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IMPACT OF πd SCATTERING DATA ON πN AMPLITUDE ANALYSIS*)

N. ZOVKO

Institute »Ruđer Bošković« Zagreb and University of Zagreb, Zagreb

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Abstract: The question to what extent πd elastic-scattering data at high energies and large angles may help in resolving some alternatives in πN amplitude analysis is studied within a simplified Glauber model.

1. Introduction

In the past few years, general features of elastic scattering of energetic pions on a deuteron target have been extensively studied and fairly well understood within Glauber's model of multiple scattering. The explanation of more subtle features, however, is still far from satisfactory. The main reason for that is the fact that the problem of πd scattering represents a meeting place of nuclear and particle physics, and a possible failure of the model may be ascribed either to an insufficiently accurate model of deuteron structure or to uncertainties in our knowledge of elementary (πN) amplitudes.

It has been a general opinion so far that nuclear properties of the deuteron play a decisive role in the analysis of scattering data, the impact of πN amplitudes being not strong enough to enable one to distinguish critically between different models for these amplitudes, and, eventually, to determine some of their properties in such a way. So, for example, the determination of the absolute phase of πN amplitudes by means of πd data, which was proposed a long time ago, is still at a rather discouraging stage[1].

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There are, however, other open problems in πN amplitude analysis, and it is the aim of the present work to re-examine to what extent πd data might help to resolve them. We shall study large-angle πd elastic scattering within a simple model based on the fact that at high energies and large angles only one πN amplitude (diffraction amplitude) and only the zeroth moment of the deuteron form factor play the dominant role.

To summarize the situation in πd elastic scattering, we distinguish three momentum transfer intervals:

a) — the single-scattering region ($0 \leq -t \leq 0.3$),

b) — the interference region ($0.3 \leq -t \leq 0.6$) for the description of which the d-wave component of the deuteron wave function is of crucial importance, and

c) — the region $0.6 \leq -t$, where double scattering dominates (t is in units of GeV^2).

At the present stage of experimental accuracy the agreement between theory and data in regions a) and b) is very good[2]. Because of the dominance of deuteron form factors, the t-dependence of πN amplitudes is of no practical importance here, and that is the reason why the essential progress in the absolute phase determination may not be expected in the near future. In region c) however, elementary amplitudes enter the differential cross section biquadratically and it is in this region that slight variations of different parametrizations of πN amplitudes become more pronounced and may, therefore, be studied. It has already been shown[3] that the serious disagreement[4] between the prediction of Glauber's model and πd data at high energies and large angles, reported recently, may be ascribed to the uncertainty of existing solutions for πN amplitudes. Moreover, the πd differential cross section in this kinematic region shows a sensitive dependence on these amplitudes and may serve as a critical test for them. It is an interesting question to what extent πd data might justify some of high-energy assumptions that underlie existing πN amplitude analyses. This is the subject of our consideration.

2. *The model*

As long as region c) of momentum transfer is considered, we may to a good approximation neglect the single-scattering component and restrict our attention only to the double scattering amplitude. In this case, however, the momentum transfer has to be kept still sufficiently low so that relativistic corrections do not get pronounced.

Forgetting about such corrections, one would naively expect that at appropriately high-momentum transfers only that component of the pion-light nucleus elastic amplitude where the pion scatters on all constituent nucleons (so that the light nulceus does not disintegrate) will be of importance. This picture would, of course, work better for less stable nuclei, the form factors of which are sharply peaked in the forward direction. The differential cross section, if measurable in such a kinematic region, would then be proportional to a high power (twice the number of constituent nucleons) of elementary (πN) amplitudes and would, therefore, be an extremely sensitive test for them.

Let us now turn back to the case of the deuteron, for which the number of scattering steps is not large; however, due to its unstability, its form factors are very sharply peaked in the forward direction.

The following assumptions can be made for the region of large-angle scattering:

— πN charge-exchange amplitudes may be neglected when compared with diffraction amplitudes;

— the remaining πN amplitudes may be (due to the forward peaking of deuteron form factors) taken out of the integral in the double-scattering amplitude and, consequently,

— the deuteron structure may be approximated by the lowest moment of the s-wave form factor.

