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VERTEX CORRECTIONS IN VECTOR-MESON PHOTOPRODUCTION

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Abstract: The size of the one-pion-exchange (OPE) contribution to $\gamma p \to V^o p$ processes is analysed. Effects of virtuality of the π meson are taken into account in the form of vertex corrections. A suitable parametrization of the $V\pi\gamma$ form factor $F_V(t)$ is obtained under the assumption of equality between radiative decay widths of V when the pion is on and slightly off the mass shell. For the πNN form factor K(t), a simple ansatz is used according to which K(t) is given by the sum (not necessarily infinite) of poles with a linearly rising mass trajectory. Detailed comparison between several parametrizations of K(t) found in the literature leads to a three-parameter expression for K(t). In special cases, K(t) can be reduced to the form factors considered by Hamilton, Pilkuhn and Wolf. Some other behaviours of K(t) (dipole or Veneziano-like) are also considered. Effects of the different parametrizations of K(t) have on the OPE contribution to the differential cross section for the $\gamma p \to \omega p$ process at large t are illustrated.

1. Introduction

High-energy photoproduction of vector mesons $\gamma p \to V^o p$ ($V^o \equiv \rho^o, \omega, \phi$ ...) is of interest for several reasons[1]. These processes belong to the class of diffractive processes commonly thought of to proceed by pomeron ($\mathbb{P}$) exchange in the t-channel. The main unnatural-parity competitor to $\mathbb{P}$ exchange is expected to be π^o exchange.

The energy dependence of the cross section and the linear polarization of the beam allow to establish that the term in the cross section with energy dependence $\sim E_\gamma^{-2}$ should be associated with unnatural-parity exchange in the t-channel. The E_{γ} and t dependence and the spin density matrix of unnatural-parity exchange contributions are found to be consistent with the one-pion-exchange (OPE) model.

Numerous OPE calculations2) have been carried out using different techniques for estimating virtual effects of elementary exchange in the t-channel. These effects are usually taken into account in the form of vertex corrections. The assumed factorization into form factors is already a limitation, which, however, is quite commonly made. The frequently used modification of elementary exchange is due to the Benecke-Dürr (BD)3) parametrization of form factors; it is based on the relativistic generalization of the nonrelativistic penetration factor4).

In this paper we propose an alternative approach to $V\pi\gamma$ and πNN vertex functions. First, we consider $V\pi\gamma$ and πNN vertices separately. The reason for this is that $V\pi\gamma$ may become a real process, e.g. in $V\to\pi\gamma$ radiative decay, while πNN exists only as a virtual process. On the other hand, the πNN vertex, which is related to the basic πNN interaction coupling constant, appears also in many other theoretical as well as experimental analyses, e.g. in the Goldberger-Trieman relation5) or in the analysis of the π-meson form factor in the space-like region6). Its parametrization is, therefore, of fundamental importance. Comparing different parametrizations commonly used in the literature for the πNN form factor, we have been able to obtain a general three-parameter expression for it. In special cases, the πNN form factor can be reduced to the ones proposed, for example, by Hamilton7), Pilkuhn8) and Wolf2).

In Section 2 we give some details of our treatment of OPE. In Section 3 we discuss the $V\pi\gamma$ vertex and its parametrization. In Section 4 we give a close examination of different parametrizations used for the πNN form factor and discuss the properties of the newly proposed one. An estimate of the OPE contribution tc the

$\gamma p \to \omega p$ process is considered in Section 5.

