson, and A. Salop, Phys. Rev. **A 16** (1977) 531;
16) V. A. Belyaev, B. G. Brezhnev and E. M. Erastor, Sov. Phys. — JETP, **25** (1967) 777;
17) H. S. W. Massey and H. B. Gilbody, *Electronic and Ionic Impact Parameter*, Vol. 4 (Oxford—Clarendon), 1974;
18) W. L. Fite, A. C. H. Smith, R. F. Stebbings and J. A. Rutherford, J. Geophys. Res. **68** (1963) 3225;
19) M. B. Shah and H. B. Gilbody, J. Phys. **B 7** (1974) 636;
20) M. B. Shah and H. B. Gilbody, J. Phys. **B 11** (1978) 121; ibid. p. 1459.
21) M. J. Fulton and M. H. Mittelman, Proc. Phys. Soc. **87** (1966) 669;
22) N. B. Fedorenko, Sov. Phys. — JTP, **15** (1971) 1947;
23) R. A. Mapleton, Phys. Rev. **122** (1961) 528;
24) Dž. S. Belkić, and R. K. Janev, J. Phys. **B 6** (1973) 1020;
25) P. Hvelplund, J. Hienemeier, E. Horsdal Pedersen and F. R. Simpson, IX ICPEAC, Seattle, p. 186 (Univ. Wash. Press. 1975);
26) R. A. Mapleton, Phys. Rev. **124** (1962) 1477;
27) I. M. Cheshire, D. F. Gallagher and A. J. Taylor, J. Phys. **B 3** (1970) 813;
28) D. R. Bates and A. Dalgarno, Proc. Phys. Soc. **A 66** (1953) 972;
29) J. P. Coleman and S. Trelease, J. Phys. **B 1** (1968) 172;
30) R. A. Young, R. F. Stebbings and J. W. McGowan, Phys. Rev. **171** (1968) 85;
31) T. G. Winter and C. C. Lin, Phys. Rev. **10** (1974) 1241;
32) J. S. Risley, F. J. de Heer and C. B. Kerkdijk, J. Phys. **B 11** (1978) 1759;
33) E. O. Andreev, V. A. Ankudinov and S. V. Bobashev, Sov. Phys. — JETP, **50** (1965) 565;
34) M. B. Shah, T. V. Goffe and H. B. Gilbody, J. Phys. **B 11** (1978) L 233;
35) C. D. Lin, J. Phys. **B 11** (1978) L 185;
36) H. Tawara, P. Richard, T. Gray, J. Newcomb, K. A. Jamison, C. Schmiedekamp and J. M. Hall, Phys. Rev. **A 18** (1978) 1373.

UNITARIZOVANI PRESECI IZMENE NAELEKTRISANJA U IMPAKT-PARAMETARSKOJ REPREZENTACIJI

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Koristeći relaciju između amplituda prvog reda za proces izmene naelektrisanja u kvantno-mehaničkoj i impakt-parametar reprezentaciji, izvršena je unitarizacija (normiranje) efikasnog preseka ovog procesa. Demonstrirano je da se normalizovani efikasni preseci dobro slažu sa eksperimentalnim podacima i u oblasti niskih energija.