Under the above assumptions the unpolarized πd differential cross section reads[3)]

$$\frac{d\sigma_{\pi d}(t)}{dt} = \frac{\pi I^2 (1 - t/16m^2)^2 J(t, m_d)}{J^2(t/4, m)} \left| \frac{C(t/4)}{4\pi k} \right|^4 G(t/4). \tag{1}$$

Here m and m_d are the nucleon and the deuteron masses, respectively, k is the lab momentum of the incoming pion, $C(t)$ is the crossing even pion-nucleon amplitude[5)] (diffraction amplitude); the symbol J stands for the Jacobian

$$J(k, t; m) \equiv \frac{k^2}{\pi} \frac{d\Omega_L}{dt} = \frac{k}{k'} \left[1 - \left(\frac{\omega}{m} + \frac{t}{2m^2} \right) \left(\frac{\omega\omega'}{\omega'^2 - m_\pi^2} - \frac{\omega}{\omega'} + \frac{t - 2m_\pi^2}{2k'^2} \right) \right] \cong$$

$$\cong \left(1 + \frac{t}{2mk} \right)^{-2} \quad \text{for } k \gg m_\pi. \tag{2}$$

Here I is the zeroth moment of the deuteron form factor

$$I = \frac{1}{2} \int_0^\infty dq^2 \, \Phi(q^2) \equiv \frac{1}{2} \int_0^\infty dq^2 \int_0^\infty dr \, j_0(qr) \, |u(r)|^2.$$

Numerically, using the Gartenhaus wave function (and taking into account the lost d-wave probability), we obtain

$$I = 0.011\ \mathrm{GeV}^2. \tag{3}$$

This value is by about 15% lower than that obtained by means of a simple Gaussian distribution. The function $G(t)$ is defined by

$$G(t) = 1 + \frac{1}{3} g(t) \{4 \,|\, R(t) \,|^2 - \mathrm{Re} \,|\, R^2(t) \,|\} + \frac{1}{4} g^2(t) \,|\, R(t) \,|^4, \tag{4}$$

$$g(t) = -(t/2m^2)(1 - t/4m^2)^{-2}, \tag{5}$$

$$R(t) = kB(t)/C(t), \tag{6}$$

where $B(t)$ is the second pion-nucleon (crossing even) invariant amplitude[5]. Writing the πN lab amplitude as

$$F_{\pi N} = a + \vec{\sigma}(\vec{q} \times \vec{k}_0)\, b, \tag{7}$$

where $\vec{k}_0$ is the unit vector in the direction of k and

$$a = (1 - t/4m^2)^{\frac{1}{2}}\, C/(4\pi J^{\frac{1}{2}}), \tag{8}$$

$$b = (1 - t/4m^2)^{-\frac{1}{2}}\, ikB/(8\pi m J^{\frac{1}{2}}), \tag{9}$$

it is obvious that $G(t)$ measures the effect of the nucleon spin. What remains when $G(t) = 1$ $(R = B = 0)$ is πd scattering as generated by pure diffractive πN scattering.

3. *πN amplitude analyses*

It is a well-known fact that the presently accessible set of πN experiments is not sufficient to fix πN amplitudes with satisfactory accuracy[6]. (We do not even mention the absolute phase problem.) Existing solutions for these amplitudes fit available data on measurable quantities in a more or less satisfactory way. The amplitudes themselves however, possess in some cases remarkably different characterist cs, depending on the way they were determined. It is then an interesting idea* to search whether πd data might a posteriori justify some dynamical assumptions underlying the performed πN analysis.

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*) Suggested by G. Höhler in private communications.

There are at present three different types of approach to the determination of πN amplitudes from experimental data at high energies and nonzero angles:

— phase shift analysis,

— analysis in terms of Regge poles, and

— the analysis that maximally exploits analyticity constraints.

Among the most popular phase-shift solutions are those called CERN-71 (Ref.[7]) and Saclay-72 (Ref.[8]). They cover the energy range up to 2.1 GeV and 2.8 GeV, respectively. Aparat from the well-known ambiguities of phase-shift solutions with many partial waves, recently studied in details by Martin[9], their test outside the analyzed set of πN experiments would be welcome from a practical point of view.

Although πN amplitudes describe πN data in a more or less satisfactory way, they, when constructed out of phase shifts, posses different properties. It has been found in a recent paper[6] that above 1.5 GeV and at momentum transfers of about -1 GeV^2 the CERN solution has the dominant spin-flip amplitude, F_{+-}, while the Saclay solution has the dominant spin-nonflip amplitude, F_{++}. If the property shared by the CERN solution were closer to reality, then, as emphasized in Ref.[6], the diffraction-pattern explanation of the observed minima and maxima in the πN angular distribution above 1.5 GeV would be in trouble. The suggested measurements of spin-rotation parameters would definitely decide this question; however, they are at present inaccessible. It will be discussed later whether πd data can give some evidence in favour of one of the above two cases.

