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LETTER TO THE EDITOR

ON THE INSTABILITY MECHANISM IN A COLLISIONLESS SHOCK WAVE

BHIMSEN K. SHIVAMOGGI

Department of Theoretical Physics, Research School of Physical Sciences, The Australian National University, Canberra, A. C. T. 2600, *Australia*

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The Krall-Book model of ion-acoustic instability in an inhomogeneous plasma is extended to include the anisotropic effects in the plasma. It is found that whereas a greater thermal motion parallel to the magnetic field enhances the instability, a greater thermal motion perpendicular to the magnetic field reduces it.

The instability mechanism within a collisionless shock wave propagating perpendicular to a strong magnetic field has been a topic of much discussion. Sagdeev[1] suggested an ion-acoustic instability in an inhomogeneous plasma, and this was explored further by Krall and Book[2]. Lashmore-Davies[3] proposed that the instability is due to the negative-energy character of the Bernstein modes in the presence of an $\vec{E}_0 \times \vec{B}_0$ drift on the electrons and the ensuing resonance of these Bernstein modes with the ion-acoustic waves. As a consequence of this instability, the leading edge of the shock would have a laminar structure, but, on moving further into the shockfront, there would be an increasing level of turbulence.

Now, a strongly magnetised plasma ceases to have isotropic properties, and there will be an anisotropy in the velocity distribution perpendicular and parallel to the magnetic field. The purpose of this note is to extend the Krall-Book model by including the anisotropic effects in the plasma.

Consider the wave propagation in the negative x-direction. Let the plasma be confined by a magnetic field

$$\vec{B}_0 = B_0 (1 + \hat{\varepsilon}\, x)\, \vec{i}_\beta \tag{1}$$

in the β-direction. Let the particle distribution be given by

$$F_{s0} = n_0 \left(\frac{m_s}{2\pi k_B T_{s\perp}}\right) \left(\frac{m_s}{2\pi k_B T_{s_{||}}}\right)^{1/2} \left[1 + \varepsilon_s \left(x + \frac{v_y}{\Omega_s}\right)\right] \times$$

$$\times \, \mathrm{e}^{-\frac{m_s}{2k_B}\left(\frac{v_\perp^2}{T_{s\perp}} + \frac{v_{||}^2}{T_{s||}}\right)} \tag{2}$$

where

$$\Omega_s = \frac{e_s B_0}{m_s c},$$

the subscript s refers to ions and electrons, and $||$ and $\perp$ refer to directions parallel and perpendicular, respectively, to the magnetic field. Assuming the perturbations, about the equilibrium given by (2), of the form $\mathrm{e}^{i(\kappa y - \omega t)}$, considering $\omega \gg \Omega_i$ so that the ions are unmagnetised, and following the procedure given by Krall[4], one obtains the dispersion relation

$$0 = \sum_s \left[1 + \left(\omega - \frac{\varepsilon_s k k_B T_{s_{||}}}{m_s \Omega_s}\right) \times \right.$$

$$\left. \times \int \frac{\mathrm{e}^{-z}\left(1 - \frac{k^2 k_B T_{s\perp}}{m_s \Omega_s^2} z\right)}{-\omega + \frac{k \hat{\varepsilon} k_B T_{s\perp}}{m_s \Omega_s} z} \mathrm{d}z \right] \tag{3}$$

where we have assumed

$$\frac{k^2 k_B T_{s\perp}}{m_s \Omega_s^2} \ll 1, \quad \text{and} \quad z = \frac{m_s v_\perp^2}{2 k_B T_{s\perp}}$$

and we have used the result:

$$\sum_{n=-\infty}^{\infty} I_n(\sigma)\, \mathrm{e}^{-\sigma} \approx I_0(\sigma)\, \mathrm{e}^{-\sigma} \approx 1 - \sigma \qquad \sigma \ll 1 \tag{4}$$

$I_n(\sigma)$ being the modified Bessel function of order n, and $\sigma = \frac{k^2 k_B T_{s\perp}}{m_s \Omega_s^2} z$.

Using the Plemelj formula, one obtains from (3)

$$0 = \sum_s \left[1 + (\omega - \hat{\omega}_{s_{||}}) \left(\frac{1 - k^2 a_s^2}{-\omega} + \mathrm{i}\,\gamma_s\right)\right] \tag{5}$$

where

$$\gamma_s = \frac{\pi}{|\hat{\omega}_{s\perp}|} \mathrm{e}^{-\frac{\omega}{\hat{\omega}_{s\perp}}}$$

$$\hat{\omega}_{s_{||}} = \frac{\varepsilon_s k k_B T_{s_{||}}}{m_s \Omega_s}, \ \hat{\omega}_{s\perp} = \frac{\hat{\varepsilon} k k_B T_{s\perp}}{m_s \Omega_s}$$

$$a_s^2 = \frac{k_B T_{s\perp}}{m_s \Omega_s^2}$$

and we have assumed $|\omega| \ll |\Omega_i|$.

Putting

$$\omega = \omega_r + \mathrm{i}\,\omega_i \tag{6}$$

and assuming $T_i \ll T_e$, (5) gives

$$\omega_r \approx \hat{\omega}_{e_{||}} \qquad \omega_i \approx \frac{\frac{\hat{\omega}_{e_{||}}^2}{\hat{\omega}_{e\perp}}}{1 - k^2 a_e^2} \mathrm{e}^{\frac{\omega}{|\omega_{e\perp}|}}. \tag{7}$$

(7) shows that the parallel and perpendicular thermal motions of the electrons play quite different roles in the instability mechanism. Whereas a greater parallel thermal motion enhances the instability, a greater perpendicular thermal motion reduces it.

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O MEHANIZMU NESTABILNOSTI U BESKOLIZIONOM UDARNOM VALU

BHIMSEN K. SHIVAMOGGI

Department of Theoretical Physics, Research School of Physical Sciences, The Australian National University, Canberra, A. C. T. 2600, *Australia*

UDK 533.951.

Oıiginalni znanstveni rad

Krall-Bookov model ionsko akustičke nestabilnosti u nehomogenoj plazmi proširen je na uključenje anizotropnih efekata. Pokazano je da za razliku od termičkog gibanja u smjeru magnetskog polja koje pojačava nestabilnost, termičko gibanje u okomitom smjeru proizvodi smanjenje nestabilnosti.

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