YU ISSN 0015—3206
FZKAAA 12 (3) 187 (1980)

INVARIANT AMPLITUDES FOR DESCRIPTION OF NUCLEAR REACTION $\vec{1} + \frac{\vec{1}}{2} \rightarrow \frac{\vec{1}}{2} + \frac{\vec{1}}{2} + \frac{\vec{1}}{2}$ WITH POLARIZED BEAMS

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Received 3 June 1980

.UDC 539.17

Original scientific paper

The formalism for description of nuclear reactions of the type $\vec{1} + \frac{\vec{1}}{2} \rightarrow \frac{\vec{1}}{2} + \frac{\vec{1}}{2} + \frac{\vec{1}}{2}$ induced by polarized beams has been constructed. An explicit form of the M-matrix was found and effects of parity conservation were included.

1. *Introduction*

Experiments with polarized beams are of great importance for investigating the spin structure of cross section. The general formalism for description of polarized phenomena in the processes of the type (1) + (2) → (3) + (4) + (5) is available[1]. However this formalism does not lead to a straightforward explicit construction of the M-matrix for a particular process. Particular formalisms were developed for some types of interactions[2] that have been treated experimentally. Processes like $H(d, 2p)\, n$ require the formalism for the reactions of the type $\vec{1} + \frac{\vec{1}}{2} \to \frac{\vec{1}}{2} + \frac{\vec{1}}{2} + \frac{\vec{1}}{2}$. Before we expose it, we will give some formulas and definitions that will be used in the following text.

In the uncoupled representation[2, 3], incoming and outgoing beams have folowing spin structures. For the incoming beam we have

$$\chi_i = \sum_{j=1}^{6} a_j \Phi_j. \tag{1}$$

a_j are components of composite states Φ_j:

$$\begin{aligned} &\Phi_1 = |11\rangle|\tfrac{1}{2}\tfrac{1}{2}\rangle \qquad \Phi_2 = |10\rangle|\tfrac{1}{2}\tfrac{1}{2}\rangle \qquad \Phi_3 = |1-1\rangle|\tfrac{1}{2}\tfrac{1}{2}\rangle \\ &\Phi_4 = |11\rangle|\tfrac{1}{2}-\tfrac{1}{2}\rangle \qquad \Phi_5 = |10\rangle|\tfrac{1}{2}-\tfrac{1}{2}\rangle \qquad \Phi_6 = |1-1\rangle|\tfrac{1}{2}-\tfrac{1}{2}\rangle. \end{aligned} \tag{2}$$

$|1, m\rangle$ are eigenstates of spin-1 operator $S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$, and $|\tfrac{1}{2}, m\rangle$ are eigenstates of spin $-\tfrac{1}{2}$ operator $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. The final state consists of three independent spin $-\tfrac{1}{2}$ particles and it's spin structure is

$$\chi_f = \sum_{j=1}^{8} b_j \Phi'_j \tag{3}$$

where b_j are amplitudes of composite states Φ'_j:

$$\Phi'_j = |\tfrac{1}{2} m_1\rangle|\tfrac{1}{2} m_2\rangle|\tfrac{1}{2} m_3\rangle; \quad m_{1,2,3} = \pm\tfrac{1}{2}. \tag{4}$$

The amplitude of this process is an element of the T-matrix:

$$T_{fi} = \langle f|T|i\rangle. \tag{5}$$

Initial and final states are characterized by the momenta, spins and spin projections of incoming and outgoing particles:

$$\begin{aligned} |i\rangle &= |\vec{p}_1\, \vec{p}_2\, \sigma_1\, \sigma_2\, M_1\, M_2\rangle \equiv |\vec{p}_1\, \vec{p}_2\, M_1\, M_2\rangle \\ |f\rangle &= |\vec{k}_1\, \vec{k}_2\, \vec{k}_3\, s_1\, s_2\, s_3\, m_1\, m_2\, m_3\rangle \equiv |\vec{k}_1\, \vec{k}_2\, \vec{k}_3\, m_1\, m_2\, m_3\rangle. \end{aligned} \tag{6}$$

