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THE EFFECT OF KINK ON STRING MODEL IN DISLOCATION DAMPING

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Abstract: The kink model in the light of string model in dislocation is solved by the operational method due to Heaviside. The expression for damping introduces a correction term over the normal as predicted by Seeger and Schiller.

1. Introduction

The attenuation of stress waves travelling through a solid due to dislocation motion is discussed by many authors. The most suitable, interesting and heuristic in approach is the vibrating string model of dislocation motion. The theory of the vibrating string model as given by Koehler[1] and developed by Granato and Lücke[2] has been further refined by Bhattacharya and Ghosh[3-5] following Haeviside's[6] operational method. It is based on the assumption that the dislocations have a mass per unit length

$$A = \varrho\, b^2,$$

where ϱ is the density of the solid and b is the Burgers vector. Dislocations also have a drag co-efficient B per unit length which represents the dragging force due to phonons and electrons. This drag is proportional to the dislocation velocity through the crystals. Also there is a line tension eflect $c \cong \frac{Gb^2}{2}$, where G is the

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shear elastic constant in the glide plane which tends to keep the dislocation straight between two pinning points. In the string model any interaction of the dislocations with the Peierls energy barrier is neglected. In the low amplitude region there is considerable evidence that dislocations do not lie in a straight line between pinning points but rather lie partly in Peierls valleys and partly across the Peierls valleys in the form of kinks. However, the kinks are closely spaced and merely put small wiggles on the dislocation line. During a dislocation displacement, the kinks move sidewise but the over-all motion is much like that of a string. When the kinks are not free but interact with each other in the dislocation segment, they repel each other when they get close together and this force takes the place of the line tension force $c = \frac{Gb^2}{2}$ in the string model. The kink has a mass and a viscous drag co-efficient and if we neglect the lattice vibration mode set up by traversing the negative slope of the kink barrier, the kink model gives the same internal friction as the string model except for minor numerical term as observed by Seeger and Schiller[7]. Mason[8] assumed the value of internal friction due to dislocation in the string model and introduced the idea of kink in the string model. He also assumed the internal friction value in kink model without any calculation.

2. *Solution of the problem*

To obtain the combined effect of the lattice vibrational dissipation and drag co-efficient B it is easiest to apply an average kink force per unit length to the string model. There will be no force applied to stretching the string until a kink has crossed a barrier and this force is $m\,\sigma_k\, ab = \sigma\, b,$ (1)

hence

$$m = \frac{\sigma}{\sigma_k\, a},$$

where σ_K is the stress required to cross the kink barrier, a is the height of the kink and σ is the applied stress. In addition to the conservative force applied by the kinks there is a dissipative force equal to $m\,\sigma_{dp}\, a\, b,$ where σ_{dp} is the dynamic Peierls stress. So the average force applied to the string model by the motion of the kinks is

$$\sigma\, b + i\, m\, \sigma_{dp}\, ab = \sigma\, b\, [1 + i\, \beta], \tag{2}$$

where $\beta = \frac{\sigma_{dp}}{\sigma_k}$ and its value calculated by Weiner[9] is 0·01 and Atkinson and Cabrera's[10] value is 0·1 but experimental value obtained by Mason and Wehr[11] is 0.03.

If we apply this force to the equation for string model we have

$$A\frac{\partial^2 \xi}{\partial t^2} + B\frac{\partial \xi}{\partial t} - C\frac{\partial^2 \xi}{\partial y^2} = \sigma b(1 + i\beta), \tag{3}$$

where ξ, y and t stand for the displacement of the dislocation from equilibrium position, coordinate of an element of the dislocation line and time respectively.

Since the dislocaton motion is always over damped — neglecting the mass term in Equ. (3), we have

$$B\frac{\partial \xi}{\partial r} - C\frac{\partial^2 \xi}{\partial y^2} = \sigma b(1 + i\beta). \tag{4}$$

Let us assume the applied stress σ to be of the form

$$\sigma = \sigma_0 \exp(-\alpha x)\exp\left[i\omega\left(t - \frac{x}{v}\right)\right], \tag{5}$$

where α is the damping constant, ω, the applied frequency, x, the varible co--ordinate along which the stress is applied, v_1 the velocity of the elastic wave and σ_0 a constant.

With the help of Equ. (5) the Equ. (3) in operational form is

$$\frac{\partial^2 \xi}{\partial y^2} - \frac{D}{m_1}\xi = -\sigma_1 \exp(i\omega t), \tag{6}$$

where $D = \frac{\mathrm{d}}{\mathrm{d}t}$, the operator, $m_1 = \frac{C}{B}$,

a constant and

$$\sigma_1 = \frac{b(1 + i\beta)}{C}\sigma_0 \exp(-\alpha x - i\omega x/v).$$

The general solution of Equ. (6) is

$$\xi = A\cosh py + B\sinh py + \frac{\sigma_1}{p^2}\exp(i\omega t), \tag{7}$$

where $p^2 = \frac{D}{m_1}$ and A and B are two arbitrary constants.

