YU ISSN 0015-3206
FZKAAA 13 (3) 199 (1981)

# A SEARCH ON THE ENERGY DEPENDENCE OF THE PION INELASTICITY IN THE MODEL OF COCCONI-KOESTER-PERKINS

D. P. BHATTACHARYYA and S. C. MUKHERJEE

*Department of Theoretical Physics*

*Indian Association for the Cultivation of Science*

*Jadavpur, Calcutta* 700032, *India*

and

R. K. ROYCHOUDHURY

*Department of Physics*

*University of Durham*

*Durham, U. K.*

Received 4 May 1981

Revised manuscript received 25 June 1981

UDC 539.12

Original scientific paper

A search has been made on the energy dependence of the pion inelasticity $K_\pi$ in the fireball model of Cocconi, Koester and Perkins. The two sets of primary cosmic ray spectra viz. of Durham and Goddard groups along with the sea level muon spectrum of Kiel and Durham have been used to search the energy dependence of $K_\pi$. It is found that Durham primary spectrum favours the energy dependence of $K_\pi$ in the Cocconi, Koester and Perkins model reasonably. Attention has been paid on the comments made earlier by Chakraborty, Das and De who failed to realise the motivation of our previous work (Sarkar et al.).

## 1. Introduction

The two fireball CKP model[1] usually used in high energy cosmic hadron cascade calculations is ruled out by the *Interacting Storage Ring* data due to its absence of a dip near the region of the Feynman variable $x = 0$ which would be required in the model by the separation between the forward and backward fireballs of produced pions. Gaisser[2] have shown that CKP model differs markedly from the scaling distribution; in particular, there are virtually no fast secondaries. The CERN ISR data at 1000 GeV laboratory energy have shown that the CKP model to be incorrect, its use suggests that momentum distribution of fast secondaries may be at least as important as the multiplicity law in determining shower development in the atmosphere. However the CKP relation has a simple phenomenological basis and also is being widely used even today in the cosmic ray cascade calculations.

In an earlier investigation Brooke et al.[3] have estimated the primary nucleon spectrum from the sea level muon spectrum using CKP model. They have shown that the values of $K_\pi$ has an energy dependence in the range 5 to 50 TeV and the value of pion inelasticity increases from 0.2 to 0.32. This fact resurrected our interest with the use of recent cosmic ray spectra along with the machine yield interaction parameters at high energies. In an earlier work[4] we have used the primary cosmic ray nucleon spectrum of Ryan et al.[5] (to be referred to as ROB) whose spectral index was 2.75 which is very high as has been found from the later analysis of the direct measurement of the energy spectrum of primary cosmic ray particles and the associated problems of mass composition by Olejniczak, Wdowczyk and Wolfendale[6] (to be referred to as OWW). Later Popova[7] estimated a composite spectrum of the primary cosmic ray particles proposed by OWW which is found to follow the form:

$$N(E)\,\mathrm{d}E = (0.551\,E^{-2.6} + 0.19\,E^{-2.6} + 0.342\,E^{-2.6} + 0.266\,E^{-2.6} + \\ + 0.551\,E^{-2.6})\,\mathrm{d}E \qquad (1)$$

where $E$ is the nucleon energy expressed in GeV and $N(E)\,\mathrm{d}E$ is the differential primary intensity expressed in $(\mathrm{cm}^2\,\mathrm{sec\,sr\,GeV})^{-1}$. The above relation represents the contribution to the energy spectrum of five groups of primaries with the following atomic numbers: 1, 4, 10, 26 and 56.

Since a consistent theory of high energy interaction is lacking, the simple phenomenological model like CKP has been used widely in the cosmic ray propagation inspite some obvious drawbacks of the model. In a recent paper Chakraborty et al.[8] have analysed our earlier preliminary investigation[4] critically but they were rather confused as to the real motivation of our application of CKP model. They stressed about the agreement of CKP model with the correlation of cosmic ray data viz. with the correlation of primary nucleon spectrum of Ryan et al.[5] with the ground level muon spectrum of Allkofer et al.[9] and Ayre et al.[10]. Our main treatment was on the energy dependence of the pion inelasticity which can throw some light on the sharing of more energy from the incident nucleons at high energies.