Generaly, T is a matrix in spin space, and we can write:

$$\langle f|T|i\rangle = \langle m_1\, m_2\, m_3|\mathcal{M}(\vec{p}_1\, \vec{p}_2\, \vec{k}_1\, \vec{k}_2\, \vec{k}_3)|M_1\, M_2\rangle \ldots \tag{7}$$

where $\mathcal{M}(\vec{p}_1 \vec{p}_2 \vec{k}_1 \vec{k}_2 \vec{k}_3)$ expresses momentum dependence of the T-matrix. In our particular case, the M-matrix is an 8×6 matrix in the spin space. It is convenient to describe polarized beams with the density matrix, which is given, by definition[3], for the incoming beam with:

$$(\varrho^i)_{jk} = \frac{1}{N} \sum_{n=1}^{N} a_j^{(n)} a_k^{(n)*} \tag{8}$$

and for the outgoing beam

$$(\varrho^f)_{jk} = \frac{1}{N} \sum_{n=1}^{N} b_j^{(n)} b_k^{(n)*}. \tag{9}$$

N is the number of particles in the beam. In addition $X_f = M X_i$, so $b_j = M_{jk} a_k$ inserted in (9) gives:

$$\varrho^f = M \cdot \varrho^i \cdot M^{\dagger}. \tag{10}$$

In our special case ϱ^i is a 6×6 matrix and ϱ^f is an 8×8 matrix. We can obtain the basis of 6×6 matrix space by means of direct products of a 2×2 matrix space basis and a 3×3 matrix space basis. For the base of a 2×2 matrix space we can choose Pauli's matrices[2] $I = \sigma_0, \sigma_1, \sigma_2, \sigma_3$ and for the basis of a 3×3 matrix space it is convenient to choose[2] $I = \Omega_0, \Omega_1, \ldots, \Omega_8$. Each of these basis has a feature[3]:

$$\begin{aligned} Tr\, \sigma_i \sigma_j &= 2\, \delta_{ij} \\ Tr\, \Omega_i \Omega_j &= 3\, \delta_{ij} \end{aligned} \tag{11}$$

so we can write

$$\varrho^i = \frac{1}{6} \sum_{lm} \langle \Omega_l \rangle \langle \sigma_m \rangle \Omega_l \sigma_m. \tag{12}$$

Insertion of (12) into (10) gives

$$\varrho^f = \frac{1}{6} \sum_{lm} \langle \Omega_l \rangle \langle \sigma_m \rangle M \Omega_l \sigma_m M^{\dagger}. \tag{13}$$

The differential cross section $I(\zeta)$, where ζ represents a set of quantities that describe the outgoing beam, is given by the trace of the density matrix ϱ^f.

$$I(\zeta) = Tr\, \varrho^f = \frac{1}{6} \sum_{lm} \langle \Omega_l \rangle \langle \sigma_m \rangle Tr \{ M \Omega_l \sigma_m M^{\dagger} \}. \tag{14}$$

In the special case in which target and projectile are not polarized: $\langle \Omega_l \rangle = \delta_{ol}$, $\langle \sigma_m \rangle = \delta_{om}$, we have

$$\varrho^f = \frac{1}{6} M M^\dagger \tag{15}$$

or

$$I_0(\zeta) = Tr\, \varrho^f = \frac{1}{6}\, Tr\, M M^\dagger. \tag{16}$$

From (14) and (16) follows the expression for the cross section $I(\zeta)$:

$$I(\zeta) = I^0(\zeta) \left[1 + \sum_{l \neq 0,\, m \neq 0} \langle \Omega_l \rangle \langle \sigma_m \rangle \frac{Tr \{M\, \Omega_l \sigma_m M^\dagger\}}{Tr\, M M^\dagger} \right]. \tag{17}$$

2. *The construction of the M-matrix for reaction of the type:* $\vec{1} + \frac{\vec{1}}{2} \to \frac{\vec{1}}{2} + \frac{\vec{1}}{2} + \frac{\vec{1}}{2}$.

In our reaction the M-matrix is an 8 × 6 matrix. We can form a basis of an 8 × 6 matrix space by direct products of the following quantities:

$$\chi_+ = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \chi_0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \chi_- = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\Psi_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \Psi_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \Psi_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \Psi_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{18}$$

$$I = \sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Thus the general term of a 8 × 6 matrix is

$$\chi_{\oplus}^\dagger \Psi \otimes \sigma = \left[\begin{array}{c:c:c} \sigma & \sigma & \sigma \\ \hdashline \sigma & \sigma & \sigma \\ \hdashline \sigma & \sigma & \sigma \end{array} \right]. \tag{19}$$

Measurements in the outgoing chanel are often performed in such a way that the third particle is not detected, so we can put

$$\vec{k}_1 + \vec{k}_2 = \vec{k}_{out}. \tag{20}$$

In the laboratory system the target is at rest, so $\vec{p}_2 = 0$. We can also use $\vec{p}_1 = \vec{k}_{in}$.

