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COHERENCE OF PIONS IN THE LINEAR CHAIN DECAY

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Abstract: The effects of the Bose-Einstein symmetry for emitted pions in the decay of cluster with $I = 0,1$ via linear chain decay are investigated. According to the degree of coherence the process is divided into coherent and incoherent stages. Effects on integrated quantities are calculated.

1. Introduction

The effect of identical particles should manifest itself in high energy multiparticle production at least, in principle. It should, for example cause the fluctuations in a number of mesons produced and influence the correlation experiments of different types. Unfortunately, as this effect has to compete with various other effects of dynamical and kinematical origin, the question of isolating and distinguishing it from the other phenomena is by no means a solved problem, especially for a system with unknown dynamics, as multiparticle production is. The first observed correlations in multiparticle production as a possible candidate for the effect was the correlation between momenta and charges of mesons, created in $p\bar{p}$ annihilation known as a Goldhaber effect[1]. In the same spirit there are attempts to explain short range effects in azimuthal correlations using Bose-Einstein symmetrization in the framework of the cluster model[2]. There were also suggestions in close analogy with Hanbury-Brown, Twiss[3] experiment, that the identical particle correlations of the second order may give information on the space-time properties of the sources of particle produced[4].

If the coherence effects are big we expect a big effect in fluctuation of particle number and in the similar integrated quantities, but we do not expect pronounced structure in correlations for two pions close in the momentum space.

Coherence effects are strong if the identical particles are produced in a limited spacial volume and with the momentum spread of the order which is consistent with uncertainty relations. Then due to the strong interaction, particles are produced nearly instantaneously and therefore they form a partially degenerate Bose gas. Recently, the pion gas was treated in a statistical approach localizing them in a »reaction volume« after which they come out freely. It was shown[5] that the clustering of the decay product pions reflects the localization in the initial state. The Bose-Einstein distribution with the initial temperature, for the inclusive spectra of pion was obtained in the limit of the infinite cluster mass. Generally, in a case when particles are populating a smaller number of elementary phase space cells then, in principle, it might be possible that coherence effects are strong and they cause broadening of the multiplicity distribution[6].

In this paper we want to discuss the possible relevance of the coherence effect of produced identical particles in the linear chain decay. There is a reason to believe that such effects could be present in so called one-dimensional processes according to the Amati-Fubini classification[7]. Those processes are characterized by one kinematical variable (that is energy in CM frame) and in such a process a cut-off in energy and isotropy of the single particle inclusive spectrum is expected[8]. If the process is assumed to go through cluster formation and its subsequent decay, as in the linear chain decay model, then there is a similarity in distribution of secondaries belonging to two steps in decay. Therefore, for produced pions we assume that they represent partially coherent gas.

2. *Definition of the degree coherence*

The question how important the coherence is could be put on quantitative ground by defining the »degree of coherence«. For two particle production amplitude $A(q_1, q_2)$ and exchange amplitude $A(q_2, q_1)$ the »degree of coherence« is naturally defined as the ratio of interference and incoherent part of the square of the amplitude

$$C^2(q_1, q_2) = \frac{2\,\mathrm{Re}\,A(q_1, q_2)\,A^*(q_2, q_1)}{|A(q_1, q_2)|^2 + |A(q_2, q_1)|^2}. \tag{2.1}$$

For the production of more particles it is more comfortable to introduce i^{th} particle production operator

$$\varphi_i^\dagger = \int \frac{d^3 p}{(2\pi)^3\, 2 p_0}\, g_i(\vec{p})\, \varphi^+(\vec{p}),$$

and define commutator

$$C_{ij} = [\varphi_i, \varphi_j^+] = \int \frac{d^3 p}{(2\pi)^3 \, 2 p_0} g_i^* (\vec{p}) \, g_j (\vec{p}). \tag{2.2}$$

It is easy to check (for example when $A(q_1, q_2)$ factorizes) that definitions (2.1) and (2.2) are equivalent.

Unitary symmetries will introduce additional indices in (2.2).