## 2. *Formulation*

A) *Pion spectrum derived from the sea level muon spectrum*

Recently Chakraborty et al.[8] have discussed the conventional solution of the pion atmospheric diffusion equation which relates the sea level muon intensity $\mu(E, x_0)\,\mathrm{d}E$ with the pion spectrum $P(E/r)\,\mathrm{d}E$ at the top of the atmosphere (near atmospheric depth 100 g-cm$^{-2}$) by the following relation

$$\mu(E, x_0)\,\mathrm{d}E = \frac{P(E/r)\, r^{-1}\, B_\pi\, r\, h(E, \mathrm{d}E/\mathrm{d}x)\, y(K/\pi, E)\,\mathrm{d}E}{\lambda_N (B_\pi r + E)} \qquad (2)$$

where

$$\Lambda = \lambda_\pi = 120\ \text{g-cm}^{-2},\ r = 0.76,\ x_0 = 1033\ \text{g-cm}^{-2},$$

$$B_\pi = 118\ \text{GeV},\ \lambda_N = 90\ \text{g-cm}^{-2}.$$

If one fits the derived pion spectrum by a particular spectral index $\gamma_\pi$ viz. of the form $P(E)\,\mathrm{d}E = A\,E^{-\gamma_\pi}\,\mathrm{d}E$ in a specific energy interval the relation follows:

$$\mu(E, x_0)\,\mathrm{d}E = \frac{P(E)\, r^{\gamma_\pi - 1}\, B_\pi\, r\, h(E, \mathrm{d}E/\mathrm{d}x)\, y(K/\pi, E)}{\lambda_N (B_\pi r + E)}\,\mathrm{d}E. \qquad (3)$$

Using the above mentioned formulae the pion spectra at the depth 100 g-cm$^{-2}$ have been estimated (by the relation (2) and (3) with $\gamma_\pi = 3$) and has been displayed in Fig. 1. The plot shows that the derived pion spectrum is almost same above 50 GeV energy. Below this the spectrum of pions derived after relation (3) overestimates that derived by (2). This does not affect our present discussion.

B) *Pion spectrum from the primary spectrum using CKP model*

The conventional CKP model[1] (cited by Brooke et al.[3]) related the pion spectrum at production from the primary nucleon spectrum by the relation

$$P(E_\pi)\,\mathrm{d}E_\pi = \frac{6a^2 B}{K_\pi}\,\mathrm{d}E_\pi \int_{3E_\pi}^{\infty} E_p{}^{-\gamma_p - 0.5}\, \exp\left(-\frac{3a\,E_\pi}{K_\pi\, E_p^{3/4}}\right) \mathrm{d}E_p \qquad (4)$$

where $a = 0.45$, $B$ = amplitude of the primary spectrum whose spectral index is $\gamma_p$.

We have estimated the pion spectra from the primary cosmic ray spectra viz. of Ryan et al.[5] and Olejniczak et al.[6] with fixed and variable pion inelasticities $K_\pi$ and using relation (2) the sea level muon spectra have also been estimated.

The primary spectra of ROB and OWW have been displayed in Figs. 2 and 3. The pion spectrum derived after CKP model with variable $K_\pi$ (as in Fig. 4) has been displayed along with the calculated sea level muon spectra. $\mu_2$ and $\mu_1$ for fixed and variable values of $K_\pi$ (using formula (2)) have also been presented in Figs. 2 and 3. Table 1 shows the spectral amplitudes along with the spectral index $\gamma_p$, nucleon inelasticities $K_T$ and pion spectral index $\gamma_\pi$.

