FIZIKA 9 (1977) 105—115 YU ISSN 0015-3206

POTENTIALS IN THE DIRAC MASSLESS FIELD THEORY

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Received 2 February 1976

Abstract: The potential method in the theory of Dirac's field introduced in the paper¹⁾ is analysed with respect to gauge invariance of the Lagrangian density. It is shown that the requirement of the gauge invariance of the Lagrangian density is not necessary condition. Considering the Dirac's field as physical object, only final results which can be expressed completely by ψ are correct selection, regardless how they are obtained. Various Lagrangian densities are constructed from the potential and known constants of motion are obtained. The considerations are restricted to the massless Dirac's field.

1. *Introduction*

In the paper¹⁾ the potential of Dirac's field is introduced in the following way*⁾:

$$\partial_\mu \gamma^\mu \psi = 0, \quad \partial_\mu \bar{\psi} \gamma^\mu = 0, \tag{1.1}$$

$$\psi = \gamma^\nu \partial_\nu \Phi, \quad \bar{\psi}_\mu = \partial_\mu \bar{\Phi} \gamma^\nu, \tag{1.2}$$

$$\partial_\mu \partial^\mu \Phi = 0 \quad (\Box \Phi = 0), \quad \partial_\mu \partial^\mu \bar{\Phi} = 0, \tag{1.3}$$

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*⁾ It is interesting to mention that Feynman and Gell-Mann³⁾ and Feynman himself⁴⁾⁵⁾, had introduced in the Dirac field theory the quantity which corresponds to Φ, considering weak interaction problems, but with different viewpoint and corresponding consequences. However, the algebraic connection Φ and ψ does not hold for massless Dirac field. In this paper we do not discuss interaction of the Dirac field with some other field. We will do it at another place. The situation will be then more clear. Because of this we leave further comparison out.

where ψ is the Dirac's field, Φ is the potential, γ^μ Dirac's matrices

$$\begin{aligned} &\gamma^0 = \beta,\ \gamma^k = \beta\, \alpha^k, \qquad && \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}_{4\times 4}, \qquad && \alpha^k = \begin{pmatrix} 0 & \sigma^k \\ \sigma^k & 0 \end{pmatrix}, \\ &\qquad k = 1,\ 2,\ 3 && && \\ &\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2 g^{\mu\nu}, && x^\mu = (t, \vec{x}), && \partial_\mu \equiv \frac{\partial}{\partial x^\mu}, \\ &\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu = -2\, \mathrm{i}\, \sigma^{\mu\nu}, && x_\mu = (t, -\vec{x}), && \end{aligned} \tag{1.4}$$

and (1.3) is equation for the potential (c = 1).

The potential is undetermined up to a transformation

$$\Phi' = \Phi + \chi \tag{1.5}$$

with

$$\partial_\nu \gamma^\nu \chi = 0. \tag{1.6}$$

The corresponding property of Φ we call the gauge invariance (or in comparison to the Maxwell field »gradient invariance«) and (1.5) with (1.6) the gauge (or »gradient«) transformation.

Let us emphasize that Φ is a spinor which has to satisfy the D'Alambertian equation (1.3) not the Dirac equation.

It is useful to mention an other property of Φ. In order to get general solution of (1.1) it is enough to take two non zero components of Φ (similarly to the electromagnetic field theory[2]). Indeed, writing the general solution of equation (1.3) in the form

$$\Phi = \sum_{\vec{p}} \left\{ \begin{pmatrix} C^1_{1\vec{p}} \\ \cdot \\ \cdot \\ \cdot \\ C^4_{1\vec{p}} \end{pmatrix} e^{i(\vec{p}\vec{x} - pt)} + \begin{pmatrix} C^1_{2p} \\ \cdot \\ \cdot \\ \cdot \\ C^4_{2p} \end{pmatrix} e^{i(\vec{p}\vec{x} + pt)} \right\}, \tag{1.7}$$

$$p = |\vec{p}|,$$

the substitution (1.7) into (1.2) gives the general solution of the Dirac equation