3. Linear chain decay

We assume that the process happens by the formation of heavy object (»cluster«) and its subsequent decay to another »cluster« plus a secondary. As we expect that the system with fixed energy and spin will be highly degenerated[9], one step in the decay can be represented by effective vertex operator (Fig. 1)

$$(2\pi)^{-3} \int \frac{d^3 q_i}{2 q_{0i}} \delta^{(4)} (P_{i-1} - P_i - q_i) \, F(I_i, I_{i-1} \,|\, \lambda_i, \lambda_{i-1} \,|\, P_i, P_{i-1}) \cdot$$

$$\cdot \sum_{\lambda_i} \langle 1 \, \nu_i \, I_i \, I_{3i} \,|\, I_{i-1} \, I_{3i-1} \rangle \, \varphi^+_{1 \nu_i} (q).$$

Here λ_i and λ_{i-1} denote additional labels of cluster states. The conservation laws and internal symmetry constraints are automatically satisfied by (3.1). In (3.1) also anisotropic decay is allowed[10], but keeping in mind the analogy with a decay of compound nucleus, which suggests that only low spins ($J/M \ll 1$) are important,

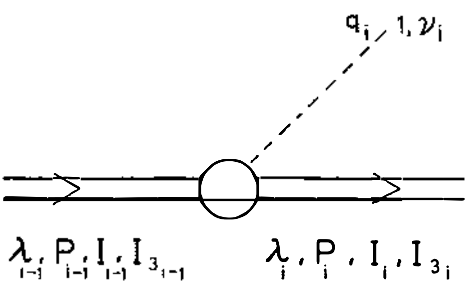

Fig. 1. Single step of the decay process.

we shall consider spin effects as inessential and consider only spin zero clusters. That means F depends only on P^2_{i-1} and P^2_i. To avoid exotic states we assume that isospin of cluster takes only values 0 and 1.

It is easy to see that recoil causes anisotropy in the distribution of secondary. To investigate this quantitatively we introduce instead of invariant variables P_i^2, P_{i-1}^2 new variables $(P_{i-1}^2)^{1/2}$, $(P_{i-1}^2 + \mu^2 - P_i^2)/(4\, P_{i-1}^2)^{1/2}$, also invariant. In the rest frame of $i-1^{th}$ cluster meaning is evident

$$\begin{aligned}(P_{i-1}^2)^{1/2} &= M_{i-1},\\ (P_{i-1}^2 + \mu^2 - P_i^{2\,2})/(4\, P_{i-1}^2)^{1/2} &= \varepsilon_i,\end{aligned} \tag{3.2}$$

mass of $i-1^{th}$cluster and energy of i^{th}particle. Now the vertex function becomes (in the CM $i-1$ frame)

$$\begin{aligned}F\,(P_i^2, P_{i-1}^2) &= f\,(M_{i-1}, \varepsilon_i)\\ F\,(P_{i+1}^2, P_i^2) &= f\,(M_i, \varepsilon_{i+1} + (\varepsilon_{i+1} - \varepsilon_i)\left(\frac{M_{i-1}}{M_i} - 1\right) +\\ &\quad + \frac{\varepsilon_i^2 - \mu^2 + (q_i - q_{i+1})^2/2)}{M_i}.\end{aligned} \tag{3.3}$$

Thus the shift of the frame due to recoil introduces anisotropy ($\vec{q}_i \cdot \vec{q}_{i-1}$term). Assuming a definit function $f\,(M, \varepsilon)$

$$f\,(M, \varepsilon)\ \alpha\ \exp\left[-\,(R + i\,I)\,\varepsilon\right]$$

a degree of coherence is

$$c^2\,(q_1, q_2) \cong \frac{\cos\,(I\,\Delta)}{\mathrm{ch}\,(R\,\Delta)},$$

$$\Delta = (\vec{q}_1 - \vec{q}_2)^2\,(\varepsilon_1 - \varepsilon_2)\,/\,(2\,M_0^2), \tag{3.4a}$$

or for

$$f\,(M, \varepsilon)\ \alpha\ \exp\,(-\,t\,\varepsilon^2)$$

a degree of coherence is

$$c^2\,(q_1, q_2) \cong 1/\mathrm{ch}\left[\frac{2\,\vec{q}_1\,\vec{q}_2}{M_0}\,(\varepsilon_2 - \varepsilon_1)\,t\right]. \tag{3.4b}$$

