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THE INTERACTION OF THE DIRAC FIELD WITH THE ELECTROMAGNETIC FIELD IN A NEW FORMULATION. THE RELATIVISTIC HYDROGEN ATOM*

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Interaction of the Dirac field with the electromagnetic field in the new formulation is considered and the results are applied to the relativistic hydrogen atom.

1. Introduction

A new formulation of the Dirac field has been recently developed[1]. This formulation differs from the standard one in the following aspects:

(1) the field is defined by two Dirac equations; the Dirac equation with positive and negative mass term as canonical equations,

(2) the definition is given in fully correct Lagrange's and canonical language,

(3) the negative energies are eliminated by the dual solution similarly to the electromagnetic field.

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In this paper we consider interaction of this Dirac field with the electromagnetic field. We introduce the interaction as in the standard theory but in a way consistent with the new formulation of the Dirac field. We apply the derived results to the relativistic hydrogen atom. The stationary states of the relativistic hydrogen atom we find without self-interaction. The self-interaction we consider elsewhere.

Section 2 contains definition of the new Dirac field in a short form. In Section 3 the interaction of this field with the electromagnetic field is considered. The definitions of the hydrogen atom and solutions of the corresponding Lagrange's and the canonical equations without self-interaction are given in Section 4. The definition of the »normal« and the »dual« solutions and corresponding consequences are given in Section 5. Constants of motion are evaluated in Section 6 and conclusions are given in Section 7.

2. *The new free Dirac field*

We start with the Lagrange's density

$$\mathcal{L} = K\,[(-i\,\partial_\alpha \overline{\Phi}\,\gamma^\alpha)\,(i\,\partial_\beta\,\gamma^\beta\,\Phi) - \varkappa^2\,\overline{\Phi}\,\Phi] \tag{1}$$

for the bispinor field Φ. K and $\varkappa$ are constants. We use the coordinates $x^\alpha = (x^0, x^1, x^2, x^3)$, the metric

$$g_{00} = -g_{11} = -g_{22} = g_{33} = 1,\; g_{\alpha\beta} = 0,\; \alpha \neq \beta,$$

the representation of the γ^μ matrices

$$\gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix},\quad \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\quad \gamma^5 = -\,i\,\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3,$$

and units $c = \hbar = 1$.

The Lagrange's equations, according to

$$\frac{\partial \mathcal{L}}{\partial \chi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \chi)} = 0, \tag{2}$$

are

$$\partial_\alpha\,\partial^\alpha\,\Phi^\dagger + \varkappa^2\,\Phi^\dagger = 0, \tag{3}$$
$$\partial_\alpha\,\partial^\alpha\,\Phi + \varkappa^2\,\Phi = 0$$

(after multiplication with $-1/K$).

The conjugate momenta to Φ and $\Phi^\dagger$ are

$$\Pi_\Phi = K\,(-\,i\,\partial_\nu\,\Phi^\dagger\,\gamma^{\nu\dagger})\,i, \tag{4}$$
$$\Pi_{\Phi^\dagger} = K\,(-\,i)\,(i\partial_\mu\,\gamma^\mu\,\Phi)$$

and the corresponding Hamiltonian density

$$\mathscr{H} = \frac{1}{K} \Pi_\Phi \gamma^0 \Pi_\Phi^\dagger - \Pi_\Phi \partial_i \alpha^i \Phi - \Phi^\dagger \partial_i \alpha^i \Pi_\Phi^\dagger + + K (\varkappa^2 \Phi^\dagger \gamma^0 \Phi). \tag{5}$$

The canonical equations, according to

$$\frac{\mathrm{d}F}{\mathrm{d}t} = \frac{\partial F}{\partial t} + \{F, H\}, \tag{6}$$

are

$$\begin{aligned} &\frac{1}{K} \Pi_\Phi \gamma^0 - \partial_\nu \Phi \gamma^\nu = 0, \\ &\frac{1}{K} \Pi_\Phi^\dagger - \partial_\mu \gamma^\mu \Phi = 0, \\ &\partial_\nu (\Pi_\Phi^\dagger \gamma^0) \gamma^\nu + K \varkappa^2 \overline{\Phi} = 0, \\ &\partial_\nu \gamma^\nu \Pi_\Phi^\dagger + K \varkappa^2 \Phi = 0 \end{aligned} \tag{7}$$

or in the matrix form

$$\begin{aligned} &\begin{pmatrix} \mathrm{i}\, \partial_\nu \gamma^\nu & -\varkappa \\ -\varkappa & \mathrm{i}\, \partial_\nu \gamma^\nu \end{pmatrix} \begin{pmatrix} \varkappa \Phi \\ \frac{\mathrm{i}}{K} \Pi_\Phi^\dagger \end{pmatrix} = 0, \\ &\left(\varkappa \Phi^\dagger, -\frac{\mathrm{i}}{K} \Pi_\Phi \right) \begin{pmatrix} -\mathrm{i}\, \partial_\nu \gamma^{\nu\dagger} & -\varkappa \\ -\varkappa & -\mathrm{i}\, \partial_\nu \gamma^{\nu\dagger} \end{pmatrix} = 0. \end{aligned} \tag{8}$$

Performing the unitary transformation by the unitary operator

$$S = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \tag{9}$$

Eqs. (8) become

$$\begin{aligned} &(\mathrm{i}\partial_\nu \gamma^\nu - \varkappa) \Psi_I = 0, \\ &(\mathrm{i}\partial \gamma^\nu + \varkappa) \Psi_{II} = 0, \\ &-\mathrm{i}\partial_\nu \overline{\Psi}_I \gamma^\nu - \varkappa \overline{\Psi}_I = 0, \\ &-\mathrm{i}\partial_\nu \overline{\Psi}_{II} \gamma^\nu + \varkappa \overline{\Psi}_{II} = 0, \end{aligned} \tag{10}$$

where

$$S\begin{pmatrix} \varkappa\Phi \\ \frac{i}{K}\Pi_\Phi{}^\dagger \end{pmatrix} = \begin{pmatrix} \Psi_I \\ \Psi_{II} \end{pmatrix}, \tag{11}$$

or explicitly

$$\Psi_I = \frac{1}{\sqrt{2}}\left(\varkappa\Phi + \frac{i}{K}\Pi_\Phi{}^\dagger\right), \tag{12}$$

$$\Psi_{II} = \frac{1}{\sqrt{2}}\left(\varkappa\Phi - \frac{i}{K}\Pi_\Phi{}^\dagger\right).$$

Let us also write the inverse of Eqs. (12):

$$\Phi = \frac{1}{\varkappa\sqrt{2}}(\Psi_I + \Psi_{II}), \tag{13}$$

$$\Pi_\Phi{}^\dagger = \frac{K}{i\sqrt{2}}(\Psi_I - \Psi_{II}).$$

We take Eqs. (10) as the definition of (new) Dirac field, or coming back to Eq. (1) we take the bispinor Φ with the Lagrange's density (1) as the definition of the (new) Dirac field.

The general solutions of Eqs. (3), written in terms of plane waves with the box normalization, is

$$\Phi(x, t) = \frac{1}{L^{3/2}} \sum_{\vec{k}} \left\{ \begin{bmatrix} a^1_{\vec{k}1} \\ a^2_{\vec{k}1} \\ a^3_{\vec{k}1} \\ a^4_{\vec{k}1} \end{bmatrix} e^{i(kx - k_0 t)} + \begin{bmatrix} a^1_{\vec{k}2} \\ a^2_{\vec{k}2} \\ a^3_{\vec{k}2} \\ a^4_{\vec{k}2} \end{bmatrix} e^{i(kx + k_0 t)} \right\}, \tag{14}$$

where $a^m_{\vec{k}n}$ are arbitrary constants and $k_0 = +\sqrt{\varkappa^2 + \vec{k}^2}$.

Writing

$$\Phi = \Phi_n + \Phi_d, \tag{15}$$

where

$$\Phi_n = \begin{pmatrix} \Phi_1 \\ \Phi_2 \\ 0 \\ 0 \end{pmatrix}, \quad \Phi_d = \begin{pmatrix} 0 \\ 0 \\ \Phi_3 \\ \Phi_4 \end{pmatrix} \tag{16}$$

the solutions Φ_n and Φ_d are in the »dual« relation

$$\Phi_d \to \gamma^5 \Phi_n \tag{17}$$

and in the next we take $\Phi_d = 0$.

After elimination of the »dual« solution, we separate the negative and positive frequency terms and write

$$\Phi = \xi + \zeta, \tag{18}$$

$$\begin{aligned} \Psi_I &= \Psi_{I\xi} + \Psi_{I\zeta}, \\ \Psi_{II} &= \Psi_{II\xi} + \Psi_{II\zeta}, \end{aligned} \tag{19}$$

where ξ contains only the negative frequency terms and ζ contains only the positive frequency terms.

The scalar, four-vector and the second order tensor constants of motion are then

$$\begin{aligned} Q &= Q_\xi + Q_\zeta, \\ P^a &= P^a_\xi + P^a_\zeta, \\ M^{\beta\gamma} &= M^{\beta\gamma}_\xi + M^{\beta\gamma}_\zeta, \end{aligned} \tag{20}$$

where

$$\begin{aligned} Q_\xi &= const_Q \int (\Psi^\dagger_{I\xi} \Psi_{I\xi} - \Psi^\dagger_{II\xi} \Psi_{II\xi}) \, \mathrm{d}^3 x, \\ Q_\zeta &= const_Q \int (\Psi^\dagger_{I\zeta} \Psi_{I\zeta} - \Psi^\dagger_{II\zeta} \Psi_{II\zeta}) \, \mathrm{d}^3 x, \\ P^a_\xi &= const_P \frac{\mathrm{i}}{2} \int \{(\Psi^\dagger_{I\xi} \partial^a \Psi_{I\xi} - \partial^a \Psi^\dagger_{I\xi} \Psi_{I\xi}) - \\ &\quad - (\Psi^\dagger_{II\xi} \partial^a \Psi_{II\xi} - \partial^a \Psi^\dagger_{II\xi} \Psi_{II\xi})\} \, \mathrm{d}^3 x, \\ P^a_\zeta &= const_P \frac{\mathrm{i}}{2} \int \{ (\Psi^\dagger_{I\zeta} \partial^a \Psi_{I\zeta} - \partial^a \Psi^\dagger_{I\zeta} \Psi_{I\zeta}) - \\ &\quad - (\Psi^\dagger_{II\zeta} \partial^a \Psi_{II\zeta} - \partial^a \Psi^\dagger_{II\zeta} \Psi_{II\zeta}) \} \, \mathrm{d}^3 x, \\ M^{\beta\gamma}_\xi &= const_M \{ \int (x^\beta T^{\gamma 0}_\xi - x^\gamma T^{\beta 0}_\xi) \, \mathrm{d}^3 x + \\ &\quad + \frac{1}{2} \int (\Psi^\dagger_{I\xi} \sigma^{\beta\gamma} \Psi_{I\xi} - \Psi^\dagger_{II\xi} \sigma^{\beta\gamma} \Psi_{II\xi}) \, \mathrm{d}^3 x\}, \\ M^{\beta\gamma}_\zeta &= const_M \{(\int (x^\beta T^{\gamma 0}_\zeta - x^\gamma T^{\beta 0}_\zeta) \, \mathrm{d}^3 x + \\ &\quad + \frac{1}{2} \int (\Psi^\dagger_{I\zeta} \sigma^{\beta\gamma} \Psi_{I\zeta} - \Psi^\dagger_{II\zeta} \sigma^{\beta\gamma} \Psi_{II\zeta}) \, \mathrm{d}^3 x\} \end{aligned} \tag{21}$$

and $K = \varkappa$.

Selecting (when field components are different from zero)

$$\int (\Psi^\dagger_{I\xi} \Psi_{I\xi} - \Psi^\dagger_{II\xi} \Psi_{II\xi}) \, d^3 x = 1,$$

$$\int (\Psi^\dagger_{II\zeta} \Psi_{II\zeta} - \Psi^\dagger_{I\zeta} \Psi_{I\zeta}) \, d^3 x = 1, \tag{22}$$

and taking

$$const_Q = q, \ const_P = const_M = 1, \tag{23}$$

these constants of motion may be interpreted as the charge of a particle and its antiparticle, the average values of their four-momenta and the average values of their angular momenta, with the space-time probabilities

$$w_\xi (\vec{x}, t) = \Psi^\dagger_{I\xi} \Psi_{I\xi} - \Psi^\dagger_{II\xi} \Psi_{II\xi},$$

$$w_\zeta (\vec{x}, t) = \Psi^\dagger_{II\zeta} \Psi_{II\zeta} - \Psi^\dagger_{I\zeta} \Psi_{I\zeta} \tag{24}$$

for the particle and its antiparticle, respectively.