2. One-pion exchange

The one-pion exchange cross section with vertex corrections included has the following form

$$\frac{d\sigma^{OPE}}{dt}(E_\gamma,t) = \frac{d\sigma^{OPE}}{dt}(E_\gamma,t)_{Born}|F_V(t)|^2|K(t)|^2 , \qquad (2.1)$$

where

$$\frac{d\sigma^{OPE}}{dt}(E_\gamma,t)_{Born} = \frac{A_V}{E_\gamma^2}\, g(x_V)\, (\frac{\mu b}{GeV^2}) , \qquad (2.2)$$

$$g(x_V) = x_V(\frac{1+x_V}{c_V+x_V})^2 ; \quad x_V = \frac{-t}{m_V^2} ; \quad c_V \doteq (\frac{m_\pi}{m_V})^2 . \qquad (2.3)$$

Here, we have used Table 1 (Ref. 9) and $G^2_{\pi NN}/4\pi = 14.6$. The normalized form factors $F_V(t)$ and $K(t)$ are related to $V\pi\gamma$ and πNN vertices, respectively.

3. $V\pi\gamma$ vertex

In order to obtain a suitable parametrization of $F_V(t)$, we make a reasonable assumption of equality between radiative decay widths of vector mesons when the pion is on and slightly off the mass shell:

$$\Gamma(V \to \pi_{on}\gamma) = \Gamma(V \to \pi_{off}\gamma) . \qquad (3.1)$$

The statement (3.1) is rather arbitrary from the physical point of view. Arguments for its validity are difficult to find unless we argue on general grounds that decay rates should be stable against small variations of the values of the outgoing particle masses. For dispersion calculations of the transition form factor $F_{\pi\omega\gamma}(q_\gamma^2)$ we refer to Ref. 10. As a consequency of (3.1), the on-shell $g_{V\pi\gamma}$ coupling constant is converted into a form factor

$$g_{V\pi\gamma}(t) = g_{V\pi\gamma}F_V(t) , \qquad (3.2)$$

where now

TABLE 1

Parameters to be used in the OPE-Born cross section

V	ρ(770)	ω(783)	φ(1020)
A_V (μb)	1.6	37.6	0.2
$\Gamma(V\to\pi\gamma)$ (MeV)	0.036	0.88	0.006

TABLE 2

Some characteristics of the πNN form factor in different parametrizations

K(t)	K(0)	$\langle r^2_{\pi NN}\rangle^{\frac{1}{2}}$ (fm)	K(t) $(-t)\to\infty$	ρ(r) $r\to 0$
Wolf	0.91	1.06	$\sim(-t)^{-1}$	$\frac{1}{r}$
Hamilton	0.93	0.91	$\sim(-t)^{-1}$	$\frac{1}{r}$
Pilkuhn	0.95	0.72	$\sim(-t)^{-\frac{1}{2}}$	$\frac{1}{r^2}$
Dipole $m_a^2+m^2=m_{A_1}^2$	0.96	0.62	$\sim(-t)^{-2}$	$\sim$const.
Equation (4.11)			$(-t)^{-\mu}$	$(\frac{1}{r})^{\frac{3}{2}-\mu+\lvert\frac{3}{2}-\mu\rvert}$ $0<\mu<\frac{5}{2}$
Equation (5.5)	0.97	0.54	$(-t)^{-2}$	$\sim$const.

$$F_V(t) = \left(\frac{m_V^2-m^2}{m_V^2-t}\right)^{\frac{3}{2}} \quad , \tag{3.3}$$

Owing to the smallness of the pion mass we expect (3.3) to be correct at least for small $|t|$ values. If interpreting $F_V(t)$ for negative t values as a Fourier transform of a spherically symmetric density distribution, the root-mean-square (rms) radius $\langle r_V^2\rangle$ can be obtained from

$$\langle r_V^2\rangle = 6\, F_V'(m_\pi^2) \quad . \tag{3.4}$$

We find that (3.3) gives reasonable values for the radii

$$\begin{aligned} \langle r_\rho^2\rangle^{\frac{1}{2}} &= 0.78 \text{ fm} \ , \\ \langle r_\omega^2\rangle^{\frac{1}{2}} &= 0.77 \text{ fm} \ , \\ \langle r_\phi^2\rangle^{\frac{1}{2}} &= 0.58 \text{ fm} \ . \end{aligned} \tag{3.5}$$