We can conclude that in the region where the mass of the cluster is much bigger than particle energies there will be almost no distortion of coherence due to the recoil. As a consequence momentum correlation functions will be broad with a poor structure. This suggests to look at the effects of coherence on integrated quantities as neutral to charge energy ratio ($\overline{E}_0/\overline{E}_{ch}$), $\langle n_{ch}\,(n_{ch} - 1)\rangle$, $\langle n_{ch}\, n_0\rangle$. For low mass of cluster recoil will destroy coherence almost completely and the only possible effects will be narrow structures in momentum correlation functions. To these two cases we shall refer as to coherent and incoherent stage, respectively.

Apart from recoil there could be other causes of incoherence, for example, structure of cluster, symbolized by a label λ_i, can change significantly as to give a different distribution functions for a two steps in decay. If

$$f(\lambda_1, \lambda_0 \mid \varepsilon)\ \alpha \exp\left[-(R_1 + i I_1)\,\varepsilon\right]$$

$$f(\lambda_2, \lambda_1 \mid \varepsilon)\ \alpha \exp\left[-(R_2 + i I_2)\,\varepsilon\right]$$

the degree of coherence is

$$c^2(q_1, q_2) = \frac{\cos\{(\varepsilon_1 - \varepsilon_2)\,[I_1 + I_2(\varepsilon_1 + \varepsilon_2)]\}}{\mathrm{ch}\{(\varepsilon_1 - \varepsilon_2)\,[R_1 + R_2(\varepsilon_1 + \varepsilon_2)]\}}. \tag{3.5}$$

In further we shall concentrate on features of coherent stage of decay for fixed total multiplicity n so we shall neglect energy momentum conservation constraint. For a given coherent state effect of this constraint on multiplicity distribution and dispersion was considered by Monda et al.[11].

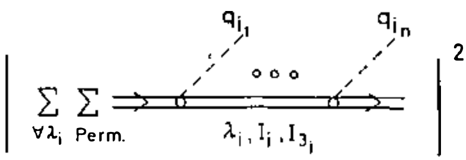

Fig. 2. n-pion production cross-section including Bose-Einstein symmetry.

The n-pion production cross-section is described graphically as (Fig. 2). Now the RPA means simply taking the sum over $\lambda_1, \lambda_2, \ldots$ out of the absolute value bracket and evidently does not spoil coherence properties.

In the coherent stage of decay we can define wave packet production operators

$$(2\pi)^{-3}\int \frac{d^3 q_l}{2\, q_{0l}}\, F(I_l, I_{l-1} \mid \lambda_l, \lambda_{l-1} \mid P_i^2, P_{i-1}^2) =$$

$$= f_l(I_l, I_{l-1} \mid \lambda_l\, \lambda_{l-1} \mid M_l)\, \varphi^{+}_{1\nu l}\,(\lambda_l, \lambda_{l-1}, M_l). \tag{3.6}$$

Keeping in mind the discussion related to (3.5) we can conclude that the degeneracy of level enters effectively by changing the degree of coherence for a two steps in chain decay. This vertex operator can be written as an matrix operator

$$[\tau_l]^{II'}_{I_3 I'_3} = \Sigma f_l(I', I)\,\langle 1\,\nu\, I'\, I'_3 \mid I\, I_3\rangle\, \varphi^{+}_{1\nu l}, \tag{3.7}$$

with commutation relations

$$[\varphi_{1\nu i}, \varphi^{+}_{1\mu j}] = \delta_{\mu\nu}\,\delta_{ij} + \delta_{\mu\nu}\,(1 - \delta_{ij})\, c_{ij}, \tag{3.8}$$

where i and j refer to the position along the chain, and c_{ij} is the effective degree of coherence for wave packets produced at i and j. $f_l(I', I)$ are invariant isospin couplings with initial (final) cluster isospin $I\,(I')$.