In the nonrelativistic limit ($k \ll \varkappa$)

$$\Psi_{I\zeta} \to 0, \ \Psi_{II\xi} \to 0, \tag{25}$$

$$\Psi_{I\xi} = e^{-i\varkappa t} \begin{pmatrix} \chi_{I1} \\ \chi_{I2} \end{pmatrix}, \quad \Psi_{II\zeta} = e^{i\varkappa t} \begin{pmatrix} \chi_{II1} \\ \chi_{II2} \end{pmatrix}$$

and Eqs. (10) and (22) become

$$i \partial_0 \chi_{I1} = -\frac{1}{2\varkappa} \Delta \chi_{I1},$$

$$-i \partial_0 \chi_{II1} = -\frac{1}{2\varkappa} \Delta \chi_{II} \quad \text{and h. c.,} \tag{26}$$

$$\int \chi^\dagger_{I1} \chi_{II1} \, d^3 x = 1, \tag{27}$$

$$\int \chi^\dagger_{II1} \chi_{II1} \, d^3 x = 1.$$

The constants of motion for $\chi_{I1} \neq 0$ and $\chi_{II1} = 0$ are then

$$Q_{nonrel} = q,$$

$$P^0{}_{nonrel} = \varkappa + \int \chi^\dagger_1 \left(-\frac{1}{2\varkappa} \Delta \chi_{I1} \right) d^3 x, \tag{28}$$

$$P^J{}_{nonrel} = \int \chi^\dagger_1 (-i \partial_J) \chi_{I1} \, d^3 x,$$

$$S_{r\ nonrel} = \frac{1}{2} \int \chi^\dagger_1 \sigma_r \chi_{I1} \, d^3x,$$

where in the last constant the spin part is written only. These constants of motion reveal the statistical interpretation of the field.

At the end let us make a remark with respect to the separation of Φ on Φ_n and Φ_d. In the representation of γ-matrices where γ^5 is diagonal the bispinor components (Φ_1, Φ_2) transform under the Lorentz's transformation independently from (Φ_3, Φ_4). The unitary transformation which realises transition to these γ-matrices is

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

Consequently, $\Phi_n = \binom{u}{0}$, $\Phi_d = \binom{0}{v}$ go over into $\tilde{\Phi}_n \sim \binom{u}{u}$, $\tilde{\Phi}_d \sim \binom{v}{-v}$. We may then define the normal solution as that with equal upper two and with lower two components. In Section 5 we return to this equestion in more detail.

3. *Interaction with the electromagnetic field*

We introduce the electromagnetic interaction in the system consisting of the new Dirac field and the electromagnetic field similarly as in the standard theory, i. e. by the substitution

$$\mathrm{i}\,\partial_\alpha \rightarrow \mathrm{i}\,\partial_\alpha - eA_\alpha, \quad e = -|e|. \tag{29}$$

We apply also the standard treatment of the electromagnetic field in order to avoid unnecessary misunderstanding. We consider theory with analogous canonical description of the electromagnetic field elsewhere.

The Lagrange's density (1) then becomes

$$\mathscr{L}_{free\ D\cdot f\cdot} + \mathscr{L}_{int} = \varkappa\,\{[(-\mathrm{i}\,\partial_\alpha - e\,A_\alpha)\,\overline{\Phi}\,\gamma^\alpha]\,[(\mathrm{i}\,\partial_\beta - eA_\beta)\,\gamma^\beta\,\Phi] - \varkappa^2\,\overline{\Phi}\Phi\}, \tag{30}$$

where from now on we take $K = \varkappa$.

The total Lagrange's density is

$$\mathscr{L}_{free\ D\cdot f\cdot} + \mathscr{L}_{int} + \mathscr{L}_{em} \tag{31}$$

where

$$\mathscr{L}_{em} = -\frac{1}{16\pi}\,(\partial_\alpha\,A_\beta - \partial_\beta\,A_\alpha)\,(\partial^\alpha\,A^\beta - \partial^\beta\,A^\alpha). \tag{32}$$

The Lagrange's equations for A^μ and Φ are

$$\partial_\alpha\,\partial^\alpha\,A^\mu = -\,4\,\pi\,\{2e^2\,\varkappa\,\overline{\Phi}\,\Phi\,A^\mu + \mathrm{i}\,e\,\varkappa\,[-\overline{\Phi}\,\gamma^\alpha\,(\partial_\alpha\,\gamma^\alpha\,\Phi) + (\partial_\alpha\,\Phi\,\gamma^\alpha)\,\gamma^\mu\,\overline{\Phi}]\}, \tag{33}$$

$$\partial_\alpha\,\partial^\alpha\,\Phi + \varkappa^2\,\Phi + \mathrm{i}\,e\,[\partial_\mu\,\gamma^\mu\,(A_\beta\gamma^\beta\,\Phi) + A_\alpha\,\gamma^\alpha\,\partial_\beta\,\gamma^\beta\,\Phi] - e^2 A_\alpha\,A^\alpha\,\Phi = 0, \tag{34}$$

where we have used in Eq. (33) also the Lorentz's condition

$$\partial_\alpha A^\alpha = 0. \tag{35}$$

For the sake of later calculations and comparison with the standard theory we give Eq. (34) also in some other forms. First we write

$$[(\mathrm{i}\,\partial_\alpha - eA_\alpha)\,\gamma^\alpha\,(\mathrm{i}\,\partial_\beta - eA_\beta)\,\gamma^\beta - \varkappa^2]\,\Phi = 0. \tag{36}$$

By making use of

$$(\mathrm{i}\,\partial_\alpha - eA_\alpha)\,(\mathrm{i}\,\partial^\beta - eA^\beta)\,\gamma^\alpha\,\gamma =$$

$$(\mathrm{i}\,\partial - eA_\alpha)\,(\mathrm{i}\,\partial^\alpha - eA^\alpha) - \frac{\mathrm{i}}{2}\,[(\mathrm{i}\,\partial_\alpha - eA_\alpha), (\mathrm{i}\,\partial_\beta - eA_\beta)]\,\sigma^{\alpha\beta}, \tag{37}$$

and

$$[\mathrm{i}\,\partial_\alpha, A_\beta] = \mathrm{i}\,\frac{\partial A_\beta}{\partial x_\alpha}, \tag{38}$$

$$F_{\alpha\beta} = \partial_\alpha\,A_\beta - \partial_\beta\,A_\alpha, \tag{39}$$

where

$$\sigma^{\alpha\beta} = \frac{\mathrm{i}}{2}\,(\gamma^\alpha\,\gamma^\beta - \gamma^\beta\,\gamma^\alpha), \tag{40}$$

Eq. (36) becomes[2)]

$$\{(\mathrm{i}\,\partial_\alpha - eA_\alpha)\,(\mathrm{i}\,\partial^\alpha - eA^\alpha) - \frac{1}{2}\,e\,\sigma^{\alpha\beta}\,F_{\alpha\beta}\}\,\Phi = \varkappa^2\,\Phi, \tag{41}$$

with

$$\frac{1}{2}\,\sigma^{\alpha\beta}\,F_{\alpha\beta} = \vec{\Sigma}\,\vec{H} - \mathrm{i}\,\vec{\alpha}\,\vec{E}, \tag{42}$$

where

$$\frac{1}{2}\,\Sigma = \frac{1}{2}\begin{pmatrix}\vec{\sigma} & 0\\ 0 & \vec{\sigma}\end{pmatrix} \tag{43}$$

is the spin momentum, $\vec{E}$ and $\vec{H}$ are the electric and magnetic field vectors

$$\vec{\alpha} = \begin{pmatrix}0 & \vec{\sigma}\\ \vec{\sigma} & 0\end{pmatrix} \tag{44}$$

and σ_i are the Pauli matrices.

The term with the magnetic vector is quasidiagonal while it not refers to the term with the electric vector. The unitary transformation

$$S\Phi = \tilde{\Phi},\ S = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}, \tag{45}$$

makes all terms quasidiagonal:

$$\{ (\mathrm{i}\,\partial^{a} - eA_{a})(\mathrm{i}\,\partial^{a} - eA^{a} - \begin{pmatrix}\vec{\sigma} & 0\\ 0 & \vec{\sigma}\end{pmatrix} e\vec{H} + \mathrm{i}\begin{pmatrix}\vec{\sigma} & 0\\ 0 & -\vec{\sigma}\end{pmatrix} e\vec{E}\}\tilde{\Phi} = \varkappa^{2}\tilde{\Phi}. \tag{46}$$

Writing

$$\Phi = \begin{pmatrix}\tilde{\Phi}_{I}\\ \tilde{\Phi}_{II}\end{pmatrix} \tag{47}$$

one gets separated equations in components $\tilde{\Phi}_{I}$, $\tilde{\Phi}_{II}$:

$$\{(\mathrm{i}\,\partial_{a} - eA_{a})(\mathrm{i}\,\partial^{a} - eA_{a}) - e\vec{\sigma}\vec{H} + \mathrm{i}\,e\vec{\sigma}\vec{E}\}\tilde{\Phi}_{I} = \varkappa^{2}\tilde{\Phi}_{I}, \tag{48}$$

$$\{(\mathrm{i}\,\partial_{a} - eA_{a})(\mathrm{i}\,\partial^{a} - eA^{a}) - e\vec{\sigma}\vec{H} - \mathrm{i}\,e\vec{\sigma}\vec{E}\}\tilde{\Phi}_{II} = \varkappa^{2}\tilde{\Phi}_{II}. \tag{49}$$

The last equation is just the one which Feynman and Gell-Mann suggested for description of electrons and positrons[3-5]. In our description it is almost so but not completely. The difference is already evident but will be seen better in a due course.

Now, we come back to the Lagrange's and canonical formalism of the field $(\Phi, \Phi^{\dagger})$.

The conjugate momenta to Φ and $\Phi^{\dagger}$ are

$$\Pi_{\Phi} - \frac{\partial \mathscr{L}}{\partial(\partial_{0}\Phi)} = \{-(\mathrm{i}\,\partial_{a} + eA_{a})\bar{\Phi}\gamma^{a}\,\mathrm{i}\,\gamma^{0}\}\,\varkappa,$$

$$\Pi_{\Phi}^{\dagger} = \frac{\partial \mathscr{L}}{\partial(\partial_{0}\Phi^{\dagger})} = \varkappa\{-\mathrm{i}(\mathrm{i}\,\partial_{\beta} - eA_{\beta})\gamma^{\beta}\Phi\}, \tag{50}$$

and the Hamiltonian density

$$\mathscr{H} = \frac{1}{\varkappa}\Pi_{\Phi}\gamma^{0}\Pi_{\Phi}^{\dagger} - \Pi_{\Phi}(\partial_{j}\gamma^{0}\gamma^{j}\Phi + \mathrm{i}\,eA_{\beta}\gamma^{0}\gamma^{\beta}\Phi) -$$

$$- (\partial_{j}\bar{\Phi}\gamma^{j} - \mathrm{i}\,eA_{a}\bar{\Phi}\gamma^{a})\Pi_{\Phi}^{\dagger} + \varkappa^{3}\bar{\Phi}\Phi. \tag{51}$$

The canonical equations are then

$$\partial_0 \Pi_\Phi = -\varkappa^3 \overline{\Phi} + \Pi_\Phi \,\mathrm{i}\, eA_\beta \gamma^0 \gamma^\beta + \partial_j (-\Pi_\Phi \gamma^0 \gamma^j),$$

$$\partial_0 \Pi_\Phi \dagger = -\varkappa^3 \gamma^0 \Phi - \partial_j (\gamma^0 \gamma^j \Pi_\Phi \dagger) - \mathrm{i}\, eA_a \gamma^0 \gamma^a \Pi_\Phi \dagger, \tag{52}$$

$$\partial_0 \Phi = \frac{1}{\varkappa} \gamma^0 \Pi_{\bar{\Phi}} - (\partial_j \gamma^0 \gamma^j \Phi + \mathrm{i}\, eA_\beta \gamma^0 \gamma^\beta \Phi),$$

$$\partial_0 \Phi \dagger = \frac{1}{\varkappa} \Pi_\Phi \gamma^0 - (\partial_j \overline{\Phi} \gamma^j - \mathrm{i}\, eA_a \overline{\Phi} \gamma^a), \tag{53}$$

or

$$(\partial_a - \mathrm{i}\, eA_a) \Pi_\Phi \gamma^{a\dagger} + \varkappa^3 \Phi \dagger = 0,$$

$$(\partial_a + \mathrm{i}\, eA_a) \gamma^a \Pi_\Phi \dagger + \varkappa^3 \Phi = 0, \tag{54}$$

$$(\partial_a + \mathrm{i}\, eA_a) \gamma^a \Phi - \frac{1}{\varkappa} \Pi_\Phi \dagger = 0,$$

$$(\partial_a - \mathrm{i}\, eA_a) \overline{\Phi} \gamma^a - \frac{1}{\varkappa} \Pi_\Phi \gamma^0 = 0. \tag{55}$$

These equations can also be written in the form

$$\begin{pmatrix} \mathrm{i}\,(\partial_a + \mathrm{i}\, eA_a) \gamma^a & -\varkappa \\ -\varkappa & \mathrm{i}\,(\partial_a + \mathrm{i}\, eA_a) \gamma^a \end{pmatrix} \begin{pmatrix} \varkappa \Phi \\ \frac{\mathrm{i}}{\varkappa} \Pi_\Phi \dagger \end{pmatrix} = 0 \tag{56}$$

and h. c.