4. πNN vertex

Surprisingly, little is known about the precise form of this vertex. From the general theory of form factors[11)], K(t) can be described by the sum of all possible πNN vertices. It is also expected that K(t) becomes very small as t becomes large and negative; this is because for negative t the form factor has the nature of a Fourier transform of some spatial density, the nucleon acting as a pion source. In the literature, there are several models[2,7,8,12)] for K(t) based on a rather different underlying dynamical picture. It is a common procedure to assume that the dominance of a single pole or of several poles in K(t) is valid at least for small t values. We retain this general recipe and consider K(t) as given by the sum[13)]

$$K(t) = \sum_n \frac{K_n}{n-\alpha(t)} \quad . \tag{4.1}$$

We take $\alpha(t)$ to be a linear mass trajectory

$$\alpha(t) = \alpha_o + \alpha' t \qquad (4.2)$$

subject to the condition

$$\alpha(m_\pi^2) = 0 \ .$$

This gives

$$\alpha(t) = \alpha'(t - m_\pi^2) \ . \qquad (4.3)$$

For a reggeized pion, the question is still not quite settled; it is expected that the range of α' is between $(0.5 \div 0.9)$ GeV^{-2} (Ref. 14). The residues K_n, which can, in general, be t-dependent, are such that the formal power series expansion

$$\sum_n K_n z^n = f(z) \qquad (4.4)$$

can be written with K_n as numerical coefficients.

The knowledge of the function $f(z)$ makes it possible to rewrite $K(t)$ for $t<0$ in the form

$$K(t) = \int_0^1 du\, u^{-\alpha(t)-1} f(u) \ . \qquad (4.5)$$

All dynamical expectations about $K(t)$ are now contained in the function $f(u)$.

Normalization $K(m_\pi^2) = 1$ gives

$$\int_0^1 \frac{du}{u} f(u) = 1 \ . \qquad (4.6)$$

At $t=0$,

$$K(0) = \int_0^1 du\, u^{\alpha' m_\pi^2 - 1} f(u) \ . \qquad (4.7)$$

The pionic radius of the nucleon is obtained from

$$\langle r^2_{\pi NN} \rangle = 6\alpha' \int_0^1 du \left(\ln \frac{1}{u}\right) u^{\alpha' m_\pi^2 - 1} f(u) \ . \qquad (4.8)$$

If interpreting the Fourier transform of K(t) as the spherically symmetric density distribution $\rho(r)$, we find

$$\rho(r) = (2\pi^2 r)^{-1} \int_0^\infty q\,dq\,\sin(qr)K(-q^2). \qquad (4.9)$$

Different models and parametrizations of K(t) (Refs. 2,7,8) suggest the following ansatz for the function f(u)

$$f(e^{-x}) = Ne^{-ax}\,I_0(bx)x^{\mu-1}\ ;\ 0<b<a,\ \mu\neq 0\ . \qquad (4.10)$$

Here, $I_0(bx)$ is the modified Bessel function of zero order, N is the normalization factor and a, b and μ are some parameters to be specified later.

This particular form of f(u) leads to

$$K(t) = \left[\frac{m_a^4-m_b^4}{(m_a^2+m_\pi^2-t)^2-m_b^4}\right]^{\frac{1}{2}\mu} \frac{P_{\mu-1}(x(t))}{P_{\mu-1}(x(m_\pi^2))}\ , \qquad (4.11)$$

where

$$x(t) = (m_a^2+m_\pi^2-t)\left[(m_a^2+m_\pi^2-t)^2-m_b^4\right]^{-\frac{1}{2}}\ , \qquad m_a^2 = a/\alpha',\quad m_b^2 = b/\alpha' \qquad (4.12)$$

and $P_{\mu-1}(x)$ is the Legendre function of the first kind.