Two coordinate systems are needed for the description of our reaction: S and S'. The axes of these coordinate systems we choose in the following way:

$$\hat{n} = \frac{\vec{k}_{in} \times \vec{k}_{out}}{|\vec{k}_{in} \times \vec{k}_{out}|} \quad (y \text{ or } y' \text{ axis})$$

$$\hat{k}' = \frac{\vec{k}_{out}}{|\vec{k}_{out}|} \quad (z' \text{ axis})$$

$$\hat{k} = \frac{\vec{k}_{in}}{|\vec{k}_{in}|} \quad (z \text{ axis}) \tag{21}$$

$$\hat{p} = \frac{\hat{n} \times \hat{k}}{|\hat{n} \times \hat{k}|} \quad (x \text{ axis})$$

$$\hat{p}' = \frac{\hat{n} \times \hat{k}'}{|\hat{n} \times \hat{k}'|} \quad (x' \text{ axis}).$$

These unit vectors together with the quantities given in (18) are needed for the construction of the M-matrix. $\hat{p}', \hat{p}, \hat{k}'$ and $\hat{k}$ are polar vectors and $\hat{n}$ is an axial vector. If we regard χ_+, χ_0, χ_- as the components of a three dimensional vector $\vec{\chi}$, and σ_x, σ_y and σ_z as the components of $\vec{\sigma}$, then $\vec{\chi}$ and $\vec{\sigma}$ are also axial vectors.

Parity conservation:

$$\mathrm{PMP}^{-1} = \mathrm{M} \tag{22}$$

requires that the M-matrix be built of scalars. It can easily be seen that:

$$\begin{aligned} &A^J (\vec{\chi}^\dagger \cdot \hat{n}) \Psi_j \sigma_0 \\ &B^J (\vec{\chi}^\dagger \cdot \hat{n}) \Psi_j (\vec{\sigma} \cdot \hat{n}) \\ &C^J (\vec{\chi}^\dagger \cdot \hat{p}) \Psi_j (\vec{\sigma} \cdot \hat{p}') \\ &D^J (\vec{\chi}^\dagger \cdot \hat{p}) \Psi_j (\vec{\sigma} \cdot \hat{k}') \\ &E^J (\vec{\chi}^\dagger \cdot \hat{k}) \Psi_j (\vec{\sigma} \cdot \hat{p}') \\ &F^J (\vec{\chi}^\dagger \cdot \hat{k}) \Psi_j (\vec{\sigma} \cdot \hat{k}') \end{aligned} \qquad j = 1, 2, 3, 4. \tag{23}$$

have the required properties.

With:

$$\chi_x = \frac{1}{\sqrt{2}}(\chi_- - \chi_+) = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\chi_z = \chi_0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tag{24}$$

$$\chi_y = \frac{i}{\sqrt{2}}(\chi_+ + \chi_-) = \frac{i}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

and with respect to (21) we obtain from (23)

$$\begin{array}{l} A^j\, \chi_y^\dagger \Psi_j \sigma_0 \\ B^j\, \chi_y^\dagger \Psi_j \sigma_{y'} \\ C^j\, \chi_x^\dagger \Psi_j \sigma_{x'} \\ D^j\, \chi_x^\dagger \Psi_j \sigma_{z'} \\ E^j\, \chi_z^\dagger \Psi_j \sigma_{x'} \\ F^j\, \chi_z^\dagger \Psi_j \sigma_{z'}. \end{array} \qquad j = 1, 2, 3, 4. \tag{25}$$

Notice that parity conservation restricts the $6 \times 8 = 48$ elements of the M-matrix to $6 \times 4 = 24$ independent elements. Therefore we can write:

$$M = \sum_{j=1}^{4} [A_j \chi_y^\dagger \Psi_j \sigma_0 + B_j \chi_y^\dagger \Psi_j \sigma_{y'} + C_j \chi_x^\dagger \Psi_j \sigma_{x'} + D_j \chi_x^\dagger \Psi_j \sigma_{z'} + $$
$$+ E_j \chi_z^\dagger \Psi_j \sigma_{x'} + F_j \chi_z^\dagger \Psi_j \sigma_{z'}]. \tag{26}$$