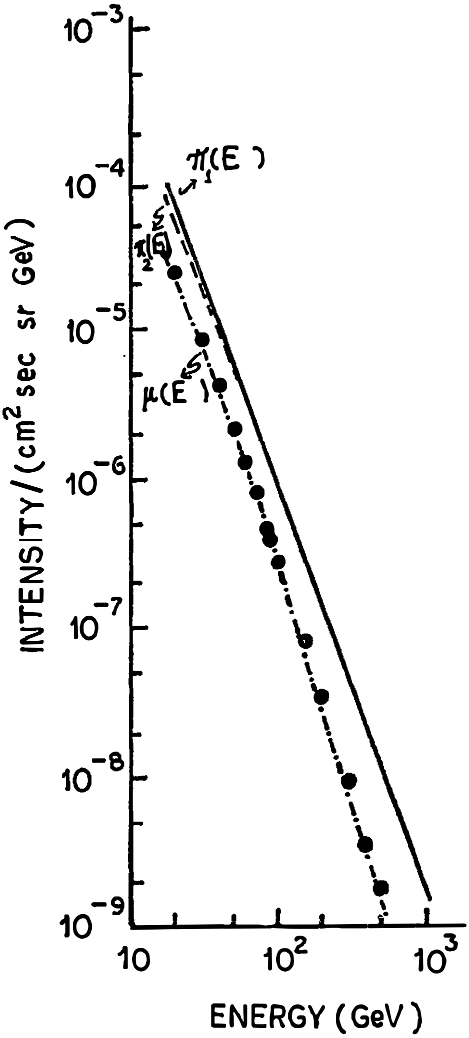


Fig. 1. The spectra of muons and pions: Fit to the experimental muon intensity data of Durham[10] chain curve; the derived pion spectra after relations (2) and (3) are the full and broken curves, respectively. Magnetic spectrograph data[10]—● · .

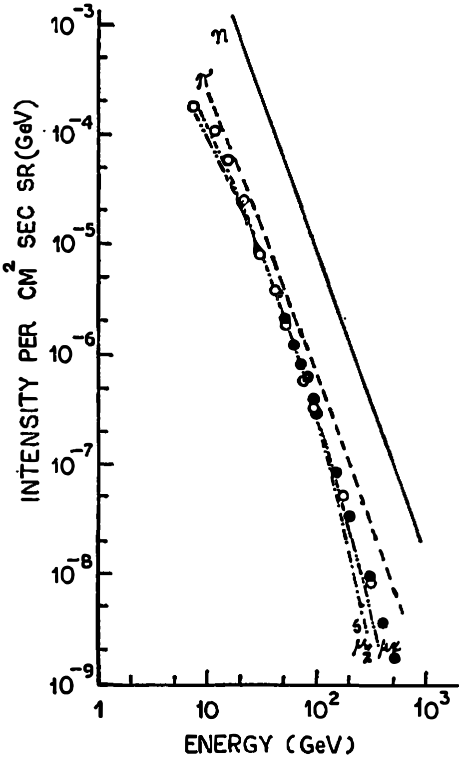


Fig. 2. The spectra of primary nucleons, pions and muons: ($n$) Primary nucleon spectrum after Ryan et al.[5] ————; ($\pi$) the derived pion spectrum from CKP model[1] with energy dependence of $K_\pi$ (as shown in Fig. 4) – – – –; $\mu_1$ is the derived sea level muon spectrum with energy dependence of $K_\pi$ — · · — · · — · · —; $\mu_2$ represents the muon spectrum for $K_\pi =$ $= 0.36$ —·—·—·; Magnetic spectrograph data: 0 — Allkofer et al.[9] and ● — Ayre et al.[10].

Table 1

| Author | $B$ | $\gamma_p$ | $\gamma_\pi = 4(\gamma_p - 0.5)/3$ | $K_T$ |
|---|---|---|---|---|
| Ryan et al.[5] | 2.66 | 2.75 | 3.0 | 0.547 |
| Olejniczak et al.[6] | 1.90 | 2.60 | 2.8 | 0.580 |

The derived muon spectrum for fixed $K_\pi = 0.36$ ($\mu_2$) lies appreciably below the measured spectra of Allkofer et al.[9] and Ayre et al.[10]. The calculated muon spectrum ($\mu_1$) (for variable pion inelasticity $K_\pi$) agrees with the magnetic spectrograph data[9,10] satisfactorily only when one assumes an energy dependence in pion inelasticity $K_\pi$ (as in Fig. 4). Earlier Brooke et al.[3] have found such increase in the range 0.5 — 50 TeV. But the present study indicates that this phenomena arises even at lower energies. The comparison of the derived muon spectra after

CKP model (with energy dependence of $K_\pi$ in Fig. 4) from the Durham muon spectrum[10] has been displayed in Fig. 5. The plot indicates that for fixed pion inelasticities $K_\pi = 0.36$ the deviation of the calculated muon spectrum from the ROB and OWW primary spectra inputs are appreciable. But the agreement between the theoretical results and measured ones is satisfactory only when the $K_\pi$ was varied from 0.26 — 0.36 (in OWW spectrum) in the spectral range 10 — 400 GeV. ROB hadron source spectrum yield rather higher values of $K_\pi$ viz. 0.3 — 0.52 which is incompatible with the machine data and not in accord with $K_T = 0.547$.