Looking at (4.11), we immediately notice the following:

i) If $m_b^2=0$, $\mu=1$, then

$$K(t) \doteq \frac{m_a^2}{m_a^2 + m_\pi^2 - t} \qquad (4.13)$$

and the single-pole-like behaviour of K(t) is obtained. Setting $m_a^2 = 14\ m_\pi^2$, the Hamilton-type parametrization of K(t) follows⁷⁾.

ii) If $m_b^2=0$, $\mu=\frac{1}{2}$, then

$$K(t) = \left[\frac{m_a^2}{m_a^2+m_\pi^2-t}\right]^{\frac{1}{2}}$$

and we obtain the Pilkuhn-type[8)] πNN form factor with $m_a^2 = 11.25\ m_\pi^2$.

iii) If $\mu=1$, then

$$K(t) = \left[\frac{m_a^4-m_b^4}{(m_a^2+m_\pi^2-t)^2-m_b^4}\right]^{\frac{1}{2}} , \tag{4.15}$$

and the Wolf-type parametrization[2)] is obtained. According to Wolf, we choose the masses m_a^2 and m_b^2 to be

$$m_a^2 = 2m_p^2 - m_\pi^2 ,$$

$$m_b^2 = \frac{4m_p^2}{R_N^2} (m_p^2 R_N^2-1) ; \quad R_N = 0.57 \text{ fm} .$$

iv) If $m_b^2=0$, $\mu=2$, then

$$K(t) = \left(\frac{m_a^2}{m_a^2+m_\pi^2}\right)^2 \frac{1}{\left(1 - \frac{t}{m_a^2+m_\pi^2}\right)^2} , \tag{4.16}$$

and the characteristic dipole-like behaviour is obtained.

In Table 2 we list some general characteristics of the form factors mentioned in i)-iv). The numbers shown in the table agree well with the approximate relation between K(0) and the pionic radius of the nucleon $\langle r_{\pi NN}^2 \rangle$

$$\langle r_{\pi NN}^2 \rangle \underset{\sim}{\sim} 6\ \frac{1 - K(0)}{m_\pi^2} , \tag{4.17}$$

This relation is valid approximately because of the smallness of the pion mass.

For the sake of comparison, Fig. 1 shows the visual relation between different parametrizations of K(t).

The asymptotic behaviour of K(t) for large and negative t

$$K(t) \xrightarrow[(-t)\to\infty]{} (-t)^{-\mu} \tag{4.18}$$

is controlled by the parameter μ. Note that μ has to be positive in order to have a decreasing form factor in the space-like region. What can be said about the possible value of μ? The origin of the asymptotic behaviour of the form factors is one of the central theoretical questions, yet unsolved. Assuming that the constituent-counting rule[15)] is correct, it predicts $\mu=2$, for example. A significant experimental determination of μ and m_a^2 is not feasible at present.

5. Conclusions and remarks

In the preceding sections we analysed vertex corrections in $\gamma p\to V^0 p$ reactions due to elementary-pion exchange in the t-channel.

Since an exact theory of form factors does not exist yet, it is clear that different parametrizations of form factors will give different estimates of the size of the OPE contribution. As an illustration, Figs. 2a,b and c show the OPE contribution to the $\gamma p\to\omega p$ process at $E_\gamma = 2.8$, 4.7 and 9.3 GeV/c according to the different parametrizations used for the K(t) form factor. The experimental points are from Ref. 16. The forward t peak shown by the data is known to be diffractive in nature and leads to a dependence of $14.5\ e^{8t}$ (μb/GeV^2) [16,17)]. Beyond $|t|\sim 1$ GeV^2, the data break away from the characteristic diffraction peak. In this region ($|t|\geq 1$ GeV^2), diffraction gives a negligible contribution and some other non-diffractive contribution should be present. It may well be that the tail of the $F_V(t)$ and K(t) form factors is seen in the data when t is large and negative. If this is true, the data would suggest that μ is smaller than $\frac{1}{2}$. The corresponding density distribution $\rho(r)$ in (4.9) (see Table 2 will be more singular at the origin ($r\to 0$) than the centrifugal barrier behaviour with the $\sim r^{-2}$ feature. Note that $\mu = \frac{1}{2}$ leads to the Pilkuhn parametrization of the K(t) form factor. For small t values, a very crude choice of different form factors is possible.