$$\begin{pmatrix} \psi^1 \\ \psi^2 \\ \psi^3 \\ \psi^4 \end{pmatrix} = \begin{pmatrix} \partial_0 & 0 & \partial_z & \partial_x - i\,\partial_y \\ 0 & \partial_0 & \partial_x + i\,\partial_y & -\partial_z \\ -\partial_z & -\partial_x + i\,\partial_y & -\partial_0 & 0 \\ -\partial_x - i\,\partial_y & \partial_z & 0 & -\partial_0 \end{pmatrix} \begin{pmatrix} \Phi^1 \\ \Phi^2 \\ \Phi^3 \\ \Phi^4 \end{pmatrix} \tag{1.8}$$

(where γ^μ are defined with (1.4)), or

$$\Psi = \sum_{\vec{p}} \left\{ \left[C^1_{1\vec{p}} \begin{pmatrix} p \\ 0 \\ p_z \\ p_x + ip_y \end{pmatrix} + C^2_{1\vec{p}} \begin{pmatrix} 0 \\ p \\ p_x - ip_y \\ -p_z \end{pmatrix} + C^3_{1\vec{p}} \begin{pmatrix} -p_z \\ -p_x \quad ip_y \\ -p \\ 0 \end{pmatrix} + \right.\right.$$

$$\left. + C^4_{1\vec{p}} \begin{pmatrix} -p_x + ip_y \\ p_z \\ 0 \\ -p \end{pmatrix} \right] e^{i(\vec{p}\vec{x} - pt)} + \left[C^1_{2\vec{p}} \begin{pmatrix} -p \\ 0 \\ p_z \\ p_x + ip_y \end{pmatrix} + C^2_{2\vec{p}} \begin{pmatrix} 0 \\ -p \\ p_x - ip_y \\ -p_z \end{pmatrix} + \right. \tag{1.9}$$

$$\left.\left. + C^3_{2\vec{p}} \begin{pmatrix} -p_z \\ -p_x - ip_y \\ p \\ 0 \end{pmatrix} + C^4_{2\vec{p}} \begin{pmatrix} -p_x + ip_y \\ p_z \\ 0 \\ p \end{pmatrix} e^{i(\vec{p}\vec{x} - pt)} \right\}.$$

One can easily see that two columns in the first row are linearly dependent from other two. This is also the case in the second row. Therefore, we can keep only two columns from each row. We take the first two columns from each row.

$$\Psi = \sum_{\vec{p}} \left\{ \left[C^1_{1\vec{p}} \begin{pmatrix} p \\ 0 \\ p_z \\ p_x + ip_y \end{pmatrix} + C^2_{1\vec{p}} \begin{pmatrix} 0 \\ p \\ p_x - ip_y \\ -p_z \end{pmatrix} \right] e^{i(\vec{p}\vec{x} - pt)} + \left[C^1_{2\vec{p}} \begin{pmatrix} -p \\ 0 \\ p_z \\ p_x + ip_y \end{pmatrix} + \right.\right.$$

$$\left.\left. + C^2_{2\vec{p}} \begin{pmatrix} 0 \\ -p \\ p_x - ip_y \\ -p_z \end{pmatrix} \right] e^{i(\vec{p}\vec{x} - pt)} \right\}. \tag{1.10}$$

The corresponding potential reads

$$\Phi = \begin{pmatrix} \Phi^1 \\ \Phi^2 \\ \Phi^3 \\ \Phi^4 \end{pmatrix} = \sum_{\vec{p}} \left\{ \begin{pmatrix} C^1_{1\vec{p}} \\ C^2_{1\vec{p}} \\ 0 \\ 0 \end{pmatrix} e^{i(\vec{p}\vec{x} - pt)} + \begin{pmatrix} C^1_{2\vec{p}} \\ C^2_{2\vec{p}} \\ 0 \\ 0 \end{pmatrix} e^{i(\vec{p}\vec{x} + pt)} \right\}. \tag{1.11}$$

It is evident that here $\Phi_3 = \Phi_4 = 0$.

