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NOTE ON THE FREQUENCY-DEPENDENT NON-RPA CONTRIBUTIONS TO THE ELECTRONIC DIELECTRIC FUNCTION

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The aim of the present paper is to study the behaviour of a degenerate high-density electron gas of concentration N immersed in a uniform background of positive charge. The dielectric function of such a system may be written as[1)]

$$\varepsilon(q, \omega) = 1 + 2V_q \sum_k F_{k+qk}(\omega)\, G_{k+qk}(\omega)\, (N_{k+q} - N_k). \tag{1}$$

N_k is the Fermi distribution function, V_q is the Fourier transform of the Coulomb potential

$$V_q = \frac{4\pi e^2}{q^2}, \tag{2}$$

and $G_{k+qk}(\omega)$ is given by

$$G_{k+qk}(\omega) = \frac{1}{\hbar\omega + E_k - E_{k+q}}, \tag{3}$$

E_K being the energy of a free electron

$$E_k = \frac{\hbar^2 k^2}{2m}. \tag{4}$$

The function $F_{k+qk}(\omega)$ plays the main role in the theory; it is determined by coupled equations of motion for particle-hole pairs. Confining ourselves to the approximation which extends the RPA only to lowest-order corrections, we have

$$F_{k+qk}(\omega) = 1 + \sum_{k'} V_{k-k'} (N_{k'+q} - N_{k'}) \left[G_{k'+qk'}(\omega) - G_{k+qk}(\omega)\right]. \tag{5}$$

Now Equ. (1) becomes

$$\varepsilon(q,\omega) = \varepsilon_L(q,\omega) + \varepsilon_{cx}(q,\omega), \tag{6}$$

where $\varepsilon_L(q,\omega)$ is Lindhard's[2] dielectric function

$$\varepsilon_L(q,\omega) = 1 + 2V_q \sum_k G_{k+qk}(\omega)(N_{k+q} - N_k), \tag{7}$$

and $\varepsilon_{cx}(q,\omega)$ represents the contribution describing exchange scattering

$$\varepsilon_{cx}(q,\omega) = 2V_q \sum_{kk'} V_{k-k'}(N_{k+q} - N_k)(N_{k+q} - N_{k'})\, G'_{k+qk}(\omega)\,[G'_{k+qk}(\omega) - G_{k'+qk'}(\omega)]. \tag{8}$$

Neglecting higher-order terms in the series expansion for $F_{k+qk}(\omega)$, we have tacitly assumed that the second term on the right-hand side of Equ. (5) is much smaller than the first one. In other words, our basic assumption is

$$\varepsilon_L(q,\omega) \gg \varepsilon_{cx}(q,\omega). \tag{9}$$

Starting from expressions (7) and (8), we calculate the real part of the long--wavelength dielectric function at zero temperature. If q is much smaller than the Fermi wave number k_F, we may write

$$N_{k+q} - N_k = -\frac{\vec{q}\,\vec{k}}{k}\,\delta(k - k_F), \tag{10}$$

$$E_{k+q} - E_k = \frac{\hbar^2}{m}\,\vec{k}\,\vec{q}. \tag{11}$$

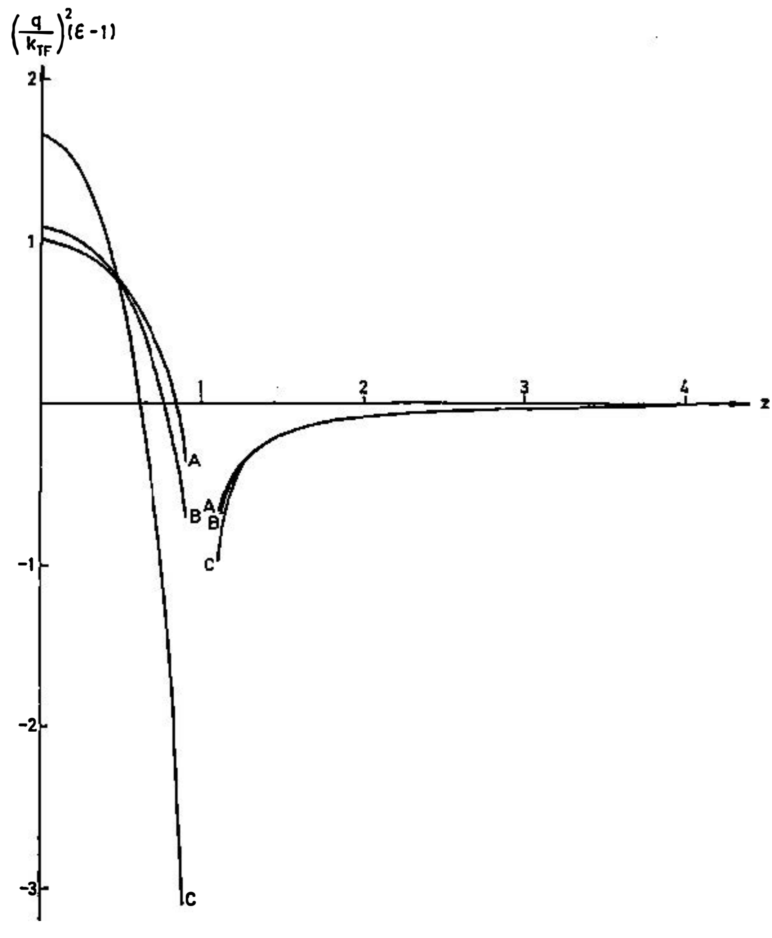
In this approximation $\varepsilon_L(q,\omega)$ is given by