As a final remark, let us consider the Veneziano-like[13)] parametrization of K(t). It is achieved by

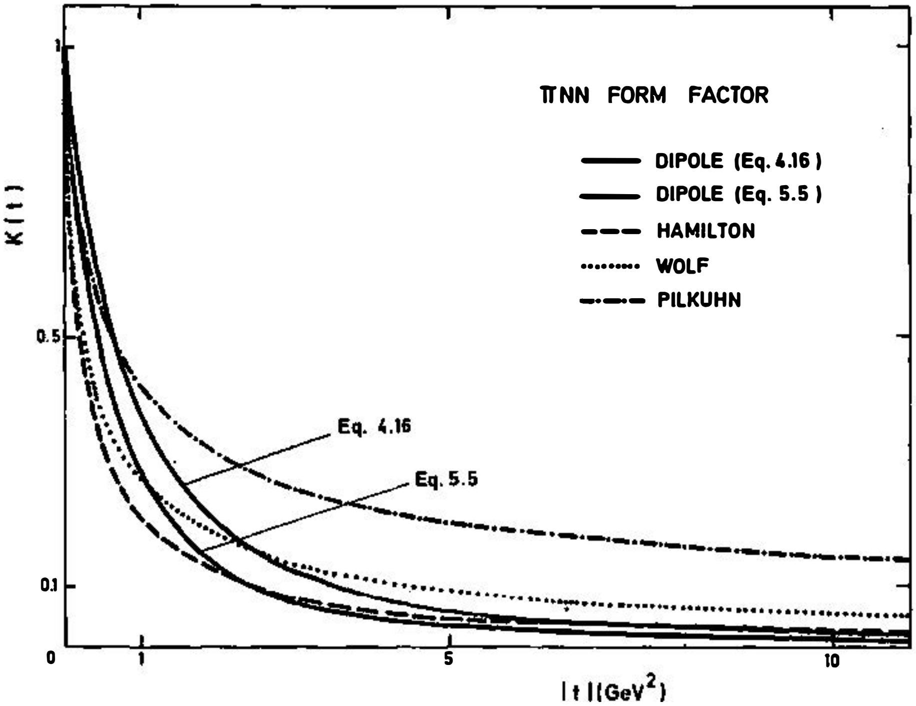
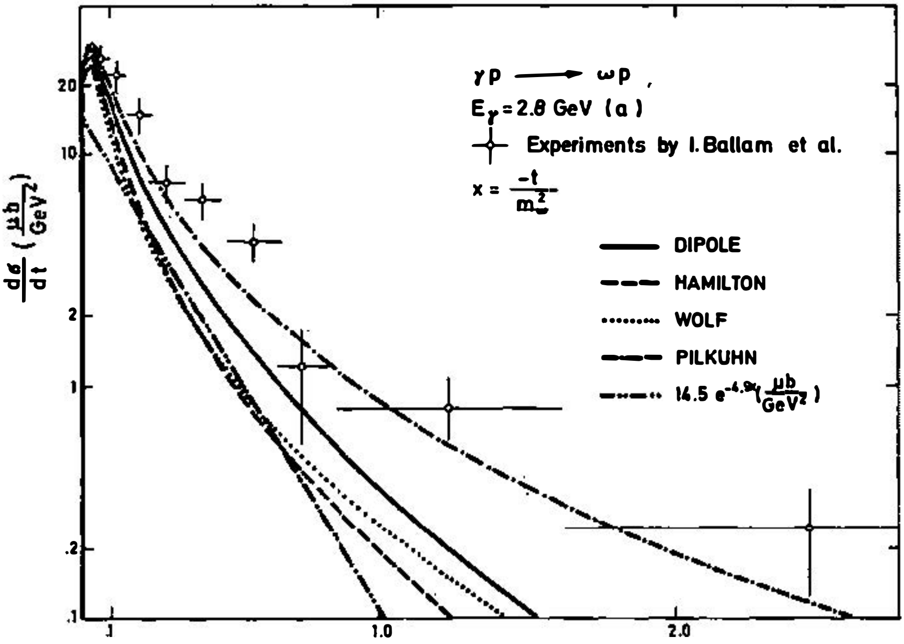