Let's write Equ. (1.2) explicitly with this choice of the potential

$$\begin{aligned} \Psi^1 &= \partial_0 \Phi^1, \\ \Psi^2 &= \partial_0 \Phi^2, \\ \Psi^3 &= -\partial_z \Phi^1 + (-\partial_x + i\,\partial_y)\Phi^2, \\ \Psi^4 &= -(\partial_x + i\,\partial_y)\Phi^1 + \partial_z \Phi^2. \end{aligned} \tag{1.12}$$

Having introduced the potential one can develop the Lagrangian formalism based on the potential. The question arises how the results of this procedure are related to the field Ψ? The answer is, however, obvious. Each final result which can be completely expressed by the field Ψ has physical meaning for the field Ψ, otherwise is of no interest to Ψ (if we take that the physical object is completely described by Ψ). Therefore, we do not require the gauge invariance of the Lagrangian[*]. We consider it here and show that various Lagrangians can be used and have to be used. The spinor's scalar with first order derivatives of Φ gives the constant of motion $\int dv\, \Psi^+ \Psi$, the spinor's scalar with second order derivatives of Φ gives the energy-momentum tensor for Ψ and corresponding constants of motion, and so on.

In Section 2 we give scalars constructed from Φ and select those which might be proportional to Lagrangian density. In Section 3 we consider Lagrangian formalism for these densities. Conclusions are given in Section 4.

2. *Spinor's scalars and Lagrangian densities*

Real spinor's scalars constructed from Φ and its partial derivatives are

$$\bar\Phi \Phi,$$

$$C_\pm (\bar\Phi\, \partial_\mu \gamma^\mu \Phi \pm \partial_\mu \bar\Phi \gamma^\mu \Phi) = C_\pm (\bar\Phi \Psi \pm \bar\Psi \Phi),$$

$$C_+ (\partial_\nu \bar\Phi \gamma^\nu)(\partial_\mu \gamma^\mu \Phi) = C_+ \bar\Psi \Psi,$$

$$C_\pm (\bar\Phi\, \partial^\mu \partial_\mu \Phi \pm \partial_\mu \partial^\mu \bar\Phi \Phi) = C_\pm (\bar\Phi \gamma^\mu \partial_\mu \Psi \pm \partial_\mu \bar\Psi \gamma^\mu \Phi),$$

$$C_+ (\partial_\mu \bar\Phi\, \partial^\mu \Phi),$$

$$C_\mp [(\partial_\nu \partial_\lambda \bar\Phi \gamma^\lambda \gamma^\nu)(\gamma^\mu \partial_\mu \Phi) \mp (\partial_\mu \bar\Phi \gamma^\mu)(\gamma^\nu \partial_\nu \gamma^\lambda \partial_\lambda \Phi)] =$$

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[*] However, it turns out that physical concentration can be put on the gauge invariant Lagrangians. The potential method then makes possible the usage of the simplest scalar $\bar\psi \psi$ constructed from ψ as a Lagrangian density.

$$= C_{\mp} [\partial_\nu \bar{\Psi} \gamma^\nu \Psi_{\mp} \bar{\Psi} \gamma^\nu \partial_\nu \Psi], \tag{2.1}$$

$$(\partial_\nu \partial_\lambda \bar{\Phi} \gamma^\lambda \gamma^\nu) (\gamma^\mu \gamma^\eta \delta_\mu \partial_\eta \Phi) = (\partial_\nu \bar{\Psi} \gamma^\nu) (\gamma^\mu \partial_\mu \Psi),$$

$$C_\pm \; [(\partial_\nu \bar{\Phi} \gamma^\nu) (\partial_\mu \partial^\mu \gamma^\alpha \partial_\alpha \Phi) \pm (\partial_\mu \partial^\mu \partial_\alpha \bar{\Phi} \gamma^\alpha) (\gamma^\nu \partial_\nu \Phi)] =$$

$$= C_\pm [\bar{\Psi} \partial_\mu \partial^\mu \Psi \pm \partial_\mu \partial^\mu \bar{\Psi} \Psi],$$

and so on,

where $\Phi = \Phi^+ \gamma^\circ$, C_+ is a real and C_- an imaginary constant.