The production of n-pions is now described by the matrix product

$$|\, \text{n pions} + \text{final cluster} \,\rangle = \prod_{i=1}^{n} \tau_i \,|\, \text{initial cluster} \,\rangle . \tag{3.9}$$

Final cluster is the end of coherent stage of decay. We want to study the initial cluster being a mixture of $I = 0$ and $I = 1$ components

$$|\, \text{initial cluster} > = \alpha \,|\, 00 \rangle + \beta \,|\, 10 \rangle . \tag{3.10}$$

We first discuss fully coherent case $c_{ij} = 1$. Omitting the details of calculation[12] we merely state results: in this case instead of variety of chains only pure isovector chain and the strictly alternating isovector isoscalar chains are contributing.

For N outgoing pions

$$\frac{\langle N_0 \rangle_N}{N} = \begin{cases} \dfrac{N\,[2 + |\varkappa|^N (5\,\varrho/\lambda + 3/\varrho) + 2\,[-1 + |\varkappa|^N/\varrho]}{5N\,[2 + |\varkappa|^N (3\,\varrho/\lambda + 1/\varrho)]} & N \text{ odd,} \\[2ex] \dfrac{N^2 [2 + |\varkappa|^N (5/\lambda + 3)] + 8N\,[|\varkappa|^N - \mathrm{Re}\,\varkappa^{N/2}] + 2\,|1 - \varkappa|^N}{5\,N\,\{N\,[2 + |\varkappa|^N (3/\lambda + 1)] + 2\,|1 - \varkappa|^N\}} & N \text{ even.} \end{cases} \tag{3.11}$$

For $N \to \infty$, the asymptotic values of $\overline{N}_0/N$ are

$$\left[\frac{\langle N_0 \rangle_N}{N}\right]_{\mathrm{as}} = \begin{cases} \dfrac{5\varrho^2 + 3\,\lambda}{5\,(3\varrho^2 + \lambda)} & N \text{ odd} \\[2ex] \dfrac{5 + 3\,\lambda}{5\,(3 + \lambda)} & N \text{ even} \end{cases} \quad |\varkappa| > 1, \qquad 0.2 \quad |\varkappa| < 1. \tag{3.12}$$

The parameters are related to invariant vertex functions

$$\varkappa = -\frac{2}{\sqrt{3}} \frac{F^\star(0,1)\,F^\star(1,0)}{F^2(1,1)}, \qquad \varrho = \frac{1}{3}\left|\frac{F(0,1)}{F(1,0)}\right|^2, \qquad \lambda = \left|\frac{\beta}{\alpha}\right|^2 . \tag{3.13}$$

For purely isovector (alternating) chain we obtain lower (upper) isospin bound for neutral to charge energy ratio.

Parameters extracted from Chan-Paton model[13] lead to the exact charge symmetry. Dominance of isovector chain gives »surplus of charged energy« and dominance of alternating chain gives »energy crisis«. Integrated correlations are for $\lambda \to \infty$

$$\frac{\langle N_0 (N_0 - 1) \rangle_N}{N^2} = \frac{(N-1)}{N} \cdot \begin{cases} [1 - 6/(5\,N)] \cdot 3/7 & |\varkappa| \to \infty \\ [1 - 2/(3N)] \cdot 3/35 & |\varkappa| \to 0 \\ [1 + 2/(3N)] \cdot 3/15 & |\varkappa| = 1 \end{cases},$$

$$\frac{\langle N_0 N_+\rangle_N}{N^2} = \frac{N-1}{N} \cdot \begin{cases} [1 - 6/(5N)]/7 & |\varkappa| \to \infty \\ [1 - 5/(4N)] \cdot 2/35 & |\varkappa| \to 0 \\ [1 - 1/N]/15 & |\varkappa| = 1 \end{cases},$$

$$\langle N_{ch}^2\rangle_N - \langle N_{ch}\rangle_N^2 = -[\langle N_0 N_{ch}\rangle_N - \langle N_0\rangle_N \langle N_{ch}\rangle_N] =$$

$$= (N-1)(N+4) \begin{cases} 12/175 & |\varkappa| \to \infty \\ 8/175 & |\varkappa| \to 0 \\ 4/45 & |\varkappa| = 1 \end{cases}. \tag{3.14}$$