Performing the unitary transformation with the unitary matrix

$$S = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{57}$$

and introducing the notation

$$S \begin{pmatrix} \varkappa \Phi \\ \frac{\mathrm{i}}{\varkappa} \Pi_\Phi \dagger \end{pmatrix} = \begin{pmatrix} \Psi_I \\ \Psi_{II} \end{pmatrix} \tag{58}$$

Eqs. (54) and (55) become

$$[(i\,\partial_\alpha - eA_\alpha)\,\gamma^\alpha - \varkappa]\,\Psi_I = 0,$$
$$[(i\,\partial_\alpha - eA_\alpha)\,\gamma^\alpha + \varkappa]\,\Psi_{II} = 0, \tag{59}$$

$$(-i\,\partial_\alpha - eA_\alpha)\,\overline{\Psi}_I\,\gamma^\alpha - \varkappa\,\overline{\Psi}_I = 0,$$
$$(-\,i\partial^\alpha - eA_\alpha)\,\overline{\Psi}_{II}\,\gamma^\alpha + \varkappa\,\overline{\Psi}_{II} = 0. \tag{60}$$

These equations are in accordance with (10) and (29). They are Dirac field equations in the electromagnetic field with negative and positive mass term.

In the next it will be sometimes useful to write

$$\frac{1}{\sqrt{2}}\,[(i\,\partial_\alpha - eA_\alpha)\,\gamma^\alpha - \varkappa] \equiv D_-$$
$$\frac{1}{\sqrt{2}}\,[(i\,\partial_\alpha - eA_\alpha)\,\gamma^\alpha + \varkappa] \equiv D_+. \tag{61}$$

Eqs. (59) then read

$$D_-\,\Psi_I = 0, \tag{62}$$
$$D_+\,\Psi_{II} = 0,$$

and from (58) we also have

$$\Psi_I = D_+\,\Phi,$$
$$\Psi_{II} = -\,D_-\,\Phi. \tag{63}$$

From the canonical equations (59) and (60) follows

$$\partial_\alpha j^\alpha = 0, \tag{64}$$

$$j^\alpha = e\,(\overline{\Psi}_I\,\gamma\;\;\Psi_I - \overline{\Psi}_{II}\,\gamma^\alpha\,\Psi_{II}). \tag{65}$$

By making use of this four-vector, $\mathscr{L}_{int}$ can be written in the form

$$\mathscr{L}_{int} = \varkappa\{\,[\,(-i\,\partial_\alpha - eA_\alpha)\,\overline{\Phi}\,\gamma^\alpha]\,[(i\,\partial_\beta - eA_\beta)\,\gamma^\beta\,\Phi] -$$
$$-\,(-\,i\,\partial_\alpha\,\overline{\Phi}\,\gamma_\alpha)\,(i\,\partial\;\gamma_\beta\,\Phi)\} = -j_\alpha\,A^\alpha - e^2\,\varkappa\,\overline{\Phi}\,\overline{\Phi}\,A_\alpha\,A^\alpha \tag{66}$$

and Eq. (33) in the form

$$\partial_\alpha\,\partial^\alpha\,A^\mu = 4\,\pi\,j^\mu. \tag{67}$$

Let us notice that we have also

$$\mathscr{L}_0 + \mathscr{L}_{int} = -\varkappa(\overline{\Psi}_I \Psi_{II} + \overline{\Psi}_{II} \Psi_I). \tag{68}$$

By this we have introduced internal interaction of the Dirac field with the electromagnetic field on the basis of (29), or the Dirac field with internal electromagnetic interaction and interaction with external free electromagnetic field.

Let us now consider the external interaction with the electromagnetic field with sources. We will not go into the source structure but will only assume that the interaction among the sources and the electromagnetic field is given by

$$\mathscr{L}_{int} = -j_{s\alpha} A^\alpha, \tag{69}$$

with

$$\partial_\alpha j_s^\alpha = 0. \tag{70}$$

Disregarding any other interaction we have only the change in the Lagrange's equation for A^μ:

$$\partial_\alpha \partial^\alpha A^\mu = 4\pi (j^\mu + j_s^\mu). \tag{71}$$

The general solution of Eq. (71) is

$$A^\mu = A_h^\mu + A_p^\mu, \tag{72}$$

where A_h^μ is a general solution of the homogeneous equation and A_p^μ is a particular solution. The particular solution is taken in the form

$$A_p^\mu = A_{pj}^\mu + A_{pj_s}^\mu, \tag{73}$$

where

$$A_{pj}^\mu = \int \frac{j^\mu(\vec{x}', x_0 - |\vec{x} - \vec{x}'|)}{|\vec{x} - \vec{x}'|} \mathrm{d}^3 x', \quad A_{pj_s}^\mu = \int \frac{j_s^\mu(\vec{x}', \vec{x} - |\vec{x} - \vec{x}'|)}{|\vec{x} - \vec{x}'|} \mathrm{d}^3 x'. \tag{74}$$

The substitution of A^μ from (72) into (34) gives

$$\partial_\alpha \partial^\alpha \Phi + \varkappa^2 \Phi + \mathrm{i}e\, [e_\mu \gamma^\mu (A_{h\beta} \gamma^\beta \Phi) + A_{h\alpha} \gamma^\alpha \partial_\beta \gamma^\beta \Phi] - e^2 A_{h\alpha} A_{h\alpha} \Phi$$

$$- 2e^2 A_{p\alpha} A_h^\alpha + \mathrm{i}e\, [\partial_\mu \gamma^\mu \Phi) + A_{p\alpha} \gamma^\alpha \partial_\beta \gamma^\beta \Phi] - e^2 A_{p\alpha} A_p^\alpha \Phi = 0. \tag{75}$$

We have now the general elements of the interaction of the new Dirac field with the electromagnetic field introduced by (29) . Is this interaction correct? We answer to this question partly in the next Section. The other aspects we consider elsewhere.

4. *Relativistic hydrogen atom*

In this Section we apply Eq. (75) to the relativistic hydrogen atom. We do not consider here the self-effects as we do it elsewhere. Therefore, we take

$$A_s^\mu = 0, \; A_{pl}^\mu = 0, \; A_{pl_0}^\mu = \left(-\frac{Ze}{r} \equiv A_0, 0, 0, 0\right). \tag{76}$$

Eq. (75) then becomes

$$\partial_\alpha \partial^\alpha \Phi + \varkappa^2 \Phi + \mathrm{i}\, e \left[\partial_\mu \gamma^\mu (A_0 \gamma^0 \Phi) + A_0 \gamma^0 \partial_\beta \gamma^\beta \Phi\right] - e^2 A_0^2 \Phi = 0 \tag{77}$$

or by making use of (41) and (42)

$$\{(\mathrm{i}\, e_0 - e A_0)^2 + \Delta - \mathrm{i}\, e \vec{\alpha} \vec{E}\} \Phi = \varkappa^2 \Phi. \tag{78}$$

We seek solution of this equation in the form

$$\Phi = e^{-\mathrm{i} e k_0 t} \varphi_{k_0} (\vec{r}) \tag{79}$$

and for $0 > k_0 > \varkappa$, since we expect that it will be connected with the hydrogen atom. After substitution in (78) the equation for φ reads

$$\{(k_0 - e A_0)^2 + \Delta - \mathrm{i}\, e \vec{\alpha} \vec{E}\} \varphi_{k_0} (r) = \varkappa^2 \varphi_{k_0} (\vec{r}). \tag{80}$$

Now, we have to solve this equation. We can follow usual procedure of solving such an equation. However, we may also make use of already known solutions of the Dirac equation from the standard Dirac field theory. Due to general knowledge of this material we follow this line. By this we are solving the canonical Eqs. (59). (For direct solution of Eq. (80) see the Appendix.)

Multiplying Eq. (58) by S^{-1} one gets

$$\Phi = \frac{1}{\varkappa \sqrt{2}} (\Psi_I + \Psi_{II}) \tag{81}$$

where Ψ_I and Ψ_{II} are arbitrary solutions of the first and the second of Eqs. (59), respectively.

From the standard Dirac field theory the solutions Ψ_I are known and the solutions Ψ_{II} one can get from them by changing the sign of the mass term ($\varkappa \to -\varkappa$).

Writing

$$\Psi_I (\vec{r}, t) = e^{-\mathrm{i} k_0 t} \Psi_{I k_0} (\vec{r}), \; \Psi_{II} (\vec{r}, t) = e^{-\mathrm{i} k_0 t} \Psi_{II k_0} (\vec{r}) \tag{82}$$

and taking into account (79) and (81) it follows

$$\varphi_{Ik_0}(\vec{r}) = \frac{1}{\varkappa\sqrt{2}} [\Psi_{Ik_0}(\vec{r}) + \Psi_{IIk_0}(\vec{r})]. \tag{83}$$

From the standard theory we have[3)]

$$\Psi_{Ik_0}(\vec{r}) = \begin{pmatrix} f(r)\,\Omega_{jlm}(\vec{n}) \\ \mathrm{i}^{1+l-l'} g(r)\,\Omega_{jl'm}(\vec{n}) \end{pmatrix}$$

where $\Omega_{jlm}(\vec{n})$ is the spherical spinor, $l = j \pm \frac{1}{2}$, $l' = 2j - l$, $\vec{n} = \frac{\vec{r}}{r}$, and

$$f(r) = \sqrt{\varkappa + k_0}\, \mathrm{e}^{-\frac{\varrho}{2}} \varrho^{\gamma-1} (Q_1 + Q_2),$$

$$g(r) = -\sqrt{\varkappa - k_0}\, \mathrm{e}^{-\frac{\varrho}{2}} \varrho^{\gamma-1} (Q_1 - Q_2), \tag{84}$$

$$Q_1 = A_{n_r k} F(-n_r, 2\gamma + 1, \varrho),$$

$$Q_2 = B_{n_r k} F(1 - n_r, 2\gamma + 1, \varrho), \tag{85}$$

$$B_{n_r k} = \frac{n_r}{k - \frac{Z\alpha\varkappa}{\lambda}} A_{n_r k}, \tag{86}$$

$$k_0 = \left\{ \frac{\varkappa^2}{1 + \left(\frac{Z\alpha}{\gamma + n_r}\right)^2} \right\}^{1/2}, \tag{87}$$

$$k = \begin{cases} -\left(j + \frac{1}{2}\right) = -(l+1), & j = l + \frac{1}{2}, \\ \left(j + \frac{1}{2}\right) = l, & j = l - \frac{1}{2} \end{cases} \tag{88}$$

($k = \pm 1, \pm 2, \pm 3, \ldots$; the positive values correspond to $j = l - \frac{1}{2}$ and negative to $j = l + \frac{1}{2}$).

$$n_r = \begin{cases} 0, 1, 2, 3, \ldots & \text{for } k > 0 \\ 1, 2, 3, \ldots & \text{for } k < 0 \end{cases}, \tag{89}$$

$$\varrho = 2\lambda r,\ \lambda = \sqrt{\varkappa^2 - k_0^2},\ \gamma = \sqrt{k^2 - Z^2 - a^2}. \tag{90}$$

$F(a, b, \varrho)$ is the confluent hypergeometric function. $A_{n_r k}$ is a constant determined from the normalization of the solution, i. e. from the condition

$$\int_0^\infty (f^2 + g^2)\, r^2\, \mathrm{d}r = 1.$$

In our calculation in this Section we leave the constant $A_{n_r k}$ undetermined and multiply the functions $f(r)$ and $g(r)$ by $\sqrt{\varkappa + k_0}$. This corresponds to the free field procedure[1]. Then we have (denoting the new radial functions again by f and g)

$$\Psi_{Ik_0}(r) = \begin{pmatrix} f(r)\,\Omega_{jlm} \\ \mathrm{i}^{1+l-l'} g(r)\,\Omega_{jl'm} \end{pmatrix},$$

$$f(r) = (\varkappa + k_0)\, \mathrm{e}^{-\frac{\varrho}{2}} \varrho^{\gamma-1} (Q_1 + Q_2), \tag{91}$$

$$g(r) = -\sqrt{\varkappa^2 - k_0^2}\, \mathrm{e}^{-\frac{\varrho}{2}} \varrho^{\gamma-1} (Q_1 - Q_2).$$

We use this form of the solution of the equation $D_- \Psi_I = 0$ in the following.