From the terminal conditions

$$\xi = 0 \text{ for } y = 0 \text{ and } y = 1 \tag{8}$$

we obtain the values of A and B and have the Equ. (17) as

$$\xi = \frac{\sigma_1}{p^2} \exp(i\,\omega\,t) \left[1 - \frac{\cosh p(y - l/2)}{\cosh p\, l/2}\right]. \tag{9}$$

The integrodifferential equation satisfying the given stress wave is

$$\frac{\partial^2 \sigma}{\partial x^2} - \frac{\varrho}{G}\frac{\partial^2 \sigma}{\partial t^2} = \frac{\Lambda \varrho b}{l}\frac{\partial^2}{\partial t^2}\int_0^l \xi(y)\,\mathrm{d}y. \tag{10}$$

Combining equation (5), (9) and (10) we have

$$\left(a + \frac{i\,\omega}{v}\right)^2 \sigma + \frac{\varrho}{G}\omega^2\sigma = \frac{\Lambda \varrho b}{l} D^2 \frac{\sigma_1}{p^2}\exp(i\,\omega\,t)\left[l - \frac{2\tanh pl/2}{p}\right] \tag{11}$$

obtaining the operational solution of the right hand side of equation (11) we get the equation (11) as

$$\left(a + \frac{i\,\omega}{v}\right)^2 \sigma + \frac{\varrho}{G}\omega^2\sigma = \frac{\Lambda \varrho b^2}{B}\omega(i - \beta)\sigma - \frac{2\Lambda \varrho b^2}{l}.$$

$$\frac{(C)^{1/2}}{(B)^{3/2}} \cdot (1 + i\,\beta)\,\sigma \exp(i\,\omega\,t)\left[\frac{1}{(\pi\,t)^{1/2}} + (i\,\omega)^{1/2}\exp(i\,\omega\,t)\right.$$

$$\operatorname{erf}(i\,\omega\,t)^{1/2} - 2\left\{\frac{1}{(\pi\,t)^{1/2}}\exp\left(-\frac{l^2}{4 m_1 t}\right) + i\,\omega(4\,t)^{1/2}\,\mathrm{i\,erfc}\frac{l}{2(m_1 t)^{1/2}} -\right.$$

$$\left. - \omega^2(4\,t)^{3/2}(i)^3\operatorname{erfc}\frac{l}{2(m_1 t)^{1/2}} + \ldots\right\} + 2\left\{\frac{1}{(\pi\,t)^{1/2}}\exp\left(-\frac{4\,l^2}{4 m_1 t}\right) +\right.$$

$$\left. + i\,\omega(4\,t)^{1/2}\,\mathrm{i\,erfc}\frac{2l}{2(m_1 t)^{1/2}} - \omega^2(4\,t)^{3/2}(i)^3\operatorname{erfc}\frac{2\,l}{2(m_1 t)^{1/2}} + \ldots\right\} -$$

$$- 2\left\{\frac{1}{(\pi\,t)^{1/2}}\exp\left(-\frac{9\,l^2}{4 m_1 t}\right) + i\,\omega(4t)^{1/2}(i)\operatorname{erfc}\frac{3\,l}{2(m_1 t)^{1/2}} -\right.$$

$$\left.\left. - \omega^2(4\,t)^{3/2}(i)^3\operatorname{erfc}\frac{3\,l}{2(m_1 t)^{1/2}} + \ldots\right\} + \ldots\right]. \tag{12}$$

Since we are interested in low frequency stress pulses of short duration i. e., of microsecond order we can neglect terms with higher power of time. As the loop lengths are exceedingly short we neglect the terms containing $l/(t)^{1/2}$ also. So equation (12) becomes

$$\left(a + \frac{i\,\omega}{v}\right)^2 \sigma + \frac{\varrho}{G}\,\omega^2\,\sigma = \frac{\Lambda\,\varrho\,b^2}{B}\,\omega\,(i - \beta)\,\sigma - \frac{2\Lambda\,\varrho\,b^2}{l} \cdot \frac{(C)^{1/2}}{(B)^{3/2}} \cdot$$

$$\cdot\,(1 + i\,\beta)\,\sigma \left[\frac{1}{(\pi\,t)^{1/2}} \exp\left(-\,i\,\omega\,t\right) + (i\,\omega)^{1/2}\,\mathrm{erf}\,(i\,\omega\,t)^{1/2}\right].$$

Equating imaginary terms from both sides of Equ. (13), we obtain

$$a = \frac{v}{2} \cdot \frac{\Lambda\,\varrho\,b^2}{B} - \frac{v}{2\,\omega} \cdot \frac{2\Lambda\,\varrho\,b^2}{l} \cdot \frac{(C)^{1/2}}{(B)^{3/2}} \cdot \frac{1}{(\pi\,t)^{1/2}} \cdot (\beta + \omega\,t). \qquad (14)$$

The decrement is given by

$$\Delta = \frac{2\,\pi\,v}{\omega}\,a = \frac{\pi^2}{8}\,\Delta_0\,\Lambda\,l^2\,\frac{\omega_0^2}{\omega\,d} - \frac{(\pi)^{1/2}}{4}\,\Delta_0\,\Lambda\,l^2 \cdot \frac{\omega_0^3}{\omega^2\,(d)^{3/2}} \cdot \frac{1}{(t)^{1/2}} \cdot (\beta + \omega\,t), \qquad (15)$$

where $\Delta_0 = \dfrac{8\,G\,b^2}{\pi^3\,C}$ and $v^2 = \dfrac{G}{\varrho}$

are two constants and $\omega_0 = \dfrac{\pi}{l}\left(\dfrac{C}{A}\right)^{1/2}.$

This equation holds when all the stress is applied in the glide plane.

3. *Discussion*

Present analysis shows distinctly that the Kink model gives the same internal friction as string model as obtained by Bhattacharya and Ghosh[3-5] in the high frequency and high damping case except for a very small numerical term given by

$$\frac{(\pi)^{1/2}}{4}\,\Delta_0\,\Lambda\,l^2\,\frac{\omega_0^3}{\omega^3\,(d)^{3/2}} \cdot \frac{1}{(t)^{1/2}} \cdot (\beta + \omega\,t),$$

which justifies the comment made by Seeger and Schiller[7]. Moreover it is seen that the second term is transient in nature in out mathematical deduction we are concerned with the steady state solution which makes the correction term further small.

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