The Lagrangian equation (1.3) for Φ is second order differential equation. Consequently, the first two scalars from (2.1) can not be Lagrangian densities. All others might be. We multiply these scalars with a constant and analyze them as Lagrangian densities in the next Section.

3. Lagrangian formalism

a). Taking the third scalar from (2.1), multiplied by a constant K_a, as Lagrangian density,

$$\mathscr{L}_a = K_a (\partial_\nu \bar{\Phi} \gamma^\nu) (\gamma^\mu \partial_\mu \Phi) = K_a \bar{\Psi} \Psi, \tag{3.1}$$

the Lagrangian equations

$$\frac{\partial \mathscr{L}}{\partial \varphi} - \partial_\mu \frac{\partial \mathscr{L}}{\partial (\partial_\mu \varphi)} = 0 \tag{3.2}$$

give

$$- \gamma^\nu \gamma^\mu \partial_\nu \partial_\mu \Phi = 0$$

or

$$\partial^\mu \partial_\mu \Phi = 0 \tag{3.3}$$

and

$$\partial^\mu \partial_\mu \bar{\Phi} = 0. \tag{3.4}$$

Therefore, the $\mathscr{L}_a$ given by Equ. (3.1) is a correct Lagrangian density for Φ.

We evaluate now the energy-momentum tensor for this $\mathscr{L}_a$

$$T_\alpha^\beta = \partial_\alpha \bar{\Phi} \frac{\partial \mathscr{L}}{\partial (\partial_\beta \bar{\Phi})} + \frac{\partial \mathscr{L}}{\partial (\partial_\beta \Phi)} \partial_\alpha \Phi - \delta_\alpha^\beta \mathscr{L} =$$

$$= K_a [\partial_\alpha \bar{\Phi} \gamma^\beta \gamma^\mu \partial_\mu \Phi + \partial_\nu \Phi \gamma^\nu \gamma^\beta \partial_\alpha \Phi - \delta_\alpha^\beta \partial_\nu \bar{\Phi} \gamma^\nu \gamma^\mu \partial_\mu \Phi] =$$

$$= K_a [\partial_\alpha \bar{\Phi} \gamma^\beta \Psi + \bar{\Psi} \gamma^\beta \partial_\alpha \Phi - \delta_\alpha^\beta \bar{\Psi} \Psi],$$

or

$$T_\alpha^\beta = K_\alpha \begin{pmatrix} (\dot{\Phi}^+ \Psi + \Psi^+ \dot{\Phi} - \bar{\Psi}\Psi) & (\dot{\Phi}\gamma^1 \Psi + \bar{\Psi}\gamma^1 \dot{\Phi}) & (\dot{\Phi}\gamma^2 \Psi + \bar{\Psi}\gamma^2 \dot{\Phi}) & (\dot{\Phi}\gamma^3 \Psi + \bar{\Psi}\gamma^3 \dot{\Phi}) \\ (\partial_x \Phi^+ \Psi + \Psi^+ \partial_x \Phi) & (\partial_x \bar{\Phi}\gamma^1 \Psi + \bar{\Psi}\gamma^1 \partial_x \Phi - \bar{\Psi}\Psi) & (\partial_x \bar{\Phi}\gamma^2 \Psi + \bar{\Psi}\gamma^2 \partial_x \Phi) & (\partial_x \bar{\Phi}\gamma^3 \Psi + \bar{\Psi}\gamma^3 \partial_x \Phi) \\ (\partial_y \Phi^+ \Psi + \Psi^+ \partial_y \Phi) & (\partial_y \bar{\Phi}\gamma^1 \Psi + \bar{\Psi}\gamma^1 \partial_y \Phi) & (\partial_y \bar{\Phi}\gamma^2 \Psi + \bar{\Psi}\gamma^2 \partial_y \Phi - \bar{\Psi}\Psi) & (\partial_y \bar{\Phi}\gamma^3 \Psi + \bar{\Psi}\gamma^3 \partial_y \Phi) \\ (\partial_z \Phi^+ \Psi + \Psi^+ \partial_z \Phi) & (\partial_z \bar{\Phi}\gamma^1 \Psi + \bar{\Psi}\gamma^1 \partial_z \Phi) & (\partial_z \bar{\Phi}\gamma^2 \Psi + \bar{\Psi}\gamma^2 \partial_z \Phi) & (\partial_z \bar{\Phi}\gamma^3 \Psi + \bar{\Psi}\gamma^3 \partial_z \Phi - \bar{\Psi}\Psi) \end{pmatrix} \tag{3.5}$$

where α are rows and β are columns, and $\dot{\Phi} = \partial_0 \Phi$.