Fig. 1. The behaviour of the πNN form factor K(t) in different parametrizations.



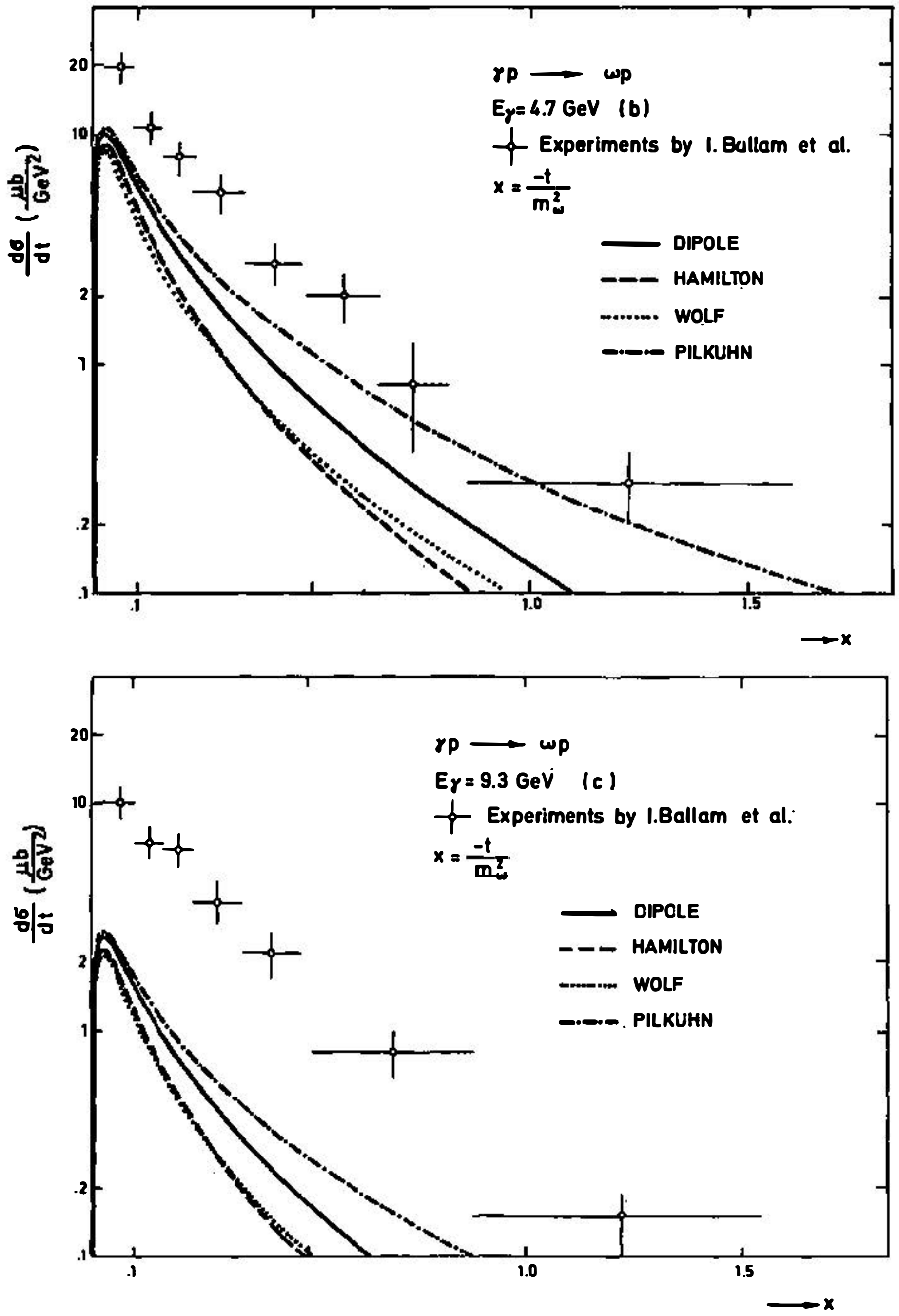


Fig. 2. The OPE contribution to the γp→ωp process at energies a) E_γ = 2.8 GeV, b) E_γ = 4.7 GeV and c) E_γ = 9.3 GeV, using different parametric forms of the πNN form factor.

choosing the function f(u) in (4.5) to have the form

$$f(u) = b\, u(1-u)^{b-1} \quad . \qquad (5.1)$$

This gives

$$K(t) = \frac{\Gamma(1-\alpha(t))\,\Gamma(1+b)}{\Gamma(1+b-\alpha(t))} \quad . \qquad (5.2)$$

The residues K_n are readily found from (4.4) to be

$$K_0=0,\ K_n= -b(-)^n\binom{b-1}{n-1};\ n\geq 1 \quad . \qquad (5.3)$$

The asymptotic behaviour of K(t) for large and negative t

$$K(t)\xrightarrow[(-t)\to\infty]{}\Gamma(1+b)\,(-\alpha' t)^{-b} \qquad (5.4)$$

requires b to be positive in order to have a decreasing form factor in the space-like region. The case b=2 seems to be of some interest, since it gives the behaviour of K(t) in the form

$$K(t) = \frac{(m_{\pi'}^2-m_\pi^2)\,(m_{\pi''}^2-m_\pi^2)}{(m_{\pi'}^2-t)\,(m_{\pi''}^2-t)} \quad , \qquad (5.5)$$

where

$$m_{\pi'}^2 = m_\pi^2 + 1/\,\alpha' \ ,$$
$$m_{\pi''}^2 = m_\pi^2 + 2/\,\alpha' \ ,$$