Let us write now the solution of the equation $D_+ \Psi_{II} = 0$ of the form (82). In order to get these solutions we rely on equation for $f(r)$ and $g(r)$ from (91) which are[6]

$$\frac{\mathrm{d}}{\mathrm{d}r}(fr) + \frac{k}{r}(fr) - \left(k_0 + \varkappa + \frac{Za}{r}\right) gr = 0,$$

$$\frac{\mathrm{d}}{r}(gr) - \frac{k}{r}(gr) + \left(k_0 - \varkappa + \frac{Za}{r}\right) fr = 0. \tag{92}$$

The substitution $\varkappa \to -\varkappa$ gives

$$\frac{\mathrm{d}}{\mathrm{d}r}(\widetilde{fr}) + \frac{k}{r}(\widetilde{fr}) - \left(k_0 - \varkappa + \frac{Za}{r}\right) \widetilde{gr} = 0,$$

$$\frac{\mathrm{d}}{r}(\widetilde{gr}) - \frac{k}{r}(\widetilde{gr}) + \left(k_0 + \varkappa + \frac{Za}{r}\right) \widetilde{fr} = 0. \tag{93}$$

We see that

$$\tilde{f} \to -g,\ \tilde{g} \to f,\ k \to -k \tag{94}$$

transforms the system (93) into (92). Consequently, we have

$$\tilde{f} = \sqrt{\varkappa^2 - k_0^2}\, e^{-\frac{\varrho}{2}} \varrho^{\gamma-1} (\tilde{Q}_1 - \tilde{Q}_2),$$

$$\tilde{g} = (\varkappa + k_0)\, e^{-\frac{\varrho}{2}} \varrho^{\gamma-1} (\tilde{Q}_1 + \tilde{Q}_2),\ \tilde{Q}_1 \leftarrow Q_1(-k), \tilde{Q}_2 \leftarrow Q_2(-k). \quad (95)$$

After multiplication by $\frac{k_0 - \varkappa}{\sqrt{\varkappa^2 - k_0^2}}$ and denoting the new radial function again by f and g it becomes

$$\tilde{f} = (k_0 - \varkappa)\, e^{-\frac{\varrho}{2}} \varrho^{\gamma-1} (\tilde{Q}^1 - \tilde{Q}_2),$$

$$\tilde{g} = -\sqrt{\varkappa^2 - k_0^2}\, e^{-\frac{\varrho}{2}} \varrho^{\gamma-1} (\tilde{Q}_1 + \tilde{Q}_2). \quad (96)$$

Thus, we find

$$\Psi_{IIk_0}(r) = \begin{pmatrix} \tilde{f}(r)\ \Omega_{jlm} \\ i^{1+l-l'} \tilde{g}(r)\ \Omega_{jl'm} \end{pmatrix} \quad (97)$$

where $\tilde{f}$ and $\tilde{g}$ are given by (96).

Let us write also explicitly the $\tilde{Q}$ functions:

$$\tilde{Q}_1 = A_{n_r k} F(-\tilde{n}_r, 2\gamma + 1, \varrho),$$

$$\tilde{Q}_2 = -\tilde{A}_{n_r k} \frac{n_r}{k + \frac{Z\alpha\varkappa}{\lambda}} F(1 - \tilde{n}_r, 2\gamma + 1, \varrho), \quad (98)$$

$$n_r = \begin{cases} 0, 1, 2, 3, \dots & \text{for } k > 0 \\ 1, 2, 3, \dots & \text{for } k < 0, \end{cases}$$

where $\tilde{A}_{n_r k}$ are arbitrary constants. In contrast to Ψ_{Ik_0}, $\tilde{n}_r$ at Ψ_{IIk_0} cannot have zero values for $k < 0$. One gets this solution from (91) after the change $\varkappa \to -\varkappa$.

The substitution of (91) and (97) into (81) gives Φ. The adjoint momentum follows from Eq. (58). By this, the solution of the Lagrange's equation (78) and the canonical equations (59) are found.

5. *Definition of „normal" and „dual" solutions*

The solution Φ is decomposed into the »normal« and »dual« at the free field case according to upper and down components (see Eq. (16)). The question is how to do it now?

We gave the solution of Eq. (78) in terms of Ψ_I and Ψ_{II}. Due to this reason we express the »normal« and »dual« solutions of the free field in terms of the corresponding Ψ_I and Ψ_{II} functions. Inspection of Ref. 1, Eqs. (86) and (87), shows

$$(\Phi_n)_{free\,field} = \frac{1}{\varkappa\sqrt{2}} [\Psi_{In}(\varkappa) - \Psi_{In}(-\varkappa)], \tag{99}$$

$$(\Phi_d)_{free\,field} = \frac{1}{\varkappa\sqrt{2}} [\Psi_{IId}(\varkappa) - \Psi_{IId}(-\varkappa)], \tag{100}$$

where $\Psi_{In}(\varkappa)$ has large upper and $\Psi_{IId}(\varkappa)$ lower components in the nonrelativistic limit. According to $\Phi_d \to \gamma^5 \Phi_n$, we take that it holds true in the case of the hydrogen atom.

Consequently, we write

$$\Phi = \Phi_n + \Phi_d = \frac{1}{\varkappa\sqrt{2}} [(\Psi_{In} + \Psi_{IIn}) + (\Psi_{Id} + \Psi_{IId})] \tag{101}$$

and define

$$\Phi_n = \frac{1}{\varkappa\sqrt{2}} [\Psi_{In}(\varkappa) - \Psi_{In}(-\varkappa)], \tag{102}$$

where $\Psi_{In}(\varkappa)$ is given by Eq. (91), $\Psi_{In}(-\varkappa)$ by the corresponding equation of (97) (let us point out that there are no functions in this solution for $n_r = 0, k > 0$), and

$$\Phi_d = \frac{1}{\varkappa\sqrt{2}}, [\Psi_{IId}(\varkappa) - \Psi_{IId}(-\varkappa)], \tag{103}$$

where $\Psi_{IId}(\varkappa)$ is in dual relation to $\Psi_{In}(\varkappa)$ and $\Psi_{IId}(-\varkappa)$ is determined similarly as $\Psi_{In}(-\varkappa_n)$ with respect $\Psi_{In}(\varkappa)$.

Due to the same reason as in the free field case[1)] we take in the following $\Phi_d = 0$. Keeping only Ψ_n we then leave out the index n. Therefore, we take as physically interesting solution:

$$\Phi = \frac{1}{\varkappa\sqrt{2}} [\Psi_I(\varkappa) - \Psi_I(-\varkappa)], \tag{104}$$

$$\Psi_I(\varkappa) = \begin{pmatrix} f_\varkappa(r)\,\Omega_{jlm} \\ \mathrm{i}^{1+l-l'}\, g(r)\,\Omega_{jl'm} \end{pmatrix}, \tag{105}$$

$$f_\varkappa(r) = A_{n_r k}(\varkappa + k_0)\,\mathrm{e}^{-\frac{\varrho}{2}}\,\varrho^{\gamma-1}\Bigg[F(-n_r, 2\gamma+1, \varrho) +$$

$$+ \frac{n_r}{k - \frac{Z\alpha\varkappa}{\lambda}} F(1 - n_r, 2\gamma+1, \varrho)\Bigg],$$

$$g_{\varkappa}(r) = -A_{n_r k}\sqrt{\varkappa^2 - k_0^2}\,\mathrm{e}^{-\frac{\varrho}{2}}\,\varrho^{\gamma-1}\left[F(-n_r, 2\gamma+1, \varrho) - \right. \tag{106}$$

$$\left. - \frac{n_r}{k - \frac{Z\alpha\varkappa}{\lambda}} F(1-n_r, 2\gamma+1, \varrho)\right],$$

$$n_r = \begin{cases} 0, 1, 2, 3, \ldots, & k < 0, \\ 1, 2, 3, \ldots, & k > 0, \end{cases}$$

$$\Psi_I(-\varkappa) = \begin{pmatrix} f_{-\varkappa}(r)\,\Omega_{jlm} \\ \mathrm{i}^{1+l-l'} g_{-\varkappa}(r)\,\Omega_{jl'm} \end{pmatrix}, \tag{107}$$

$$f_{-\varkappa}(r) = A_{n_r k}(k_0 - \varkappa)\,\mathrm{e}^{-\frac{\varrho}{2}}\,\varrho^{\gamma-1}\left[F(-n_r, 2\gamma+1, \varrho) + \right.$$

$$\left. + \frac{n_r}{k + \frac{Z\alpha\varkappa}{\lambda}} F(1-n_r, 2\gamma+1, \varrho)\right],$$

$$g_{-\varkappa}(r) = -A_{n_r k}\sqrt{\varkappa^2 - k_0^2}\,\mathrm{e}^{-\frac{\varrho}{2}}\,\varrho^{\gamma-1}\left[F(-n_r, 2\gamma+1, \varrho) - \right.$$

$$\left. - \frac{n_r}{k + \frac{Z\alpha\varkappa}{\lambda}} F(1-n_r, 2\gamma+1, \varrho)\right], \tag{108}$$

$$n_r = \begin{cases} 1, 2, 3, \ldots, & k > 0, \\ 1, 2, 3, \ldots, & k < 0. \end{cases}$$

Nonrelativistic limit

In the nonrelativistic limit

$$f(r) \to 2\varkappa \mathrm{e}^{-\frac{\varrho}{2}}\,\varrho^{\gamma-1}(Q_1 + Q_2),\ g(r) \to 0,$$

$$\tilde{f}(r) \to 0,\ \tilde{g}(r) \to 0$$

and, according to (91) and (97),

$$\Psi_I(\varkappa) \to \begin{pmatrix} f(r) & \Omega_{jlm} \\ & 0 \end{pmatrix},\ (f = f_{\varkappa}),$$

$$\Psi_I(-\varkappa) \to 0. \tag{109}$$

Thus, in the nonrelativistic limit the system is completely described by the function $\Psi_I(\varkappa)$.

To the nonrelativistic limit we may come also directly from Eq. (80). Writing

$$k_0 = \varkappa + \varepsilon \tag{110}$$

and keeping only large terms with respect to ε and $\varkappa$ Eq. (80) becomes

$$(2\varkappa\,\varepsilon - 2\varkappa\, e\, A_0 + \Delta)\,\varphi = 0$$

or

$$\left(-\frac{1}{2\varkappa}\Delta + eA_0\right)\varphi = \varepsilon\,\varphi. \tag{111}$$

The solution of this equation for given $\varepsilon < 0$ is

$$\varphi = \sum_{lm} \begin{bmatrix} c^1_{nlm} \\ c^2_{nlm} \\ c^3_{nlm} \\ c^4_{nlm} \end{bmatrix} \chi_{nlm} \tag{112)}$$

where χ_{nlm} are the Schrödinger's wave functions of the hydrogen atom.

In comparisson to the free field case we can define the »normal« and »dual« solutions as

$$\varphi_n = \sum_{lm} \begin{bmatrix} c^1_{nlm} \\ c^2_{nlm} \\ 0 \\ 0 \end{bmatrix} \chi_{nlm},$$

$$\varphi_d = \sum_{lm} \begin{bmatrix} 0 \\ 0 \\ c^3_{nlm} \\ c^4_{nlm} \end{bmatrix} \chi_{nlm}. \tag{113}$$

This is in agreement with (109), since φ_n contains two spin states for given »quantum numbers« *nlm* as it does also (109) when one takes the sum over the corresponding »quantum numbers« for given k_0.