The case $c_{ij} = c < 1$ can be treated by the introduction of new operators

$$\varphi_{\nu i} = \gamma\, \varphi_\nu + \delta\, \psi_{\nu i}$$
$$\gamma^2 + \delta^2 = 1, \quad c = \gamma^2 \tag{3.15}$$
$$[\varphi_\nu, \psi_{\mu i}^+] = 0, \qquad [\varphi_\nu, \varphi_M^+] = \delta_{\mu\nu}, \qquad [\psi_{\mu i}, \psi_{\nu j}^+] = \delta_{\mu\nu}\, \delta_{ij}.$$

$\varphi_{\nu i}$ defined this way satisfy (3.8) with $c_{ij} = c$. Following the method of calculation developed in Ref.[12] we calculate $\overline{N}_0/N_T$ for N_T odd and $\beta = 1$. In Fig. 3 we see that deviations from $c = 1$ affect strongly only low N_T region. Asymptotic region remains the same.

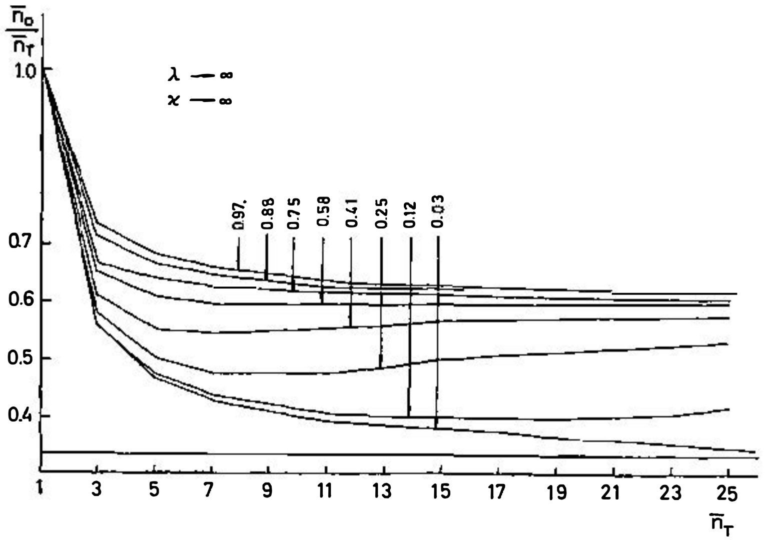

Fig. 3. $\frac{\overline{n_0}}{n}$ for the partially coherent case plotted for various values of c in the case when $|\varkappa| \to \infty$ and $\lambda \to \infty$.

To conclude if we believe that produced clusters are heavy enough to develop coherent stage of decay we can expect strong effects on integrated quantities. For $\frac{\langle N_0\rangle_N}{N} - \frac{1}{3}$ this effect will depend in sign and magnitude on isospin couplings, and for integrated correlations only the magnitude will be sensitive. Such situations could be realized in $e^+e^- \to$ hadrons and (or) $\bar{p}p$ annihilation.

For low mass clusters structure in the energy-momentum correlations-functions are to be expected. Such features are discussed in pp and πp scattering[2,4].

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KOHERENCIJA PI-MEZONA U RASPADU PREKO LINEARNOG LANCA

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Sadržaj

U radu je istraživan utjecaj Bose-Einsteinove simetrije na raspad »klastera« izospina $I = 0,1$ preko linearnog lanca. Ispitan je utjecaj kinematičkih i dinamičkih faktora na stupanj koherencije, te je proces podijeljen na koherentnu i nekoherentnu fazu.

Pokazano je da u koheretnoj fazi procesu doprinose samo dva lanca: lanac izospina 1, te lanac sa strogo alternirajućim izospinom 0 i 1. Za slučaj da je početno stanje linearna kombinacija stanja izospina 1 i 0 proračunati su $\langle n \rangle$, $\langle n_0 \rangle$ te integrirane funkcije korelacije. Diskutiran je i slučaj parcijalne koherencije.