$$\varepsilon_L(q,\omega) = 1 + \frac{3\,\omega_p^2}{q^2 v_F^2}\left(1 - \frac{z}{2}\ln\left|\frac{1+z}{1-z}\right|\right), \tag{12}$$

where ω_p is the electron plasma frequency $\omega_p = (4\pi Ne^2/m)^{1/2}$ and z is the ratio between the phase velocity ω/q and the Fermi velocity v_F

$$z = \frac{\omega}{q\,v_F}. \tag{13}$$

Expression (12) shows that our approximate treatment automatically introduces singularity into the calculation. This occurs if the phase velocity of the wave is equal to the maximum particle velocity. Having in mind that singularity does not appear in the exact Lindhard's result, we must exclude the point $z = 1$ from our consideration.



Curve A = RPA. Curve B = Extended RPA for $r_s = 0.5$. Curve C = Extended RPA for $r_s = 4$.

In a similar manner we can perform the calculation of the exchange contribution to the dielectric function. The result is

$$\varepsilon_{ex}(q, \omega) = \left(\frac{k_{TF}}{q}\right)^2 \frac{\gamma}{2}\left[2 - \frac{4z^2}{1-z^2} + z\left(\frac{z^2}{1-z^2} - 2\right)\ln\left|\frac{1+z}{1-z}\right|\right], \tag{14}$$

where k_{TF} is the Thomas-Fermi wave number

$$k_{TF} = \frac{\omega_p}{v_p}\sqrt{3} \tag{15}$$

and γ is defined by

$$\gamma = \left(\frac{k_{TF}}{2k_F}\right)^2 = \frac{r_s}{\pi}\left(\frac{4}{9\pi}\right)^{1/3}, \tag{16}$$

r_s being the dimensionless measure for the interparticle spacing

$$r_s = \frac{me^2}{\hbar^2}\left(\frac{3}{4\pi N}\right)^{1/3}. \tag{17}$$

The exchange terms are proportional to the parameter of the perturbation expansion γ. Of course, for a high-density gas it must satisfy the condition

$$\gamma \ll 1. \tag{18}$$

Now it is easy to find the behaviour of the dielectric function in the limiting cases $z \gg 1$ and $z \ll 1$. For large frequencies, the dielectric function becomes

$$\varepsilon(q, \omega) = 1 - \frac{\omega_p^2}{\omega^2}\left[1 + \frac{3}{5z^2}\left(1 - \frac{\gamma}{3}\right)\right] \quad z \gg 1, \tag{19}$$

in agreement with the results obtained previously[3-5]. In the opposite limit of low frequencies, by virtue of (18) we may write

$$\varepsilon(q,\omega) = 1 + \left(\frac{k_{TF}}{q}\right)^2 \frac{1}{1-\gamma} \quad z \ll 1, \tag{20}$$

which is in accordance with the result derived by v. Ross[6].

Expression (19) shows that for $z \gg 1$ the correction contribution does not influence the leading term of $[\varepsilon(q,\omega) - 1]$. Hence we expect that RPA works better in the high-frequency than in the low-frequency limit. This is shown in the figure. It is worth noting that the RPA and the extended RPA are already in close agreement at frequencies which are of the order of qv_F. For example, choosing $r_s = 4$, we find that at $z = 2$ exchange scattering influences the RPA value by 6% and at $z = 3$ only by 1%.

References

1) D. Pines and P. Nozieres, The Theory of Quantum Liquids I (W. A. Benjamin, New York —Amsterdam, 1966);
2) J. Lindhard, Kgl. Danske Videnskab. Selskab. Matt-fys. Medd. **28** (1954) 8;
3) P. Nozières and D. Pines, Phys. Rev. **111** (1958) 442;
4) H. Kanazawa, S. Misawa and E. Fujita, Progr. Theoret. Phys. **23** (1960) 426;
5) O. v. Roos and J. S. Zmuidzinas, Phys. Rev. **121** (1961) 941;
6) O. v. Roos, Phys. Rev. **124** (1961) 71.

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