With the definition:

$$\frac{1}{\sqrt{2}}(D_i \pm i A_i) = G_i^{(\pm)} \tag{27}$$

$$\frac{1}{\sqrt{2}}(C_i \pm B_i) = H_i^{(\pm)}$$

expression (26) gets an explicit form:

$$M = \begin{vmatrix} -G_1^{(+)} & -H_1^{(-)} & F_1 & E_1 & -G_1^{(+)} & H_1^{(-)} \\ -H_1^{(+)} & G_1^{(-)} & E_1 & -F_1 & H_1^{(+)} & G_1^{(-)} \\ -G_2^{(+)} & -H_2^{(-)} & F_2 & E_2 & -G_2^{(+)} & H_2^{(-)} \\ -H_2^{(+)} & G_2^{(-)} & E_2 & -F_2 & H_2^{(+)} & G_2^{(-)} \\ -G_3^{(+)} & -H_3^{(-)} & F_3 & E_3 & -G_3^{(+)} & H_3^{(-)} \\ -H_3^{(+)} & G_3^{(-)} & E_3 & -F_3 & H_3^{(+)} & G_3^{(-)} \\ -G_4^{(+)} & -H_4^{(-)} & F_4 & E_4 & -G_4^{(+)} & H_4^{(-)} \\ -H_4^{(+)} & G_4^{(+)} & E_4 & -F_4 & H_4^{(+)} & G_4^{(-)} \end{vmatrix} \tag{28}$$

The consequence of parity conservation is for instance $M_{14} = M_{23} = E_1$. The experimental testing of this equality in fact tests parity conservation in strong interactions.

Expression (28) which is the main result of this paper can be used for the calculation of analysing powers[2] in expression (17).

In the recently done experiment[5], H $(\vec{d}, 2p)$ n, the events were selected in which the final state neutron was at rest in CMS. Our formulae can be specialized for that case by introducing

$$\vec{k}_{out} = \frac{1}{2} \vec{k}_{in},$$

or:

$$\hat{n} = 0 \qquad \hat{p} = 0 \qquad \hat{p}' = 0.$$

Therefore, in (23) survive only terms with F^j; and in M-matrix (28) only F_j $j = 1, 2, 3, 4$ are nonvanishing. Thus, for instance

$$\frac{1}{6} Tr\, M\, M^{\dagger} = I_0(\zeta) = \frac{1}{3} (|F_1|^2 + |F_2|^2 + |F_3|^2 + |F_4|^2)$$

here F_j $j = 1, 2, 3, 4$ remain momentum-dependent. Even in this simple case expression (17) contains 36 terms.

3. Conclusion

The formalism for description of nuclear reaction of the type $\vec{1} + \frac{\vec{1}}{2} \to \frac{\vec{1}}{2} + \frac{\vec{1}}{2} + \frac{\vec{1}}{2}$ has been constructed in such a way that effects of parity conservation were included in the construction procedure. The general formalism for description of polarized phenomena in the process of the type (1) + (2) → (3) + (4) + (5) is available elsewhere[1]. As we have already stated in the introduction, this formalism does not lead to a straighforward explicit construction of the M-matrix. The main purpose of this paper was the construction of the M-matrix for the reactions of the type $\vec{1} + \frac{\vec{1}}{2} \to \frac{\vec{1}}{2} + \frac{\vec{1}}{2} + \frac{\vec{1}}{2}$.

Acknowledgments

The author expresses his gratitude to Ivo Šlaus for having suggested the problem and for helpful discussions, and to Dubravko Tadić for providing the stimulus whenever necessary.

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INVARIJANTNA AMPLITUDA ZA OPIS NUKLEARNE REAKCIJE $\vec{1}+\frac{\vec{1}}{2}\rightarrow\frac{\vec{1}}{2}+\frac{\vec{1}}{2}+\frac{\vec{1}}{2}$ S POLARIZIRANIM SNOPOVIMA

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UDK 539.17

Originalni znanstveni rad

Konstruiran je formalizam za opis nuklearne reakcije $\vec{1}+\frac{\vec{1}}{2}\rightarrow\frac{\vec{1}}{2}+\frac{\vec{1}}{2}+\frac{\vec{1}}{2}$ s polariziranim snopovima i nađen je explicitni oblik M-matrice. Uključeni su efekti sačuvanja pariteta.