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THE ABSENCE OF ELECTRON MASS CREATION IN THE FIRST APPROXIMATION OF THE FINITE ELECTRON PROPAGATOR

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The electron propagator in the first approximation of the Schwinger-Dyson equation in finite QED is analysed. The absence of electron mass creation for space-time dimensions $d = 2, 3, 4$ is demonstrated.

1. Introduction

One of the permanent problems of elementary particle physics is the problem of mass spectrum and accordingly the problem of the role of fundamental interactions in the particle mass generation. This is also a very important problem of modern gauge theories of fundamental interactions and their unification. According to the fact that the electron is one of the fundamental constituents of matter and that quantum electrodynamics (QED) is the simplest physical model of gauge theory it is natural to start to consider the mass problem in the framework of QED.

One of the most attractive programmes for finite QED is that of Johnson, Baker and Willey[1] (JBW). The basic features of their model of finite QED are: a) it is better to make non-ordinary perturbation expansion of the Schwinger-Dyson (SD) equations for the electron propagator, the photon propagator and the vertex function than the ordinary S-matrix perturbation expansion; b) in each order of the new perturbation expansion one has to make an appropriate choice of the gauge parameter; c) the electron self-energy is finite if and only if the bare electron mass vanishes ($m_0 = 0$) and d) the photon propagator is finite if the bare fine-structure constant is a positive finite root of the definite equation $f(\alpha_0) = 0$. (As one could already conclude, the basic meaning of the notion »finite« is: finite before applying the renormalisation procedure). Consequently, the electron mass should be totally dynamical in origin and the JBW finite QED traces the way along which we can study electron mass creation.

In this paper we present the results of the investigation of the electron mass non-appearance in the first approximation of the SD equation for the electron propagator in the JBW perturbation expansion. The analysis has been performed comparatively for three different space-time dimensions ($d = 2,3$ and 4). Section 2. contains some properties of the Dirac γ-matrices and ultraviolet divergences which are necessary in this work. The absence of electron mass creation is demonstrated in the Section 3.

2. *The Dirac γ-matrices and ultraviolet divergences*

The Dirac γ-matrices in the d-dimensional energy-momentum space are defined by the same anticommutation relations as for $d = 4$

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2 g^{\mu\nu} \; (\mu, \nu = 0, 1, \dots, d-1) \tag{1}$$

where

$$g^{00} = - g^{11} = \dots = - g^{d-1,d-1} = 1; \; g^{\mu\nu} = 0, \; \mu \neq \nu.$$

The number of linearly independent γ-matrices and their representations in $d = 4$ are well known[2]. Some usual calculations for $d = 2,3$ give:

$$d = 2 \qquad \gamma^0 = \sigma_1, \; \gamma^1 = \mathrm{i}\,\sigma_2, \; \gamma^5 = -\gamma^0\gamma^1 = \sigma_3, \tag{2}$$

$$d = 3 \qquad \gamma^0 = \sigma_3, \; \gamma^1 = \mathrm{i}\,\sigma_1, \; \gamma^2 = \mathrm{i}\,\sigma_2, \; \gamma^5 = \mathrm{i}\,\gamma^0\gamma^1\gamma^2 = 1, \tag{3}$$

where σ_i (i = 1, 2, 3) are the Pauli matrices. Note that γ-matrices for $d = 2$ and $d = 3$ have 2×2 representation. The fact that $\gamma^5 = 1$ for $d = 3$ is the consequence of the even number of the momentum (spatial) coordinates. Hence, for $d = 2$ and $d = 3$ we have

$$Tr\, \gamma^\mu = 0, \; Tr\, 1 = 2, \tag{4}$$

$$Tr\, (\gamma^\mu \gamma^\nu) = 2 g^{\mu\nu}. \tag{5}$$

For each Feynman diagram (graph) G of an arbitrary dimension d, the index of the diagram[2] $w_d(G)$ can be introduced which is suitable to estimate its ultraviolet divergences

$$w_d(G) = \frac{n}{2}(d - 4) + d - \frac{1}{2} \sum_e (r_e + d - 2) \tag{6}$$

where n is the number of the diagram vertices, e enumerates all external lines and r_e is the degree of the polinom of the numerator of the propagator in the external line. There are two possible classes: a) $w_d(G) \geq 0$, the diagrams are divergent and b) $w_d(G) < 0$, the diagrams are convergent. When $d > 4$, it follows from (6) that $w_d(G)$ increases by increase of the vertices number n and the theory (QED) becomes unrenormalized. That is why we analyse only lower dimensions ($d \leq 4$).

For QED_4, $w_4(G)$ does not depend on the number of vertices but only on the external lines (we have in mind strongly connected diagrams). According to the dependence of (6) on n, the higher-order diagrams of QED_3 and QED_2 should be convergent. What we are further intersted is the divergence of the Feynman diagrams of the electron propagator $S(p)$. Since $r_e = 1$ for two electron lines ($e = 1$, 2) of the electron propagator, it follows

$$w_d(S) = \frac{n}{2}(d - 4) + 1. \tag{7}$$

It ih evident that:

$$w_4(S) = 1$$

$$w_3(S) = \frac{2 - n}{2}. \tag{8}$$

$$w_2(S) = 1 - n.$$

We can conclude that the electron propagator is: a) divergent for $n \geq 2$ orders of the Feynman diagrams in perturbation expansion of QED_4, b) divergent for only the second order diagram of QED_3 and c) convergent for all Feynman diagrams of QED_2.