From here the constants of motion are

$$P_\alpha = C \int T_\alpha^0 \, \mathrm{d}V, \text{ (C is a constant),}$$

or explicitly

$$\begin{aligned} P_0 &= CK_\alpha \int (\dot{\Phi}^+ \Psi + \Psi^+ \Phi - \bar{\Psi}\Psi) \, \mathrm{d}V, \\ P_i &= CK_\alpha \int (\partial_i \Phi^+ \Psi + \Psi^+ \partial_i \Phi) \, \mathrm{d}V, \quad i = 1,2,3. \end{aligned} \tag{3.6}$$

Now, taking $\Phi_3 = \Phi_4 = 0$ and making use of (1.12) we get

$$\begin{aligned} P_0 &= CK_\alpha \int \Psi^+ \Psi \, \mathrm{d}V, \\ P_i &= CK_\alpha \int (\partial_i \Phi^{1*} \Psi^1 + \partial_i \Phi^{2*} \Psi^2 + \Psi^{1*} \partial_i \Phi^1 + \Psi^{2*} \partial_i \Psi^2) \, \mathrm{d}V. \end{aligned} \tag{3.7}$$

We see that P_0 is only constant of motion fully expressed by Ψ. Therefore, this constant has physical meaning for the field Ψ. The conclusion is that from the Lagrangian density (3.1) follows only one constant of motion of the field Ψ and that is P_0.

The angular momentum tensor

$$M^{\alpha,\beta\gamma} = (x^\gamma T^{\beta\alpha} - x^\beta T^{\gamma\alpha}) - \frac{\partial \mathcal{L}}{\partial(\partial_\alpha \Phi)} \left(\frac{i}{2} \sigma^{\beta\gamma} \right) \Phi - \Phi \left(-\frac{i}{2} \sigma^{\beta\gamma} \right) \frac{\partial \mathcal{L}}{\partial(\partial_\alpha \bar{\Phi})} \tag{3.8}$$

for $\mathscr{L}_a$ is

$$\begin{aligned} M^{\alpha,\beta\gamma} &= K_a x^\gamma (\partial^\beta \Phi \gamma^\alpha \Psi + \overrightarrow{\bar{\Psi}} \gamma^\alpha \partial^\beta \Phi - g^{\alpha\beta} \overleftarrow{\bar{\Psi}} \Psi) - \\ &- K_a x^\beta (\partial^\gamma \bar{\Phi} \gamma^\alpha \Psi + \bar{\Psi} \gamma^\alpha \partial^\gamma \Phi - g^{\alpha\gamma} \overleftarrow{\bar{\Psi}} \Psi) - \\ &- K_a \bar{\Psi} \gamma^\alpha \left(\frac{i}{2} \sigma^{\beta\gamma}\right) \Phi - K_a \bar{\Phi} \left(- \frac{i}{2} \sigma^{\beta\gamma}\right) \gamma^\alpha \Psi. \end{aligned} \tag{3.9}$$

The corresponding constant of motion is

$$\begin{aligned} M^{o,\beta\gamma} &= K_a \int \mathrm{d}V x^\gamma (\partial^\beta \Phi^+ \Psi + \Psi^+ \partial^\beta \Phi - g^{\circ\beta} \bar{\Psi} \Psi) - \\ &- K_a \int \mathrm{d}V x^\beta (\partial^\gamma \Phi^+ \Psi + \Psi^+ \partial^\gamma \Phi - g^{o\gamma} \bar{\Psi} \Psi) - \\ &- \frac{i}{2} K_a \int \mathrm{d}V (\Psi^+ \sigma^{\beta\gamma} \Phi - \bar{\Phi} \sigma^{\beta\gamma} \gamma^o \Psi). \end{aligned} \tag{3.10}$$