6. Constants of motion

We evaluate the basic constants of motion of the Dirac field interacting with the electromagnetic field first generally and then apply to the hydrogen atom.

The scalar constant of motion, which follows from

$$j^\mu = \mathrm{i}\left(\eta_l^\dagger \frac{\partial \mathscr{L}}{\partial(\partial_\mu \eta_l^\dagger)} - \frac{\partial \mathscr{L}}{\partial(\partial_\mu \eta_l)} \eta_l\right), \; \partial_\mu j^\mu = 0, \tag{114}$$

is given by

$$Q = const_Q \int j_0 \, \mathrm{d}^3 x. \tag{115}$$

For the Lagrangian (30—31) and (69), or explicitly,

$$\mathscr{L} = \varkappa \{ [(-\mathrm{i}\,\partial_\alpha - eA_\alpha)\bar{\Phi}\gamma^\alpha] [(\mathrm{i}\,\partial_\beta - eA_\beta)\gamma^\beta \Phi] - \varkappa^2 \bar{\Phi}\Phi\} -$$

$$-\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} - A_\alpha j^\alpha_{ext} + (\mathscr{L}_{ext}(q, \partial_\alpha q) + A_\alpha j^\alpha{}_{ext}), \tag{116}$$

where $\mathscr{L}_{ext}$ is the Lagrange's density of the external sources interacting with the electromagnetic field (q symbolize the field variables of these sources), the current density j^μ and the scalar constant of motion are

$$j^\mu = e\,i\,(\Phi\,\gamma^\mu \Pi_\Phi{}^\dagger - \Pi_\Phi\,\gamma^0\,\gamma^\mu \Phi) = e\,(\tilde{\Psi}_I\,\gamma^\mu\,\Psi_I - \tilde{\Psi}_{II}\,\gamma^\mu\,\Psi_{II}), \tag{117}$$

$$Q = const_Q\, e \int (\Psi_I^\dagger\,\Psi_I - \Psi_{II}^\dagger\,\Psi_{II})\,\mathrm{d}^3 x \tag{118}$$

or

$$Q = const_Q\, e \int \Psi^\dagger\,\tau_\dagger\,\Psi\,\mathrm{d}^3 x, \tag{119}$$

where

$$\Psi = \begin{pmatrix}\Psi_I \\ \Psi_{II}\end{pmatrix}, \quad \tau_\dagger = \begin{pmatrix}1 & 0 \\ 0 & -1\end{pmatrix}. \tag{120}$$

We do not consider here the current of the external sources.

The energy-momentum constant of motion follows from

$$T_\alpha{}^\beta = \partial_\alpha\,\eta_l^\dagger \frac{\partial \mathscr{L}}{\partial(\partial_\beta \eta_l^\dagger)} + \frac{\partial \mathscr{L}}{\partial(\partial_\beta \eta_l)}\,\partial_\alpha\,\eta_l - \delta_\alpha^\beta \mathscr{L}, \quad \partial_\beta\,T_\alpha{}^\beta = 0, \tag{121}$$

and is given by

$$P_\alpha^D = const_P \int T_\alpha{}^0 \; d^3 x. \tag{122}$$

The tensor $T_\alpha{}^\beta$ for the Lagrangian (116) is

$$T_\alpha{}^\beta = T_\alpha{}^\beta{}_D + T_\alpha{}^\beta{}_{em} + T_\alpha{}^\beta{}_{ext}, \tag{123}$$

where

$$T_a{}^\beta{}_D = \partial_a \overline{\Phi}\, \gamma^\beta \Pi_\Phi{}^\dagger + \Pi_\Phi\, \gamma^0\, \gamma^\beta\, \partial_a \Phi - \delta_a{}^\beta\, \varkappa \left[\frac{1}{\varkappa^2}\, \Pi_\Phi\, \gamma^0\, \Pi_\Phi{}^\dagger \;-\; \varkappa^2\, \overline{\Phi}\, \Phi\right], \quad (124)$$

$$T_a{}^\beta{}_{em} = -\frac{1}{4\pi} F^{\beta\lambda}\, \partial_a A_\lambda + \delta_a{}^\beta \frac{1}{16\,\pi} F_{\mu\nu} F^{\mu\nu}, \quad (125)$$

$$T_a{}^\beta{}_{ext} = \textit{the part due to external sources.} \quad (126)$$

In the following we consider only $T_a{}^\beta{}_D$. Using (58) it becomes

$$T_a{}^\beta{}_D = \frac{1}{2\mathrm{i}} [(\partial_a \overline{\Psi}_I\, \gamma^\beta\, \Psi_I - \overline{\Psi}_I\, \gamma^\beta\, \partial_a \Psi) - (\partial_a \overline{\Psi}_{II}\, \gamma^\beta\, \Psi_{II} - \overline{\Psi}_{II}\, \gamma^\beta\, \partial_a \Psi_{II})$$

$$+ \frac{\partial}{\partial x_a} (\overline{\Psi}_{II}\, \gamma^\beta\, \Psi_I - \overline{\Psi}_I\, \gamma^\beta\, \Psi_{II})] + \varkappa\, \delta_a{}^\beta\, (\Psi_I\, \overline{\Psi}_{II} + \overline{\Psi}_{II}\, \Psi_I). \quad (127)$$

The corresponding contribution to the energy-momentum constant of motion (122) is

$$P_a{}^D = const_P \frac{1}{2\mathrm{i}} \int [(\partial_a \Psi_I^\dagger\, \Psi_I - \Psi_I^\dagger\, \partial_a \Psi_I) - (\partial_a \Psi_{II}^\dagger\, \Psi_{II} - \Psi_{II}^\dagger\, \partial_a \Psi_{II}) +$$

$$+ \frac{\partial}{\partial x_a} (\Psi_{II}^\dagger\, \Psi_I + \Psi_I^\dagger\, \Psi_{II})]\, d^3 x + \varkappa \int (\overline{\Psi}_I\, \Psi_{II} + \overline{\Psi}_{II}\, \Psi_I)\, \mathrm{d}^3 x\, \delta_a^0 . \quad (128)$$

Assuming that the surface integrals vanish at infinity, the space components become

$$P_i{}^D = const_P\ \frac{\mathrm{i}}{2}\ \int [\, (\Psi_I^\dagger\, \partial_i\, \Psi_I - \partial_i\, \Psi_I^\dagger\, \Psi_I) - (\Psi_{II}^\dagger\, \partial_i\, \Psi_{II} - \partial_i\, \Psi_{II}^\dagger\, \Psi_{II})]\, \mathrm{d}^3 x \quad (129)$$

and the time component, using also equations for Ψ_I and Ψ_{II}, can be written in the form

$$P_0{}^D = const_P \frac{\mathrm{i}}{2} \int [(\Psi_I^\dagger\, \partial_0\, \Psi_I - \partial_0\, \Psi_I^\dagger\, \Psi_I) - (\Psi_{II}^\dagger\, \partial\, \Psi_{II}^\dagger - \partial_0\, \Psi_{II}^\dagger\, \Psi_{II})]\, \mathrm{d}^3 x. \quad (130)$$

Thus, we have

$$P_a{}^D = const_P \frac{1}{2} \int \{[\Psi_I^\dagger\, (\mathrm{i}\, \partial_a\, \Psi_I) - (\mathrm{i}\, \partial_a\, \Psi_I^\dagger)\, \Psi_I] -$$

$$- [\Psi_{II}^\dagger\, (\mathrm{i}\, \partial_a\, \Psi_{II}) - (\mathrm{i}\, \partial_a\, \Psi_{II}^\dagger)\, \Psi_{II}]\}\, \mathrm{d}^3 x \quad (131)$$

or

$$P_a{}^D = const_P \int \Psi^\dagger\, \tau_\dagger\, (\mathrm{i}\, \partial_a)\, \Psi\, \mathrm{d}^3 x. \quad (132)$$

The angular momentum density tensor is

$$M^{\alpha,\beta\gamma} = (x^\gamma T^{\gamma\alpha} - x^\beta T^{\gamma\alpha}) - \frac{\partial \mathcal{L}}{\partial (\partial_\alpha \eta_l)} \left(\frac{\mathrm{i}}{2} I_\eta^{\beta\gamma} \right) \eta_l - \eta^\dagger \left(- \frac{\mathrm{i}}{2} I_\eta^{\beta\gamma\dagger} \right) \frac{\partial \mathcal{L}}{\partial (\partial_\alpha \eta_l)}. \quad (133)$$

From

$$\partial_\alpha M^{\alpha,\beta\gamma} = 0$$

follows the constant of motion

$$M^{\beta\gamma} = const_M \int M^{0,\beta\gamma} \,\mathrm{d}^3 x. \quad (134)$$

We evaluate the part of $M^{\alpha,\beta\gamma}$ which corresponds to the Dirac field:

$$M_D^{\alpha,\beta\gamma} = (x^\gamma T_D^{\beta\alpha} - x^\beta T_D^{\gamma\alpha}) - \frac{\partial \mathcal{L}}{\partial (\partial^\alpha \Phi)} \left(\frac{\mathrm{i}}{2} \sigma^{\beta\gamma} \right) \Phi - \Phi^\dagger \left(- \frac{\mathrm{i}}{2} \sigma^{\beta\gamma\dagger} \right) \frac{\partial \mathcal{L}}{\partial (\partial^\alpha \Phi^\dagger)} =$$

$$= (x^\gamma T_D^{\beta\alpha} - x^\beta T_D^{\gamma\alpha}) + \frac{1}{4} (\overline{\Psi}_I - \overline{\Psi}_{II}) \gamma^\alpha \sigma^{\beta\gamma} (\Psi_I + \Psi_{II}) + \quad (135)$$

$$+ \frac{1}{4} (\overline{\Psi}_I + \overline{\Psi}_{II}) \sigma^{\beta\gamma\dagger} \gamma^\alpha (\Psi_I - \Psi_{II}).$$

The corresponding contribution to $M^{\beta\gamma}$ is

$$M_D^{\beta\gamma} = const_M \int \{x^\gamma T_D^{\beta 0} - x^\beta T_D^{\gamma 0} + \frac{1}{2} \Psi_I^\dagger \sigma^{\beta\gamma} \Psi_I - \frac{1}{2} \Psi_{II}^\dagger \sigma^{\beta\gamma} \Psi_{II}\} \,\mathrm{d}^3 x. \quad (136)$$

The space components of this tensor are

$$M_D^{ij} = const_M \int \{x^j T_D^{i0} - x^i T_D^{j0} + \frac{1}{2} \Psi_I^\dagger \sigma^{ij} \Psi_I - \frac{1}{2} \Psi_I^\dagger \sigma^{ij} \Psi_{II}\} \,\mathrm{d}^3 x. \quad (137)$$

The angular momentum vector components

$$M_k^D = \frac{1}{2} \varepsilon_{ijk} M_D^{ij} \quad (138)$$

are then

$$M_k^D = const_M \{L_k + \int \left(\Psi_I^\dagger \frac{1}{2} \Sigma_k \Psi_I - \Psi_{II}^\dagger \frac{1}{2} \Sigma_k \Psi_{II} \right) \mathrm{d}^3 x\}, \quad (139)$$

$$L_k = \int \frac{1}{2} \varepsilon_{ijk} (x^j T_D^{i0} - x^i T_D^{j0}) \,\mathrm{d}^3 x, \quad \Sigma_k = \frac{1}{2} \varepsilon_{ijk} \sigma^{ij}. \quad (140)$$

It is useful to write L_k in explicit field components form. Using (127) one gets

$$L_k = \int \left\{ \frac{1}{2} \varepsilon_{ijk} \Psi_I^\dagger \left[x^j (\mathrm{i}\, \partial^i) - x^i (\mathrm{i}\, \partial^j) \right] \Psi_I - \frac{1}{2} \varepsilon_{ijk} \Psi_{II}^\dagger \left[x^j (\mathrm{i}\, \partial^i) - \right.\right.$$

$$\left.\left. - x^j (\mathrm{i}\, \partial^j) \right] \Psi_{II} \right\} \mathrm{d}^3 x \qquad (141)$$

or introducing the operator (of orbital momentum)

$$\hat{L}_k = \frac{1}{2} \varepsilon_{ijk} (x^j p^i - x^i p^j), \qquad (142)$$

where

$$p^i = \mathrm{i}\, \partial^i, \qquad (143)$$

$$L_k = \int (\Psi_I^\dagger \hat{L}_k \Psi_I - \Psi_{II}^\dagger \hat{L}_k \Psi_{II}) \mathrm{d}^3 x. \qquad (144)$$

Eq. (139) then becomes

$$M_k^p = const_M \int \left\{ \Psi_I^\dagger \left(\hat{L}_k + \frac{1}{2} \Sigma_k \right) \Psi_I - \Psi_{II}^\dagger \left(\hat{L}_k + \frac{1}{2} \Sigma_k \right) \Psi_{II} \right\} \mathrm{d}^3 x \qquad (145)$$

or

$$M_k^p = const_M \int \Psi^\dagger \tau_+ \left(L_k + \frac{1}{2} \Sigma_k \right) \Psi \, \mathrm{d}^3 x. \qquad (146)$$

Interpretation of the constants of motion

In accordance with the free field case[1] we accept the statistical interpretation of the field functions as probability amplitudes for two particles: a particle and its antiparticle. Consequently, we separate the field Φ as well as Ψ_I and Ψ_{II} in negative and positive frequency parts:

$$\Phi = \xi + \zeta, \qquad (147)$$

$$\Psi_I = \Psi_{I\xi} + \Psi_{I\zeta}, \qquad (148)$$

$$\Psi_{II} = \Psi_{II\xi} + \Psi_{II\zeta}, \qquad (149)$$

where ξ contains only the negative frequency terms and ξ only the positive frequency terms. In the next we consider a time independent external field.