3. *The Schwinger-Dyson equations*

The unrenormalized SD equation for the complete electron propagator[1] $S(p)$ in the d-dimensional QED is

$$S^{-1}(p) = S_0^{-1}(p) - \frac{i e_0^2}{(2\pi)^d} \int d^d q \, D_{\mu\nu}(p - q) \gamma^\mu S(q) \Gamma^\nu(p,q) \tag{9}$$

where $D_{\mu\nu}(k)$ is the complete photon propagator and $\Gamma^\nu(p, q)$ is the complete vertex function. The first approximation of Eq. (9) in the JBW perturbation expansion[1] is

$$S^{-1}(p) = S_0^{-1}(p) - \frac{i e_0^2}{(2\pi)^d} \int d^d p \, D^0_{\mu\nu}(p - q) \gamma^\mu S(q) \gamma^\nu \tag{10}$$

where the following approximations are made:

$$\Gamma^{\nu}(p, q) \to \gamma^{\nu} \tag{11}$$

$$D_{\mu\nu}(k) \to D^{0}_{\mu\nu}(k) = -\frac{1}{k^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) - \frac{d_l}{k^2}\frac{k_\mu k_\nu}{k^2}. \tag{12}$$

$D^{0}_{\mu\nu}(k)$ in Eq. (12) is the bare photon propagator and the gauge parameter d_l stands to take into account the longitudinal part of this propagator. Demanding the electron propagator to be finite, one fixes the value of d_l.

We shall be interested in the solution $S(p)$ of Eq. (10) in the well known form

$$S_0^{-1}(p) = \alpha(-p)^2 - \beta(-p)^2\,\hat{p} \tag{13}$$

where α and β are unknown scalar functions and $\hat{p} = p_\mu \gamma^\mu$. When the bare electron propagator is massless ($m_0 = 0$), we have

$$S_0^{-1}(p) = -\hat{p}. \tag{14}$$

Now there is a usual procedure to obtain the equations for the scalar functions $\alpha(-p)^2$ and $\beta(-p)^2$ from Eq. (10). Since this has been done in many papers[3)] for the case $d = 4$ and since there are no new special difficulties for the cases $d =$ $= 2, 3$, we shall only mention the principal points of the usual derivation. With the help of the trace techniques we obtain a pair of coupled d-dimensional integral equations for functions $\alpha(-p)^2$ and $\beta(-p^2)$. After the Wick rotation we can make the integration over the spherical angles in Euclidean momentum space. We obtain the following coupled non-linear integral equations:

a) for space-like values of variable $x = -p^2 = \vec{p}^{\,2} + p_0^2 > 0$, $(y > 0)$:

$$\alpha(x) = \frac{\alpha_0(d - 1 + d_l)}{(2\pi)^{d-1}} \int_0^\infty \frac{\alpha(y)\, y^{\frac{d}{2}-1}\, I_2^0(x, y)\, \mathrm{d}y}{\alpha^2(y) + y\,\beta^2(y)} \tag{15.a}$$

$$\beta(x) = 1 - \frac{\alpha_0(2-d)\,d_l}{(2\pi)^{d-1}\,x} \int_0^\infty \frac{\beta(y)\, y^{\frac{d}{2}-1}\, I_2^1(x, y)\, \mathrm{d}y}{\alpha^2(y) + y\,\beta^2(y)} \tag{16.a}$$

b) for time-like values of variable $x = -p^2 = -(p_0^2 + \vec{p}^2) < 0$, $(y < 0)$:

$$\alpha(x) = \frac{\alpha_0 (d - 1 + d_l)}{(2\pi)^{d-1}} \int_{-\infty}^{0} \frac{\alpha(y)\, y^{\frac{d}{2}-1} I_2^0(-x, -y)\, dy}{\alpha^2(y) + y\,\beta^2(y)} \tag{15.b}$$

$$\beta(x) = 1 + \frac{\alpha_0 (2 - d)\, d_l}{(2\pi)^{d-1} x} \int_{-\infty}^{0} \frac{\beta(y)\, y^{\frac{d}{2}-1} I_2^1(-x, -y)\, dy}{\alpha^2(y) + y\,\beta^2(y)} \tag{16.b}$$

where $\alpha_0 = e_0^2/4\pi$ is the bare fine-structure constant. The quantities which we denoted by $I_m^n(x, y)$ in Eqs. (15—16) are related to the integrations over the spherical angles and they are defined as follows:

$$I_m^n(x, y) = \int \frac{(pq)^n\, d^d\Omega}{(p - q)^m} = \int \frac{x^{\frac{n}{2}} y^{\frac{n}{2}} \cos^n(x, y)\, d^d\Omega}{(x + y - 2\sqrt{xy}\cos(x, y))^{\frac{m}{2}}} \tag{17}$$

$$(n = 0, 1, 2, \ldots;\ m = 0, 2, 4, \ldots)$$

where $d^d\Omega$ is the differential element of the d-dimensional space angle. Note that Eqs. (15.a) and (16.a) defined in the space-like region of the variables x and y, are obtained by the usual Wick rotation ($p_0 \to ip_0$, $q_0 \to iq_0$) in the complex p_0 — and q_0 planes. On the other hand, Eqs. (15. b) and (16. b), defined in the time-like region of the variables x and y, are obtained by the corresponding rotations ($\vec{p} \to -i\vec{p}$, $\vec{q} \to -i\vec{q}$) in the complex $\vec{p}$- and $\vec{q}$-planes.