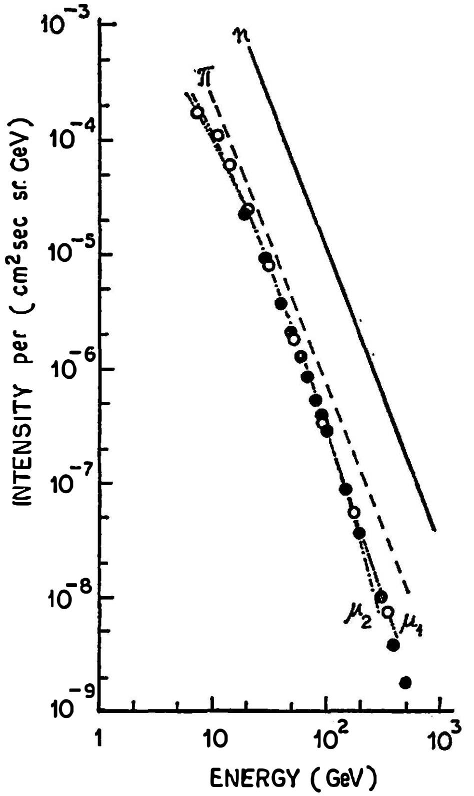


Fig. 3. The spectra of primary particles, pions and muons: ($n$) Primary OWW spectrum [6]———; ($\pi$) the derived pion spectrum from CKP model[1] with energy dependence of $K_\pi$ (as shown in Fig. 4) — — — — — ; $\mu_1$ is the derived sea level muon spectrum with energy dependence of $K_\pi$—·· —··—··—··; $\mu_2$ represents the muon spectrum for $K_\pi = 0.36$—·—·—; Magnetic spectrograph data: 0 — Allkofer et al.[9] and ● — Ayre et al. [10].

The present survey indicates that the OWW spectrum when related to the sea level muons spectrum via CKP model yield en energy dependence of $K_\pi$ which is reasonable from the experimental aspects.

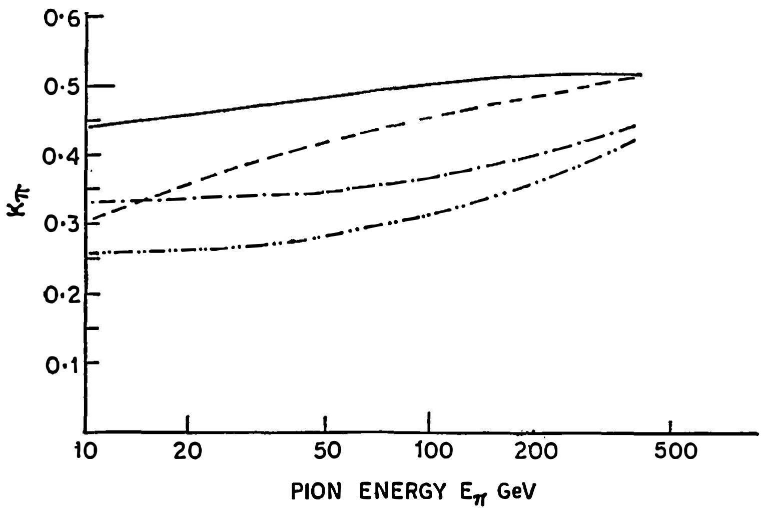


Fig. 4. Dependence of the pion inelasticity $K_\pi$ on the mean pion energy: a) Using ROB spectrum[5] and relation (2) ——— and by relation (3) – – – –; b) Using OWW spectrum[6] and relation (2) ·—·—·—· and by relation (3) —··—··—··—.