As we see this tensor can not be expressed by the Ψ. Therefore, it does not have any physical meaning for the field Ψ.

b). The next Lagrangian density, which we consider, is the fourth scalar from (2.1) multiplied by a constant K_b

$$\mathscr{L}_b = K_{b\pm} (\bar{\Phi} \partial^\mu \partial_\mu \Phi \pm \partial^\mu \partial_\mu \bar{\Phi} \Phi) = K_{b\pm} (\bar{\Phi} \gamma^\mu \partial_\mu \Psi \pm \partial_\mu \bar{\Psi} \gamma^\mu \Phi). \tag{3.11}$$

The Lagrangian equations for Lagrangians which contain second derivatives of field components φ are

$$\frac{\partial \mathscr{L}}{\partial \varphi} - \partial_\mu \frac{\partial \mathscr{L}}{\partial (\partial_\mu \varphi)} + \partial_\mu \partial_\nu \frac{\partial \mathscr{L}}{\partial (\partial_\mu \partial_\nu \varphi)} = 0, \tag{3.12}$$

and the energy-momentum tensor

$$T_\alpha^\beta = \frac{\partial \mathscr{L}}{\partial \varphi_{/\beta}} \varphi_{/\alpha} - \delta_\alpha^\beta \mathscr{L} + \frac{\partial \mathscr{L}}{\partial \varphi_{/\beta/\nu}} \varphi_{,\alpha/\nu} - \partial_\nu \frac{\partial \mathscr{L}}{\partial \varphi_{/\beta/\nu}} \varphi_{/\alpha}. \tag{3.13}$$

The equations (3.12) for $\mathscr{L}$ given by (3.11) are

$$K_{b\pm} (\partial^\mu \partial_\mu \Phi \pm \partial^\mu \partial_\mu \Phi) = 0$$

The equation with minus sign is identically zero. We conclude that only the Lagrangian with plus sign is of interest. In this case we have

$$\partial^\mu \partial_\mu \Phi = 0$$

and

$$\partial^\mu \partial_\mu \bar{\Phi} = 0.$$

Therefore, the Lagrangian (3.11) with plus sign is also a correct Lagrangian density for the field Φ.

The energy-momentum tensor for this Lagrangian, according to (3.13), is then

$$T_{\alpha}^{\beta} = K_{b+} (\bar{\Phi} \partial^{\beta} \partial_{\alpha} \Phi + \partial^{\beta} \partial_{\alpha} \bar{\Phi} \Phi - \partial^{\beta} \bar{\Phi} \partial_{\alpha} \Phi - \partial_{\alpha} \bar{\Phi} \partial^{\beta} \Phi) \tag{3.14}$$

and the constants of motion

$$\begin{aligned} P_{\alpha} &= C' \int T_{\alpha}{}^{\circ} \, \mathrm{d}V = \\ &= C' K_{b+} \int (\bar{\Phi} \partial_{\alpha} \dot{\Phi} + \partial_{\alpha} \dot{\bar{\Phi}} \Phi - \dot{\bar{\Phi}} \partial_{\alpha} \Phi - \partial_{\alpha} \bar{\Phi} \dot{\Phi}) \, \mathrm{d}V \end{aligned} \tag{3.15}$$

or

$$P_0 = C' K_{b+} \int (\bar{\Phi} \ddot{\Phi} + \ddot{\bar{\Phi}} \Phi - \dot{\bar{\Phi}} \dot{\Phi} - \dot{\bar{\Phi}} \Phi) \, \mathrm{d}V$$

$$P_{l} = C' K_{b+} \int (\bar{\Phi} \partial_{l} \dot{\Phi} + \partial_{l} \dot{\bar{\Phi}} \Phi - \dot{\bar{\Phi}} \partial_{l} \Phi - \partial_{l} \bar{\Phi} \dot{\Phi}) \, \mathrm{d}V,$$

where C' is a constant.