It can be shown that the integrals in Eqs. (16.a) and (16.b) for $\beta(x)$ function are divergent for $d \geq 2$ (since $\alpha(x) \to 0$, $\beta(x) \to 1$ in asymptotics of x). Hence, we can conclude that $d_l = 0$ (the Landau gauge) is a necessary condition for the finiteness of the function $\beta(x)$, which now becomes

$$\beta(x) = 1. \tag{18}$$

Now, we are interested in the existence of the electron mass for time-like values of x the solutions of the SD equations (15.b) and (18). This problem is equivalent to the existence of the singular point in the electron propagator

$$\hat{S}(p) = \frac{\alpha(-p^2) + \hat{p}}{\alpha^2(-p^2) - p^2} \tag{19}$$

or to the existence of the zero of the equation

$$\alpha^2(x) + x = 0 \tag{20}$$

for time-like values of x ($x < 0$).

The most important is the 4-dimensional case where $\alpha(x)$ after some transformations[3] is brought to the form

$$\alpha(x) = \alpha(0) + g^2 \int_x^0 \frac{\alpha(y)}{\alpha^2(y) + y} \left(1 - \frac{y}{x}\right) \mathrm{d}y, \quad x \in (-\infty, +\infty), \tag{21}$$

where $\alpha(0)$ is an arbitrary parameter with dimension of mass and $g^2 = 3\,\alpha_0/4\pi$ is an effective coupling constant. The integral equation (21) can be transformed into the equivalent differential boundary value problem

$$(x\,\alpha)'' = -\,g^2 \frac{\alpha}{\alpha^2 + x}$$

$$x^2\alpha' \underset{x \to 0}{\to} 0, \qquad (x\alpha)' \underset{x \to \infty}{\to} 0. \tag{22}$$

Eqs. (21) and (22) are analysed in many papers and it is demonstrated by numerical[4] and analytical[5] methods that the electron mass does not appear in this approximation. The singular points do exist in the complex x-plane.

For more complete insight in the problem of the electron mass creation we shall analyse 3- and 2-dimensional cases. In the $d = 3$ case the equation for $\alpha(x)$ in a time like region of x is

$$\alpha(x) = \frac{\alpha_0}{2\pi\sqrt{x}} \int_{-\infty}^0 \frac{\alpha(y)}{\alpha^2(y) + y} \ln\left(\frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}}\right)^2 \mathrm{d}y. \tag{23}$$

To satisfy Eq. (20), $\alpha(x)$ must be a real function of the variable x. The presence of $\sqrt{x}$ in Eq. (23) leads to the conclusion that $\alpha(x)$ cannot be a real function but a complex one. Hence, there is no electron mass creation in the first approximation of the SD equation for the 3-dimensional electron propagator as well.

The first approximation of the SD equation in 2-dimensional QED gives

$$\alpha(x) = \alpha_0 (1 + d_l) \left(\int_{-\infty}^x \frac{\alpha}{\alpha^2 + y} \frac{\mathrm{d}y}{x - y} + \int_x^0 \frac{\alpha}{\alpha^2 + y} \frac{\mathrm{d}y}{y - x} \right), \tag{24}$$

$$\beta(x) = 1.$$

From the structure of the integrands in Eq. (24) it follows that there exists only a trivial solution $\alpha(x) = 0$ and consequently the absence of the electron mass in this approximation. Moreover, QED_2 is completely soluble model and there is no electron mass apearance at all but instead of the electron mass there is dynamical »photon« mass creation ($m_\gamma = e_0/\sqrt{\pi}$).

4. Conclusion

We can conclude that no electron mass appears from the dynamics in the first JBW approximation of the SD equation for electron propagator in the 2-, 3- and 4-dimensional QED. In order to understand further the role of the electromagnetic interaction in the origin of the electron mass, it is reasonable to study the higher-order approximations of the SD equation in the JBW perturbation expansion. We have already shown how to overcome some technical problems in the investigation of these higher-order approximations[5].

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ODSUSTVO STVARANJA MASE ELEKTRONA U PRVOJ APROKSIMACIJI KONAČNOG ELEKTRONSKOG PROPAGATORA

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Analiziran je elektronski propagator u prvoj aproksimaciji Schwinger-Dysonove jednačine u konačnoj kvantnoj elektrodinamici. Pokazano je odsustvo stvaranja mase elektrona za prostorno-vremenske dimenzije $d = 2, 3, 4$.