The scalar constant of motion (118) is then also separated,

$$Q = Q_\xi + Q_\zeta, \qquad (150)$$

where

$$Q_{\xi} = const_Q\, e \int (\Psi^{\dagger}_{I\xi} \Psi_{I\xi} - \Psi^{\dagger}_{II\xi} \Psi_{II\xi})\, \mathrm{d}^3 x,$$

$$Q_{\zeta} = const_Q\, (-e) \int (\Psi^{\dagger}_{II\zeta} \Psi_{II\zeta} - \Psi^{\dagger}_{I\zeta} \Psi_{I\zeta})\, \mathrm{d}^3 x \tag{151}$$

or

$$Q_{\xi} = const_Q\, e \int \Psi^{\dagger}_{+} \tau_{+} \Psi_{+}\, \mathrm{d}^3 x,$$

$$Q_{\zeta} = const_Q\, (-e) \int \Psi^{\dagger}_{-} \tau_{-} \Psi_{-}\, \mathrm{d}^3 x, \tag{152}$$

where Ψ_{+} and Ψ_{-} are the negative and positive frequency parts of Ψ given by(120), respectively, and

$$\tau_{-} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{152}$$

Now, we select

$$const_Q = const_P = const_M = 1 \tag{153}$$

and require

$$\int (\Psi^{\dagger}_{I\xi} \Psi_{I\xi} - \Psi^{\dagger}_{II\xi} \Psi_{II\xi})\, \mathrm{d}^3 x = \int \Psi^{\dagger}_{+} \tau_{+} \Psi_{+}\, \mathrm{d}^3 x = 1, \tag{154}$$

$$\int (\Psi^{\dagger}_{II\zeta} \Psi_{II\zeta} - \Psi^{\dagger}_{I\zeta} \Psi_{I\zeta})\, \mathrm{d}^3 x = \int \Psi^{\dagger}_{-} \tau_{-} \Psi_{-}\, \mathrm{d}^3 x = 1. \tag{155}$$

The scalar constant of motion then becomes

$$Q = \begin{cases} e, & \Psi_{+} \neq 0,\ \Psi_{-} = 0, \\ -e, & \Psi_{+} = 0,\ \Psi_{-} \neq 0, \\ 0, & \Psi_{+} \neq 0,\ \Psi_{-} \neq 0. \end{cases} \tag{156}$$

The quantity

$$\Psi^{\dagger}_{I\xi} \Psi_{I\xi} - \Psi^{\dagger}_{II\xi} \Psi_{II\xi} = \Psi^{\dagger}_{+} \tau_{+} \Psi_{+} \tag{157}$$

is the probability density of finding the particle with the charge e at space-time point $(\vec{x}, t)$ and the quantity

$$\Psi^{\dagger}_{II\zeta} \Psi_{II\zeta} - \Psi^{\dagger}_{I\zeta} \Psi_{I\zeta} = \Psi^{\dagger}_{-} \tau_{-} \Psi_{-} \tag{158}$$

is the probability density of finding its antiparticle at the space-time point $(\vec{x}, t)$.

This is the same as in the free field case. The meaning of other constants of motion is analogous.

Application to the hydrogen atom

In the case of hydrogen atom there are no positive frequency terms. Thus, the scalar constant of motion (156) is

$$Q = e \tag{159}$$

and

$$\int (\Psi_I^\dagger \Psi_I - \Psi_{II}^\dagger \Psi_{II})\, d^3 x = 1 \qquad (160)$$

where Ψ_I is given by (105) and Ψ_{II} by (107). The normalization condition (160) determines the constants $A_{n_r k}$ in Eqs. (105) and (107). Using

$$\int \Omega_{jlm}^\dagger \Omega_{j'l'm'}\, d\Omega = \delta_{jj'} \delta_{ll'} \delta_{mm'} \qquad (161)$$

this condition becomes

$$\int (f^* f + g^* g - \tilde{\tilde{f}}{}^* \tilde{\tilde{f}} - \tilde{\tilde{g}}{}^* \tilde{\tilde{g}})\, r^2\, dr = 1. \qquad (162)$$

From here one gets

$$A_{n_r k} = \frac{(2\lambda)^{3/2}(2\gamma + n_r)}{2\varkappa \Gamma(2\gamma + 1)} \left[\frac{\Gamma(2\gamma + n_r)}{(n_r - 1)!\, 2 \left(\frac{Z\alpha\varkappa}{\lambda}\right)\left(k + \frac{Z\alpha k_0}{\lambda}\right)} \right]^{1/2}, \; n_r \neq 0,$$

$$A_{0k} = \frac{\lambda^2}{\Gamma(2\gamma + 1)} \left[\frac{2\Gamma(2\gamma + 1)\left(\frac{Z\alpha\varkappa}{\lambda} - k\right)}{Z\alpha\varkappa^2} \right]^{1/2}, \; n_r = 0. \qquad (163)$$

The energy of the system for the solution of given k_0, according to (132) and (160), is

$$P_0 = k_0$$

where k_0 is given by (87). From here we conclude that the energies of the relativistic hydrogen atom are equal to those of the standard theory. The space probability distribution, however, is not the same. For a given energy the space probability distribution is (157) while in the standard theory it is only the first term of this expression*. In the nonrelativistic limit $Z\alpha \to 0$, the function Ψ_{II} goes to zero and this difference disappears. The space probability distribution reduces to the standard nonrelativistic quantum theory.

These results enable us to interpret the solution (91) and (97) in terms of the hydrogen atom. Although we have already used this interpretation, it is important to state it once clearly.

For an arbitrary solution of Eqs. (59) for the hydrogen atom P_0 determined by (132) gives the average energy. Making use of the expansion

$$\Psi(\vec{r}, t) = \sum_{k_0} a_{k_0} \Psi_{k_0}(\vec{r}, t), \qquad (164)$$

* With the correspoding normalization.

where k_0 stands for all parameters characterizing a stationary state, we get

$$P^0 = \int \Psi^\dagger \tau_+ \mathrm{i}\, \partial_0 \Psi \,\mathrm{d}^3 x = \sum_{k_0} k_0\, a^\dagger_{k_0} a_{k_0}. \tag{165}$$

Due to

$$\sum_{k_0} a^\dagger_{k_0} a_{k_0} = 1 \tag{166}$$

which comes from (160), $a^\dagger_{k_0} a_{k_0}$ is the energy probability of the value k_0 and P^0 is the average energy.

Writing Eqs. (59) in the from

$$\mathrm{i}\, \partial_0 \Psi_I = (-\mathrm{i}\, \partial_j \alpha^j + eA_a \gamma^0 \gamma^a + \varkappa\beta)\, \Psi_I, \tag{167}$$

$$\mathrm{i}\, \partial_0 \Psi_{II} = (-\mathrm{i}\, \partial_j \alpha^j + eA_a \gamma^0 \gamma^a - \varkappa\beta)\, \Psi_{II},$$

or

$$\mathrm{i}\, \partial_0 \Psi = H \Psi, \tag{168}$$

where

$$H = \begin{pmatrix} -\mathrm{i}\, \partial_j \alpha^j + eA_a \gamma^0 \gamma^a + \varkappa\beta & 0 \\ 0 & -\mathrm{i}\, \partial_j \alpha^j + eA_a \gamma^0 \gamma^a - \varkappa\beta \end{pmatrix}, \tag{169}$$

the expression for P_0 becomes

$$P^0 = \int \Psi^\dagger \tau_+ H \Psi \,\mathrm{d}^3 x. \tag{170}$$

The operator H plays the role of the Hamiltonian of the standard theory.

Let us again look at the nonrelativistic limit. Taking $Z\alpha \to 0$ and Ψ_{II} in the form (97), Eq. (167) reduces to

$$\mathrm{i}\, \partial_0 \Psi_I = (-\mathrm{i}\, \partial_j \alpha^j + eA_0 + \varkappa\beta)\, \Psi_I. \tag{171}$$

This goes to the Schrödinger's equation of the hydrogen atom in the nonrelativistic limit with the corresponding Hamiltonian. Consequently, in the nonrelativistic limit P_0 becomes

$$\int \Psi_S^* H_S \Psi_S \,\mathrm{d}^3 x, \tag{172}$$

where index S denotes the quantities of the Schrödinger's quantum mechanics. Here we have used certain properties of Ψ_{II}. We may take the opposite possibility: to require the nonrelativistic limit of P_0 and establish by this the nonrelativistic behaviour of Ψ_{II}.

Similarly to the energy P^0, the spin parts of the angular momentum vector (146) has the meaning of the spin average value of the system.

At the end let us write the explicit expressions for the space probability distribution of the electron in the new and the standard theory for a given energy state. According to (157), (105) and (107) in the new theory we have

$$w_{new}(\vec{r}) \equiv \Psi_+^+ \tau_+ \Psi_+ = (f_\varkappa^* f_\varkappa - f_{-\varkappa}^* f_{-\varkappa}) \Omega_{jlm} \Omega_{jlm} + (g_\varkappa^* g_\varkappa - g_{-\varkappa}^* g_{-\varkappa}) \Omega_{jl'm} \Omega_{jl'm}, \tag{173}$$

where are

$$f_\varkappa^* f_\varkappa - f_{-\varkappa}^* f_{-\varkappa} = |A_{n_r k}^{st}|^2 \, 2k_0 \frac{\frac{Za\varkappa}{\lambda} + k}{\frac{Zak_0}{\lambda} + k} e^{-\varrho} \varrho^{2\gamma-2} \tag{174}$$

$$\times \left[Q_{n_r}^{2\gamma} + \frac{n_r}{k - \frac{Za\varkappa}{\lambda}} \left(1 + \beta \frac{Zak_0}{\lambda}\right) Q_{n_r-1}^{2\gamma} \right] \left[Q_{n_r}^{2\gamma} + \frac{n_r}{k - \frac{a\varkappa}{\lambda}} \left(1 - \beta \frac{Za\varkappa}{\lambda}\right) Q_{n_r-1}^{2\gamma} \right],$$

$$g_\varkappa^* g_\varkappa - g_{-\varkappa}^* g_{-\varkappa} = |A_{n_r k}^{st}|^2 \frac{2(\varkappa^2 - k^2)}{\varkappa} \frac{\left(-n_r \frac{Za\varkappa}{\lambda}\right)}{\left(k - \frac{Za\varkappa}{\lambda}\right)\left(k + \frac{Zak_0}{\lambda}\right)} e^{-\varrho} \varrho^{2\gamma-2}$$

$$\times Q_{n_r-1}^{2\gamma} \left[Q_{n_r}^{2\gamma} - n_r \frac{k}{k^2 - \left(\frac{Za\varkappa}{\lambda}\right)^2} \cdot Q_{n_r-1}^{2\gamma} \right], \tag{175}$$

$$|A_{n_r k}|^2 = |A_{n_r k}^{st}|^2 \frac{k + \frac{Za\varkappa}{\lambda}}{2\varkappa \left(k + \frac{Zak_0}{\lambda}\right)}, \quad \beta = \frac{1 - \frac{\varkappa}{k_0}}{k + \frac{Za\varkappa}{\lambda}}. \tag{176}$$

The corresponding standard probability is

$$w_{st}(\vec{r}) = \Psi_I^\dagger \Psi_I = f^* f \, \Omega_{jlm} \Omega_{jlm} + g^* g \, \Omega_{jl'm} \Omega_{jl'm}, \tag{177}$$

where

$$f^* f = |A_{n_r k}^{st}|^2 (\varkappa + k_0) \left[Q_{n_r}^{2\gamma} + \frac{n_r}{k - \frac{Za\varkappa}{\lambda}} Q_{n_r-1}^{2\gamma} \right]^2 e^{-\varrho} \varrho^{2\gamma-2}, \tag{178}$$

$$g^* g = |A_{n_r k}^{st}|^2 (\varkappa - k_0) \left[Q_{n_r}^{2\gamma} - \frac{n_r}{k - \frac{Za\varkappa}{\lambda}} Q_{n_r-1}^{2\gamma} \right]^2 e^{-\varrho} \varrho^{2\gamma-2}. \tag{179}$$

7. Conclusions

The obtained results show that the new Dirac field theory[1] extended to the field interacting with the electromagnetic field gives: (a) conceptually the same results for the hydrogen atom as the free field case, (b) the same energy spectrum of the hydrogen atom as is given by the standard theory but not the same corresponding electron space probability distribution, (c) the standard nonrelativistic quantum mechanics of the hydrogen atom.