The Regge-pole type analysis was performed most extensively by Barger and Phillips[10]. There are obvious reasons to suspect the validity of such a parametrization far away from the forward direction. This analysis led to a solution which had been the only explicit one for years and was used in all previous analyses of πd data above 2 GeV within Glauber's model, causing most probably a systematic overestimation of the data[4, 3]. The analyticity, taken into account, through finite-energy continuous-momentum sum rules probably had no large impact in such an approach, because the solutions themselves were not consistent with fixed-t dispersion relations[11].

The third method, namely data analysis in terms of complex amplitudes that have to satisfy the requirements of unitarity, crossing symmetry and fixed-t analyticity, seems to be at present the most reliable one. Analyses of that type have recently been carried out by Pietarinen[12], by Hecht, Jacob and Kroll[11] and by Höhler and Jacob[13]. However, in order to reduce the degree of arbitrariness in some of these analyses, additional dynamical constraints have been assumed. How πd data react to such assumptions will be another point of interest in the present paper.

4. πN alternatives at high energies

Let us now assume that at high energies there is a dynamical correlation between πN amplitudes B and C of the form (6) and that the function $R(k,t)$ is known. In this case both πN and πd differential cross sections may be expressed in terms of only one invariant amplitude C, which may be determined by πN data[5]

$$\left|\frac{C(t)}{4\pi k}\right|^4 = \left[\frac{d\sigma_{\pi N}(t)}{dt}\right]^2 \left\{1 - \frac{t}{4m^2}\frac{(1 + t/4q^2)}{(1 - t/4m^2)^2}\,|R(t)|^2\right\}^{-2} \frac{1}{(1 - t/4m^2)^2}, \tag{10}$$

where q denotes the cms momentum. Using expression (10) to eliminate the amplitude C from Equ. (1), we obtain

$$\frac{d\sigma_{\pi d}(t)}{dt} = \frac{I^2}{\pi}\left[\frac{d\sigma_{\pi N}(t/4)}{dt}\right]^2 F(t/4), \tag{11}$$

$$F(t) = \frac{\left\{1 + \frac{1}{2}g(t)\,|R(t)|^2\right\}^2 + \frac{1}{3}g(t)\left\{|R(t)|^2 - \mathrm{Re}\,R^2(t)\right\}}{\left\{1 + \frac{1}{2}g(t)\,|R(t)|^2(1 + t/4q^2)\right\}^2}, \tag{12}$$

where $g(t)$ is given by expression (5) and the Jacobians in Equ. (1) were omitted because at high energies

$$J(t, m_d) \cong J^2(t/4, m).$$

Knowing the πN cross section in the kinematic region where the present model is expected to be valid, the relation (11) enables us to study the πd cross section for different choices of $R(t)$. Here we consider the following four simple cases

$$R = 0, \tag{13}$$

which, according to Equ. (9), means that there is no spin contribution to the πN lab amplitude (7);

$$R = k(1 - t/4m^2)/(\omega + t/4m), \tag{14}$$

where ω denotes the lab energy of the incident pion. This corresponds to the vanishing of the invariant amplitude A at high energies, studied by Höhler and Strauss[14]. They have found that among amplitudes A and B it is B which dominates asymptotically. Consequently, the data analysis in Ref.[11] was performed under the constraint $|R| \cong 1$ for $k > 1.5$ GeV, $t > -0.5$ GeV^2. Up to a phase factor this coincides with the choice (14);

$$R = (1 - t/4m^2)\,(1 + m^2/q^2)^{\frac{1}{2}}/(1 + t/4q^2), \tag{15}$$

where q is the cms momentum. This case corresponds to the exact vanishing of the spin-flip amplitude[5], F_{+-}, and in practice seems to be approximately fulfilled by the Saclay phase-shift solution above $k = 1.5$ GeV and around $t = -1$ GeV^2;

$$R = k\,(4m^2 - t)/t(m + \omega), \tag{16}$$

which would be the consequence of the exact vanishing of the spin-nonflip amplitude[5], F_{++}, and approximately characterizes the CERN phase-shift solution above $k = 1.5$ GeV and around $t = -1$ GeV^2.