which is very close to the dipole-type form factor. Its general characteristics are given in Table 2 and Fig. 1 for $\alpha' = 0.84$ GeV^{-2}.

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VRŠNE POPRAVKE U FOTOPRODUKCIJI VEKTORSKIH MEZONA

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Sadržaj

Analizirana je veličina doprinosa jednopionske izmjene (OPE) u procesima $\gamma p \to V^o p$. Utjecaji virtualnosti π-mezona uzeti su u obzir u obliku vršnih popravki. Dobivena je prikladna

parametrizacija vrha $V\pi\gamma$ uz pretpostavku da postoji jednakost izmedju širina radijativnog raspada vektorskog mezona kada se pion nalazi na ljusci mase i malo izvan nje. Za pionski form faktor nukleona K(t) korištena je postavka prema kojoj je K(t) dan sumom (ne nužno beskonačnom) polova. Detaljna usporedba nekoliko parametrizacija za K(t), nadjenih u literaturi, dovodi do troparametarskog izraza za K(t). U posebnim se slučajevima K(t) reducira na form faktore koje su predložili Hamilton, Pilkuhn i Wolf. Takodjer su razmatrani i neki drugi oblici form faktora K(t)(kao npr. dipolni i Venezianov). Prikazani su utjecaji različitih parametrizacija na doprinos jednopionske izmjene diferencijalnom udarnom presjeku u procesu $\gamma p\to\omega p$ kod većih t vrijednosti.