Thus, these results support the new Dirac field theory. By this one gets quite a different view of the quantum physics and related content of the contemporary physics.

Appendix

The equation (80),

$$\{(k_0 - eA_0)^2 + \Delta - \mathrm{i}\, e\, \vec{\alpha}\, \vec{E}\}\, \varphi = 0 \tag{A1}$$

can be written in the form

$$(k_0 + H - 2eA_0)\,(k_0 - H)\,\varphi = 0 \tag{A2}$$

where

$$H = \vec{p}\,\vec{\alpha} + eA_0 + \beta\varkappa, \ \vec{p} = -\mathrm{i}\,\nabla. \tag{A3}$$

Due to

$$[H, \vec{J}^2] = [H, J_z] = 0 \tag{A4}$$

$$[A_0, \vec{J}^2] = [A_0, J_z] = 0,$$

where

$$\vec{J} = \vec{l} + \frac{1}{2}\vec{\Sigma}, \ \vec{l} = \vec{r} \times (-\mathrm{i}\,\nabla), \ \vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}, \tag{A5}$$

solutions of Eq. (A2) can be separated in radial and angular variables according to[6]

$$\varphi \to \varphi_{k_0 jlm} = \begin{pmatrix} \varphi_a \\ \varphi_b \end{pmatrix} = \begin{pmatrix} F(r)\, \Omega_{jlm}(\vec{n}) \\ (-1)^{\frac{1+l-l'}{2}} G(r)\, \Omega_{jl'm}(\vec{n}) \end{pmatrix}, \tag{A6}$$

where $\varphi_{k_0 jlM}$ is the eigenfunction of $\vec{J}^2$ and J_z,

$$\vec{J}^2\, \varphi_{k_0 jlm} = j\,(j+1)\, \varphi_{k_0 jlm},$$

$$J_z\, \varphi_{k_0 jlm} = M\, \varphi_{k_0 jlm}, \tag{A7}$$

$\Omega_{k_0 jlm}(\vec{n})$ is the spherical spinor and $l = j \pm \frac{1}{2}$, $l' = 2j - l$, $\vec{n} = \frac{\vec{r}}{r}$.

After substitution of (A7) into (A2) follows

$$(k_0 + H - 2_0 A) \begin{pmatrix} (k_0 - eA_0 - \varkappa)\,\varphi_a - \vec{\sigma}\vec{p}\,\varphi_b \\ -\vec{\sigma}\vec{p}\,\varphi_a + (k_0 - eA_0 + \varkappa)\,\varphi_b \end{pmatrix} = 0. \qquad \text{(A8)}$$

By making use of[6)]

$$\Omega_{jl'm} = i^{l-l'} (\vec{\sigma}\vec{n})\,\Omega_{jlm}, \qquad \text{(A9)}$$

$$(\vec{\sigma}\vec{p})\,\varphi_b = -\left(\frac{dG}{dr} + \frac{1-k}{r} G\right)\Omega_{jlm} \qquad \text{(A10)}$$

$$(\vec{\sigma}\vec{p})\,\varphi_k = \left(\frac{dF}{dr} + \frac{1+k}{r} F\right)\Omega_{jl'm}\, i^{1+l-l'}, \qquad \text{(A11)}$$

$$k = \begin{cases} -\left(j + \frac{1}{2}\right) = -(l+1), & j = l + \frac{1}{2}, \\ +\left(j + \frac{1}{2}\right) = l, & j = l - \frac{1}{2} \end{cases} \qquad \text{(A12)}$$

($k = \pm 1, \pm 2, \pm 3, \ldots$; positive values correspond to $j = l - \frac{1}{2}$ and negative to $j = l + \frac{1}{2}$) it becomes

$$(k_0 + H - 2eA_0) \begin{pmatrix} \left[(k_0 - eA_0 - \varkappa)\,F + \left(\frac{dG}{dr} + \frac{1-k}{r} G\right)\right]\Omega_{jlm} \\ \left[-\left(\frac{dF}{dr} + \frac{1+k}{r} F\right) + (k_0 - eA_0 + \varkappa)\,G\right] i^{1+l-l'}\,\Omega_{jl'm} \end{pmatrix} \equiv$$

$$\equiv (k_0 + H - 2eA_0) \begin{pmatrix} \widetilde{F}\,\Omega_{jlm} \\ i^{1+l-l'}\,\widetilde{G}\,\Omega_{jl'm} \end{pmatrix}. \qquad \text{(A13)}$$

The matrix on which the operator $(k_0 + H - 2eA_0)$ acts is of the same form as in (A6). Consequently the procedure repeats now with the operator $(k_0 + H - 2eA_0)$. Let us notice that

$$(k_0 + H - 2eA_0) = \begin{pmatrix} k_0 - eA_0 + \varkappa & \vec{\sigma}\vec{p} \\ \vec{\sigma}\vec{p} & k_0 - eA_0 - \varkappa \end{pmatrix} \qquad \text{(A14)}$$

As the result one gets

$$\begin{pmatrix} \left[(k_0 - eA_0 + \varkappa)\,\widetilde{F} - \left(\frac{\mathrm{d}\widetilde{G}}{\mathrm{d}r} + \frac{1-k}{r}\,\widetilde{G}\right)\right] \Omega_{jlm} \\ \left[\left(\frac{\mathrm{d}\widetilde{F}}{\mathrm{d}r} + \frac{1+k}{r}\,\widetilde{F}\right) + (k_0 - eA_0 - \varkappa)\,\widetilde{G}\right] \mathrm{i}^{1+l-l'}\,\Omega_{jlm} \end{pmatrix} = 0. \qquad \text{(A15)}$$

From here follows

$$\left[(k_0 - eA_0 + \varkappa)\,\widetilde{F} - \left(\frac{\mathrm{d}\widetilde{G}}{\mathrm{d}r} + \frac{1-k}{r}\,\widetilde{G}\right)\right] = 0, \qquad \text{(A16)}$$

$$\left(\frac{\mathrm{d}\widetilde{F}}{\mathrm{d}r} + \frac{1+k}{r}\,\widetilde{F}\right) + (k_0 - eA_0 - \varkappa)\,\widetilde{G} = 0$$

or after substitution of the expressions for $\widetilde{F}$ and $\widetilde{G}$

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r} + \left(k_0^2 - \varkappa^2 + 2k_0\frac{Ze^2}{r} + \frac{Z^2e^4}{r^2}\right) - k\,\frac{1+k}{r^2}\right] F + \frac{Ze^2}{r^2}\,G = 0,$$

$$\text{(A17)}$$

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r} + \left(k_0^2 - \varkappa^2 + 2k_0\frac{Ze^2}{r} + \frac{Z^2e^3}{r^2}\right) + k\,\frac{1-k}{r^2}\right] G - \frac{Ze^2}{r^2}\,F = 0.$$

It is useful to introduce the new variable

$$\varrho = 2\lambda r, \quad \lambda = \sqrt{\varkappa^2 - k_0^2}, \qquad \text{(A18)}$$

and the notations

$$\gamma^2 = k^2 - Z^2\alpha^2, \quad \alpha = e^2. \qquad \text{(A19)}$$

Eqs. (A17) then become

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}\varrho^2} + \frac{2}{\varrho}\frac{\mathrm{d}}{\mathrm{d}\varrho} + \left(-\frac{1}{4} + \frac{k_0}{\lambda}\frac{Z\alpha}{\varrho} - \frac{\gamma^2}{\varrho^2}\right) - \frac{k}{\varrho^2}\right] F + \frac{Z\alpha}{\varrho^2}\,G = 0,$$

$$\text{(A20)}$$

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}\varrho^2} + \frac{2}{\varrho}\frac{\mathrm{d}}{\mathrm{d}\varrho} + \left(-\frac{1}{4} + \frac{k_0}{\lambda}\frac{Z\alpha}{\varrho} - \frac{\gamma^2}{\varrho^2}\right) + \frac{k}{\varrho^2}\right] G - \frac{Z\alpha}{\varrho^2}\,F = 0.$$

The transformation

$$\begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} 1 & -\dfrac{Z\alpha}{\gamma + k} \\ \dfrac{Z\alpha}{\gamma + k} & -1 \end{pmatrix} \begin{pmatrix} P \\ Q \end{pmatrix} \qquad \text{(A21)}$$

separates Eqs. (A20) and one gets for the functions P and Q

$$\left[\frac{d^2}{d\varrho^2}+\frac{2}{\varrho}\frac{d}{d\varrho}+\left(-\frac{1}{4}+\frac{k_0}{\lambda}\frac{Z\alpha}{\varrho}-\frac{\gamma(\gamma+1)}{\varrho^2}\right)\right]P=0,$$

$$\left[\frac{d^2}{d\varrho^2}+\frac{2}{\varrho}\frac{d}{d\varrho}+\left(-\frac{1}{4}+\frac{k_0}{\lambda}\frac{Z\alpha}{\varrho}-\frac{\gamma(\gamma-1)}{\varrho^2}\right)\right]Q=0. \tag{A22}$$

We investigate first the limit $\varrho\to 0$:

$$\left[\frac{d^2}{d\varrho^2}+\frac{2}{\varrho}-\frac{\gamma(\gamma+1)}{\varrho^2}\right]P_{\varrho\to 0}=0,$$

$$\left[\frac{d^2}{d\varrho^2}+\frac{2}{\varrho}-\frac{\gamma(\gamma-1)}{\varrho^2}\right]Q_{\varrho\to 0}=0. \tag{A23}$$

The solutions of these equations are

$$P_{\varrho\to 0}=const\ \varrho^s,\ Q_{\varrho\to 0}=const\ \varrho^v, \tag{A24}$$

where s and v satisfy the equations

$$s(s-1)+2s-\gamma(\gamma+1)=0, \tag{A25}$$

$$v(v-1)+2v-\gamma(\gamma-1)=0.$$

From here we have

$$s=\begin{cases}\gamma\\-(\gamma+1)\end{cases},\quad v=\begin{cases}-\gamma\\\gamma-1\end{cases} \tag{A26}$$

and

$$P_{\varrho\to 0}=const\ \varrho^{\gamma},\ Q_{\varrho\to 0}=const\ \varrho^{\gamma-1} \tag{A27}$$

as acceptable solutions. For $|k|=1$ there is a »weak« singularity in Q not affecting the wave functions normalization.