Taking again $\Phi_3 = \Phi_4 = 0$, and making use of (1.1) we find

$$P_0 = - 2C' K_{b+} \int \mathrm{d}V \, \Psi^{+} \Psi,$$

$$P_{l} = - 2C' K_{b+} \int \mathrm{d}V (\partial_{l} \Phi^{+} \Psi + \Psi^{+} \partial_{l} \Phi).$$

Therefore, the results are the same as in the previous case.

The calculation of the angular momentum tensor requires generalisation of the Noether's theorem for the Lagrangians which depend of the second order derivatives of the field variables[6]. We do not consider it here.

The structure of the fifth scalar is similar to the fourth. Consequently one can expect the same result. It is really so. Because of this we do not give explicit calculation for this case.

c). The sixth scalar from (2.1) taken as a Lagrangian density reads

$$\begin{aligned} \mathscr{L}_c &= K_{c\mp} [(\partial_{\nu} \partial_{\lambda} \bar{\Phi} \gamma^{\lambda} \gamma^{\nu}) (\gamma^{\mu} \partial_{\mu} \Phi) \mp (\partial_{\mu} \bar{\Phi} \gamma^{\mu}) (\gamma^{\nu} \partial_{\nu} \gamma^{\lambda} \partial_{\lambda} \Phi)] = \\ &= K_{c\mp} (\partial_{\nu} \bar{\Psi} \gamma^{\nu} \Psi \mp \Psi \gamma^{\nu} \partial_{\nu} \Psi). \end{aligned} \tag{3.16}$$

From the second row we see that the Lagrangian density with minus sign is the standard Lagrangian density with the field Ψ as variable. Consequently, the standard energy-momentum tensor will be obtained and corresponding constants of motion when one uses Ψ as variable. However, we want to use Φ as variable in the Lagrangian formalism.

The Lagrangian equation (3.12) for $\mathscr{L}$ given by (3.16) are

$$K_{c\mp}\,[\partial_\mu\,\gamma^\mu\,(\partial^\nu\,\partial_\nu\,\Phi)\pm\partial_\mu\,\partial_\nu\,\gamma^\mu\,\gamma^\nu\,(\partial_\lambda\,\gamma^\lambda\,\Phi)]=0.$$

The equation with minus sign in the square bracket is identically zero. It means that the Lagrangian (3.16) with minus sign is only of interest. For this Lagrangian we have

$$\gamma^\nu\,\partial_\mu\,[\partial^\nu\,\partial_\nu\,\Phi]=0. \tag{3.17}$$

The solutions

$$\partial^\nu\,\partial_\nu\,\Phi=0$$

correspond to our problem.

The energy-momentum tensor for $\mathscr{L}_c$, according to (3.13), is

$$T_\alpha^\beta=K_{c-}\,[-\bar{\Psi}\,\partial_\alpha\,\partial^\beta\,\Phi+\partial_\alpha\,\partial^\beta\,\bar{\Phi}\,\Psi+\partial^\beta\,\bar{\Psi}\,\partial_\alpha\,\Phi-\partial_\alpha\,\bar{\Phi}\,\partial^\beta\,\Psi]. \tag{3.18}$$

The constant of motion is then

$$P_\alpha=\text{const.}\int T_\alpha^\circ\,\mathrm{d}V=\text{const}\,K_{c-}\int(-\bar{\Psi}\,\partial_\alpha\,\dot{\Phi}+\partial_\alpha\,\dot{\bar{\Phi}}\,\Psi+\dot{\bar{\Psi}}\,\partial_\alpha\,\Phi-\partial_\alpha\,\bar{\Phi}\,\dot{\Psi})\,\mathrm{d}V. \tag{3.19}$$