In all the above particular cases, R is real and the function F in Equ. (11) becomes simpler (for $|t|$ which is not too large),

$$F(t) \cong 1 - \frac{t}{4q^2}\, g(t)\, R^2(t). \tag{17}$$

Whatever value, real or complex, we choose for $R(t)$, it is obvious from Equ. (12) that $F(t/4) \geq 1$. Inclusion of spin scattering increases the cross section. However, for our cases (Equs. 13—16) this effect is, due to kinematical factors, rather small within the interval $-0.6 > t > -1.6$ GeV^2 unless R is particularly large. According to Equs. (13)—(16) the deviation of $F(t/4)$ from unity is given by

$$F(t/4) - 1 = 0, \tag{17a}$$

$$= -g(t/4)\left(\frac{k}{\omega + t/16m^2}\right)^2 \frac{t\,(1 - t/16m^2)^2}{16q^2}, \tag{17b}$$

$$= -g(t/4)\left(1 + \frac{m^2}{q^2}\right) \frac{t\,(1 - t/16m^2)^2}{16q^2\,(1 + t/16q^2)^2}, \tag{17c}$$

$$= -g(t/4)\left(\frac{k}{\omega + m}\right)^2 \frac{16m^4\,(1 - t/16m^2)^2}{tq^2}. \tag{17d}$$

It is obvious that all these cases differ qualitatively from each other. However, the first three possibilities lead to either zero or a very small effect in the region of interest, so that they cannot be of much practical use in the kinematic interval where the data are available at present. Only choice (17 d) generates a very large effect. It is by about two orders of magnitude larger than in case (17c).

The existing πd data suggest that choices (17a—17c) are rather close to reality, thus discriminating case (17d), which is the property shared by the CERN phase shift solution.

It is an unfortunate fact that there is no reliable measure of relativistic effects in the theory of the deuteron to extend our results below $t = -1.5$GeV^2, where the data also exist. As long as this is not done, πd scattering data cannot effectively help in resolving the remaining πN alternatives at high energies.

5. *Conclusion*

πN amplitude analysis may benefit from πd data in two ways. The first is to use the data in order to discriminate between different solutions for the (dominant) diffraction amplitude C by replacing the spin effect function G in Equ. (1) by unity. Owing to the fourth power of elementary amplitudes, the local structures of different solutions blow up and may be compared with the data in a more explicit way.

Another way of using πd data is via formula (11). In this case it is possible to study more subtle features of πN amplitudes. Unfortunately, the experimental accuracy and the expected rising influence of relativistic effects on the deuteron wave function limits the range of the applicability, so that clear-cut conclusions are not always possible.

Definite conclusions may be drawn concerning the ambiguities such as the one of the CERN-Saclay phase shifts. A solution for πN amplitudes with the dominant spin nonflip amplitude at high energies and momentum transfers generates spin effects in the πd cross section which are by two orders of magnitude larger than the solution with the dominant spin-nonflip amplitude.

As a final remark we remind that the present limitation to πd data in the double-scattering region is caused by the fact that there are no reliable πN amplitudes outside the interval $0 > t > -0.6$ GeV^2. A systematic computer treatment of the πd problem also includes the single-scattering term in which there is no »$t/4$-effect«, and πN amplitudes are required at arguments as large as the argument of the πd cross section.

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KORIŠTENJE EKSPERIMENTALNIH PODATAKA O πd RASPRŠENJU U ANALIZI πN AMPLITUDA

N. ZOVKO

Institut »Ruđer Bošković« i Sveučilište u Zagrebu, Zagreb

U okviru pojednostavljenog Glauberovog modela razmatrano je u kojoj mjeri πd raspršenje može pomoći pri razrješavanju nekih problema u analizi πN amplituda na visokim energijama.

U kinematičkom području gdje je invarijantni preneseni impuls dovoljno velik da se u πd problemu može ispustiti komponenta jednostrukog raspršenja, πd eksperimenti omogućuju da se razluče dileme u modelima za πN amplitude. Također je očita uloga spina.

Kako je i očekivano, spinsko raspršenje je malo u blizini uskog konusa prema naprijed. Međutim, s rastućim prenesenim impulsom raste i spinska komponenta amplitude. Model favorizira CERN-ovo rješenje za πN fazne pomake.