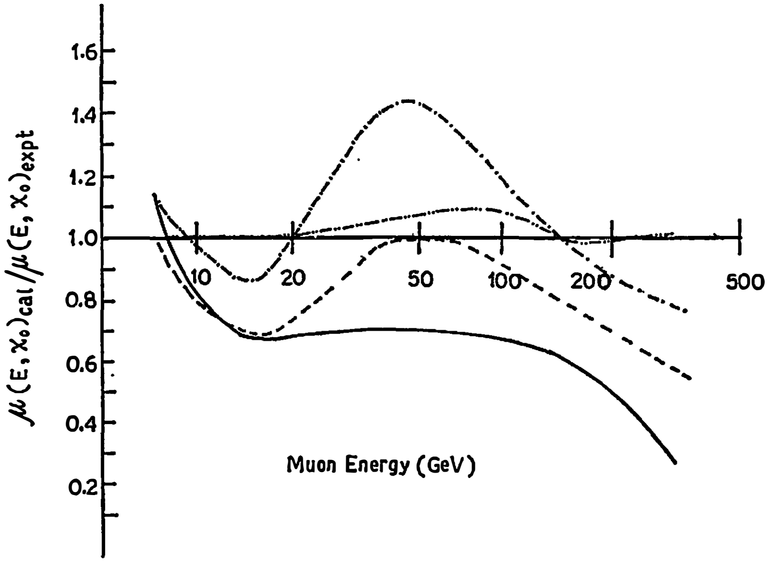


Fig. 5. Comparison of the derived muon spectra with the measured Durham muon spectrum of Ayre et al.[10], (Durham intensity ≡ 1); a) Using ROB spectrum[5] with fixed $K_\pi = 0.36$ ———; for variable $K_\pi$ – – – – –, b) Using OWW spectrum [6] with fixed $K_\pi = 0.36$ —· —·—·— for variable $K_\pi$ —··—··—.

References


1) G. Cocconi, L. J. Koester and D. H. Perkins, Lawrence Radiation High Energy Physics Seminars **28**, 2, UCIDD 144, 1—36 (1961);
2) T. K. Gaisser, Journal of Franklin Institute **298** (1974) 271;
3) G. Brook, P. J. Hayman, Y. Kamiya and A. W. Wolfendale, Proc. Phys. Soc. (London) **83** (1964) 853;
4) K. Sarkar, D. P. Bhattacharyya and D. Basu, Canadian Journal of Physics **55** (1977) 629;
5) M. J. Ryan, J. F. Ormes and V. K. Balasubrahmanyan, Phys. Rev. Lett. **28** (1972) 985;
6) J. Olejniczak, J. Wdowczyk and A. W. Wolfendale, Journal of Phys. G (Nucl. Phys) **3** (1977) 847;
7) L. Popov, Bulg. J. Phys. **5** (1978) 563;
8) A. Chakraborty, A. K. Das and A. K. De, Canad. J. Phys. **57** (1979) 373;
9) O. C. Allkofer, K. Carstensen and W. D. Dau, Phys. Lett. **36** B (1971) 425;
10) A. C. Ayre, J. M. Baxendale, C. J. Hume, B. C. Nandi, M. G. Thompson and M. R. Whalley, Journal of Phys. G (Nucl. Phys.) **1** (1975) 584.


# ISTRAŽIVANJE ENERGETSKE OVISNOSTI PIONSKOG INELASTICITETA U MODELU COCCONI-KOESTER-PERKINS


D. P. BHATTACHARYYA i S. C. MUKHERJEE

*Department of Theoretical Physics,*

*Indian Association for Cultivation of Science,*

*Jadavpur, Calcutta* 700032, *India*

i

R. K. ROYCHOUDHURY

*Department of Physics,*

*University of Durham*

*Durham, U. K.*



UDK 539.12

Originalni znanstveni rad



Istraživana je energetska ovisnost pionskog inelasticiteta $K_\pi$ u modelu »fireball« Cocconia, Koestera i Perkinsa. Da se nađe energetska ovisnost $K_\pi$ upotrebljena su dva skupa podataka spektra primarnog kozmičkog zračenja odnosno Durham i Goddard grupa. Nađeno je da Durhamovi primarni spektri favoriziraju energetsku ovisnost $K_\pi$ danu modelom Cocconia, Koestera i Perkinsa. Poklonjena je pažnja komentaru Chakrabortya i suradnika kojima je promakla motivacija našeg prethodnog rada.