Taking again $\Phi_3=\Phi_4=0$, it is

$$\begin{aligned}P_0=\text{const}\,K_{c-}\int[&-\Psi^{1\star}\,\ddot{\Phi}^1-\Psi^{2\star}\,\ddot{\Phi}^2+\ddot{\Phi}^{1\star}\,\Psi^1+\ddot{\Phi}^{2\star}\,\Psi^2+\\&\dot{\Psi}^{1\star}\,\dot{\Phi}^1+\dot{\Psi}^{2\star}\,\dot{\Phi}^2-\dot{\Phi}^{1\star}\,\dot{\Psi}^1-\dot{\Phi}^{2\star}\,\dot{\Psi}^2]\,\mathrm{d}V,\end{aligned} \tag{3.20}$$

$$\begin{aligned}P_i=\text{const}\,K_{c-}\int[&-\Psi^{1\star}\,\partial_i\,\Psi^1-\Psi^{2\star}\,\partial_i\,\Psi^2+\partial_i\,\Psi^{1\star}\,\Psi^1+\partial_i\,\Psi^{2\star}\,\Psi^2+\\&+\partial_0\,\Psi^{1\star}\,\partial_i\,\Phi^1+\partial_0\,\Psi^{2\star}\,\partial_i\,\Phi^2-\partial_i\,\Phi^{1\star}\,\partial_0\,\Psi^1-\partial_i\,\Phi^{2\star}\,\partial_0\,\Psi^2]\,\mathrm{d}V.\end{aligned}$$

By making use (1.12), (1.1) or explicitly for γ^μ given by (1.4)

$$\begin{aligned}\partial_0\,\Psi^1&=-\,\partial_z\,\Psi^3-(\partial_x-i\,\partial_y)\,\Psi^4,\\ \partial_0\,\Psi^2&=-\,(\partial_x+i\,\partial_y)\,\Psi^3+\partial_z\,\Psi^4,\\ \partial_0\,\Psi^3&=-\,\partial_z\,\Psi^1-(\partial_x-i\,\partial_y)\,\Psi^2,\\ \partial_0\,\Psi^4&=-\,(\partial_x+i\,\partial_y)\,\Psi^1+\partial_z\,\Psi^2,\end{aligned}$$

and performing some partial integrations these constants of motion can be written in the forms

$$\begin{aligned} P_0 &= \text{const}\, K_{c-} \int \mathrm{d}V\, (\dot{\Psi}^+ \Psi - \Psi^+ \dot{\Psi}), \\ P_l &= \text{const}\, K_{c-} \int \mathrm{d}\, V\, (\partial_l \Psi^+ \Psi - \Psi^+ \partial_l \Psi). \end{aligned} \tag{3.21}$$

We see that they are equal, up to a constant of proportionality, to standard expressions for energy and momentum of the Dirac field.

The calculation of the angular momentum tensor we do not consider here for the same reason as in the previous case.

We stop at this place further analysis of the scalars (2.1). In this paper we wanted to show how known constants of motion for Dirac's field can be obtained by making use of the potential method.

4. Conclusions

We have shown that the potential of the Dirac's field can be systematically used in the theory. The Lagrangians of different order with respect to derivatives of the potential have to be considered. In comparison to standard procedure, the potential method seems a little bit more difficult. However, we think that the principal point is of importance. Practical and some others values can appear when sources and interaction with other fields are present.

The performed analysis indicates existence of some new constants of motion with possible physical usefulness. At the moment it seems to us that is better to investigate this problem first directly from the Dirac's equation, the potential and the spinor's invariants. We do it at another place.

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POTENCIJALI U TEORIJI DIRACOVOG POLJA BEZ MASE MIROVANJA

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Sadržaj

U teoriju Diracovog polja uvedeni su potencijali u radu¹⁾. Nastavak tog istraživanja s obzirom na gradijentnu invarijantnost i konstante kretanja izložen je u ovom radu.

U prvom dijelu je data definicija potencijala Diracovog polja i sadržaj rada. U drugom dijelu je navedeno nekoliko skalara koji su izgrađeni iz potencijala i izbor ovih skalara za Lagrangeove gustoće. Treći dio sadrži utvrđivanje ispravnosti izabranih lagranžijana za Diracovo polje i proračun konstanti kretanja pomoću tih lagranžijana. Zaključci su izloženi u posljednjem dijelu.