In the limit $\varrho\to\infty$ Eqs. (A22) become

$$\left(\frac{d^2}{d\varrho^2}-\frac{1}{4}\right)P_{\varrho\to\infty}=0,$$

$$\left(\frac{d^2}{d\varrho^2}-\frac{1}{4}\right)Q_{\varrho\to\infty}=0. \tag{A28}$$

The physically acceptable solutions of these equations are

$$P_{\infty} = const\, e^{-\frac{\varrho}{2}}, \quad Q_{\infty} = const\, e^{-\frac{\varrho}{2}}. \tag{A29}$$

Having these asymptotic behaviours we may write

$$\begin{aligned} P &= e^{-\frac{\varrho}{2}} \varrho^{\gamma} p, \\ Q &= e^{-\frac{\varrho}{2}} \varrho^{\gamma-1} q. \end{aligned} \tag{A30}$$

After substitution in (A22) one gets equations for p and q:

$$\begin{aligned} &\frac{d^2 p}{d\varrho^2} + (2\gamma + 2 - \varrho)\frac{dp}{d\varrho} - \left(\gamma + 1 - \frac{k_0}{\lambda} Za\right) p = 0, \\ &\frac{d^2 q}{d\varrho^2} + (2\gamma - \varrho)\frac{dq}{d\varrho} - \left(\gamma - \frac{k_0}{\lambda} Za\right) q = 0. \end{aligned} \tag{A31}$$

Solutions of these equations are the confluent hypergeometric functions:

$$\begin{aligned} p &= {}_1F_1\left\{\left(\gamma + 1 - \frac{k_0}{\lambda} Za\right),\ 2(\gamma + 1),\ \varrho\right\}, \\ q &= {}_1F_1\left\{\left(\gamma - \frac{k_0}{\lambda} Za\right),\ 2\gamma,\ \varrho\right\}. \end{aligned} \tag{A32}$$

For $\varrho \gg 1$

$${}_1F_1(a, b, \varrho) \approx e^{-i\pi a} \frac{\Gamma(b)}{\Gamma(b-a)} \varrho^{-a} + \frac{\Gamma(b)}{\Gamma(a)} \varrho^{a-b} e^{\varrho}. \tag{A33}$$

The term e^{ϱ} will not be present if

$$\frac{1}{\Gamma(a)} = 0. \tag{A34}$$

The application of this condition to (A32) gives

$$\frac{1}{\Gamma\left(\gamma + 1 - \frac{k_0}{\lambda} Za\right)} = 0, \quad \frac{1}{\Gamma\left(\gamma - \frac{k_0}{\lambda} Za\right)} = 0. \tag{A35}$$

The poles of Γ-functions are negative integers and zero.

Thus, we have

$$\gamma + 1 - \frac{k_0}{\lambda} Za = -\tilde{n}_r, \ \tilde{n}_r = 0, 1, 2, \ldots, \tag{A36}$$

$$\gamma - \frac{k_0}{\lambda} Za = -n_r, \ n_r = 0,1,2, \ldots . \tag{A37}$$

From here follows

$$\tilde{n}_r = n_r - 1. \tag{A38}$$

Therefore, the series (A32) terminate, expect p for $n_r = 0$ where we take $p = 0$ and become

$$p = {}_1F_1 \{-n_r + 1, (2\gamma + 1) + 1, \varrho\}, \ p_{n_r=0} = 0, \tag{A39}$$

$$q = {}_1F_1 \{-n_r, (2\gamma - 1) + 1, \varrho\}.$$

By making use of

$$ {}_1F_1 (-s, d + 1, \varrho) = \frac{\Gamma(d+1)}{\Gamma(d+1+s)} Q_s^d (\varrho) \tag{A40}$$

where $Q_s^d(\varrho)$ is a Laguerre polynomial, they can be written in the form

$$p_{n_r k} = \frac{\Gamma(2\gamma + 1)}{\Gamma(2\gamma + n_r + 1)} Q_{n_r - 1}^{2\gamma+1} (\varrho), \ \ p_{0k} = 0, \tag{A41}$$

$$q_{n_r k} = \frac{\Gamma(2\gamma)}{\Gamma(2\gamma + n_r)} Q_{n_r}^{2\gamma-1} (\varrho), \ n_r = 0, 1, 2, \ldots . \tag{A42}$$

Returning to (A30) we find

$$P_{n_r k} = a_{n_r k} \, e^{-\frac{\varrho}{2}} \varrho^{\gamma} Q_{n_r - 1}^{2\gamma+1} (\varrho), \ a_{0k} = 0, \tag{A43}$$

$$Q_{n_r k} = b_{n_r k} \, e^{-\frac{\varrho}{2}} \varrho^{\gamma-1} Q_{n_r}^{2\gamma-1} (\varrho), \tag{A44}$$

$$n_r = 0, 1, 2, \ldots,$$

where $a_{n_r k}$ and $b_{n_r k}$ are arbitrary constants. The Γ-functions are incorporated in the constants.

The functions $F(r)$ and $G(r)$, according to (A21) are then

$$F_{n_r k} = \{a_{n_r k}\, \varrho^{\gamma}\, Q_{n_r - 1}^{2\gamma+1}(\varrho) - \frac{Z\alpha}{\gamma + k}\, b_{n_r k}\, \varrho^{\gamma-1}\, Q_{n_r}^{2\gamma-1}(\varrho)\}\, e^{-\frac{\varrho}{2}},$$

$$G_{n_r k} = \left\{\frac{Z\alpha}{\gamma + k}\, a_{n_r k}\, \varrho^{\gamma}\, Q_{n_r - 1}^{2\gamma+1}(\varrho) - b_{n_r k}\, \varrho^{\gamma-1}\, Q_{n_r}^{2\gamma-1}(\varrho)\right\} e^{-\frac{\varrho}{2}}. \tag{A45}$$

After substitution of (A45) into (A6) we get the solution of Eq. (A1).

In order to make connection with already given solution we introduce the new constants in (A45):

$$a_{n_r k} = (k + \gamma)\, A_{n_r k} - Z\alpha\, \frac{k}{|k|}\, B_{n_r k},$$

$$b_{n_r k} = -\, Z\alpha A_{n_r k} + (k + \gamma)\, \frac{k}{|k|}\, B_{n_r k}. \tag{A46}$$

The functions $F_{n_r k}$ and $G_{n_r k}$ then become

$$F_{n_r k} = A_{n_r k}\, F_{n_r k n} + B_{n_r k}\, F_{n_r k d}, \tag{A47}$$

$$G_{n_r k} = A_{n_r k}\, G_{n_r k n} + B_{n_r k}\, G_{n_r k d},$$

where

$$F_{n_r k n} = [(k + \gamma)\, \varrho\, Q_{n_r - 1}^{2\gamma+1} + (k - \gamma)\, Q_{n_r}^{2\gamma-1}]\, \varrho^{\gamma-1}\, e^{-\frac{\varrho}{2}},$$

$$G_{n_r k n} = Z\alpha\, [\varrho\, Q_{n_r - 1}^{2\gamma+1} + Q_{n_r}^{2\gamma-1}]\, \varrho^{\gamma-1}\, e^{-\frac{\varrho}{2}}, \tag{A48}$$

$$F_{n_r k d} = \frac{k}{|k|}(-Z\alpha)\, [\varrho\, Q_{n_r - 1}^{2\gamma+1} + Q_{n_r}^{2\gamma-1}]\, \varrho^{\gamma-1}\, e^{-\frac{\varrho}{2}},$$

$$G_{n_r k d} = -\frac{k}{|k|}\, [(k - \gamma)\, \varrho\, Q_{n_r - 1}^{2\gamma+1} + (k + \gamma)\, Q_{n_r}^{2\gamma-1}]\, \varrho^{\gamma-1}\, e^{-\frac{\varrho}{2}}. \tag{A49}$$

For a given k_0 $\left(n_r \text{ and } |k| = j + \frac{1}{2}\right)$ the general solution of Eq. (A1) is

$$\varphi_{k_0} = \sum_m c_{k=j+\frac{1}{2},m} \begin{pmatrix} F_{n_r k>0}\, \Omega_{j,j+\frac{1}{2},m} \\ -G_{n_r k>0}\, \Omega_{j,j-\frac{1}{2},m} \end{pmatrix} +$$

$$+ \sum_m c_{k=-\left(\frac{1}{2}\right),m} \begin{pmatrix} F_{n_r k<0}\, \Omega_{j,j-\frac{1}{2},m} \\ G_{n_r k<0}\, \Omega_{j,j+\frac{1}{2},m} \end{pmatrix},$$

$$-\left(j+\frac{1}{2}\right) < m < j + \frac{1}{2}. \tag{A50}$$

By making use of (A47) φ_{k_0} can be written in the form

$$\varphi_{k_0} = \varphi_{k_0 n}' + \varphi_{k_0 d}, \tag{A51}$$

where

$$\varphi_{k_0 n} = \sum_m \{ c_{k=j+\frac{1}{2}} A_{n_r k>0}\, \varphi_{n_r,j,k=j+\frac{1}{2},m(n)} +$$

$$+ c_{k=-\left(j+\frac{1}{2}\right),m} A_{n_r k<0}\, \varphi_{n_r,j,k=-\left(j+\frac{1}{2}\right),m(n)} \}, \tag{A52}$$

$$\varphi_{k_0 d} = \sum_m \{ c_{k=j+\frac{1}{2},m} B_{n_r k>0}\, \varphi_{n_r,j,k=j+\frac{1}{2},m(d)} +$$

$$+ c_{k=-\left(j+\frac{1}{2}\right),m} B_{n_r k>0} \varphi_{n_r,j,k=-\left(j+\frac{1}{2}\right),m(d)} \}, \tag{A53}$$

$$\varphi_{n_r,j,k=j+\frac{1}{2},m(n)} = \begin{pmatrix} F_{n_r k>0 n}\, \Omega_{j,j+\frac{1}{2},m} \\ -G_{n_r k>0 n}\, \Omega_{j,j-\frac{1}{2},m} \end{pmatrix}, \tag{A54}$$

$$\varphi_{n_r,j,k=-\left(j+\frac{1}{2}\right),m(n)} = \begin{pmatrix} F_{n_r k<0\, n}\, \Omega_{j,j-\frac{1}{2},m} \\ G_{n_r k<0\, n}\, \Omega_{j,j+\frac{1}{2},m} \end{pmatrix}, \tag{A55}$$

$$\varphi_{n_r,j,k=j+\frac{1}{2},m(d)} = \begin{pmatrix} F_{n_r k>0\, d}\, \Omega_{j,j+\frac{1}{2},m} \\ -G_{n_r k>0\, d}\, \Omega_{j,j-\frac{1}{2},m} \end{pmatrix}, \tag{A56}$$

$$\varphi_{n_r,j,k=-\left(j+\frac{1}{2}\right),m(d)} = \begin{pmatrix} F_{n_r k<0\, d}\, \Omega_{j,j-\frac{1}{2},m} \\ G_{n_r k<0\, d}\, \Omega_{j,j+\frac{1}{2},m} \end{pmatrix}. \tag{A57}$$

The solutions (A56) and (A57) are in dual relation to the solutions (A54) and (A55), i. e.

$$\varphi_{n_r,j,\,k=-\left(j+\frac{1}{2}\right),\,m(d)} \sim \gamma^5\,\varphi_{n_r,j,k=j+\frac{1}{2},\,m(n)}, \qquad (A58)$$

$$\varphi_{n_r,j,k=j+\frac{1}{2},\,m(d)} \sim \gamma^5\,\varphi_{n_r,j,\,k=-\left(j+\frac{1}{2}\right),\,m(n)}. \qquad (A59)$$

This can directly be seen if one uses the expressions (A48) and (A49) for the functions F and G.

In the nonrelativistic limit ($Z\alpha \to 0$)

$$G_{n_r kn} \to 0, \qquad (A60)$$

$$F_{n_r kd} \to 0$$

and the functions (A54—55) and (A56—57) become

$$\begin{pmatrix} \neq 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ \neq 0 \end{pmatrix}, \qquad (A61)$$

respectively. This is in accordance to the definition of the »normal« and »dual« solutions of the free field case.

The selection (A52—53) of the »normal« and »dual« solutions agrees with that one in Section 5 where we have used the solutions of the canonical equations.

At the end let us mention that one can investigate other selections of the »normal« and »dual« solutions in order to get positive definite energy.

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MEĐUDJELOVANJE DIRACOVOG POLJA SA ELEKTROMAGNETSKIM POLJEM U NOVOJ FORMULACIJI. RELATIVISTIČKI VODIKOV ATOM

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Razmatrana je i izgrađena teorija međudjelovanja Diracovog polja u novoj kanonskoj formulaciji sa elektromagnetskim poljem. Radi izbjegavanja poteškoća u razumijevanju, elektromagnetsko polje je opisano na standardan način. Utvrđena je ista fizikalna sadržina Diracovog polja kod relativističkog vodikovog atoma kao i u slučaju slobodnog polja. Nađeno je da je energetski spektar vodikovog atoma isti kao i u standardnoj teoriji. Raspodjela vjerojatnosti položaja elektrona u određenom stacionarnom stanju, međutim, nije ista. Nerelativistička aproksimacija nove teorije je standardna nerelativistička kvantna mehanika.