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LETTERS TO THE EDITOR

THE KAON RADIUS*

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Abstract:
It is shown within a low-energy approximation for absorptive parts of the amplitudes $\pi\pi \to \pi\pi$, $\pi\pi \to K\bar{K}$, $\gamma \to \pi\pi$ and $\gamma \to K\bar{K}$ that the kaon and pion electromagnetic radii are equal.

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A recent measurement of $e^+e^- \to K\bar{K}$ annihilation cross section[1] has shown that very massive photons produce both charged pion and kaon pairs at approximately equal rates. This suggests that at least in the region $2\,\mathrm{GeV}^2 < q^2 < 9\,\mathrm{GeV}^2$ moduli of pion and kaon form factors are approximately equal. Thus far there has been no experimental information on F_K at lower momentum transfers due to the well known difficulties. The kaon radius, given by the slope of $F_K(q^2)$ at $q^2 = 0$, is, therefore, completely unknown from the experimental point of view.

Some theoretical models based on ρ-universality or SU-symmetry suggest that the kaon, seen electromagnetically, should be of the size of a pion or somewhat smaller[2,3]. Here we show that the equality

$$F_K^V(q^2) = F_\pi(q^2) \tag{1}$$

follows as a solution to the low-energy approximation for the unitarity equations of the $\pi\pi \to \pi\pi$ and $\pi\pi \to K\bar{K}$ amplitudes and imaginary parts of pion and kaon form factors.

Taking only the 2π cut and neglecting the left-hand cut completely, we have

$$\mathrm{Im}\, A_{\pi\pi\to\pi\pi} = \rho_{\pi\pi}\, \Theta(q^2 - 4m_\pi^2)\, A^*_{\pi\pi\to\pi\pi} A_{\pi\pi\to\pi\pi}\,, \tag{2}$$

$$\mathrm{Im}\, F_\pi = \rho_{\pi\pi}\, \Theta(q^2 - 4m_\pi^2)\, F_\pi^*\, A_{\pi\pi\to\pi\pi}\,, \tag{3}$$

$$\rho_{\pi\pi} = \left(\frac{q^2 - 4m_\pi^2}{4q^{4/3}}\right)^{3/2}.$$

The solution of these equations is

$$F_\pi(q^2) = \frac{A_{\pi\pi\to\pi\pi}(q^2)}{A_{\pi\pi\to\pi\pi}(0)}\, P(q^2)\,, \qquad P(0) = 1\,, \tag{4}$$

where the arbitrary real polynomial $P(q^2)$ may be chosen $P(q^2) = 1$, as extensively discussed in the literature[4]. Taking into account the existence of the ρ-rsesonance, a double subtraced dispersion relation for $A^{-1}_{\pi\pi\to\pi\pi}$ leads to the famous Gounaris-Sakurai formula [5] for F_π, which describes the data over the whole measured region $-2\text{GeV}^2 < q^2 < 9\text{GeV}^2$ in a satisfactory way.

Applying the same approximation to the $\pi\pi \to K\bar{K}$ amplitude and the isovector part of the kaon form factor, we have

$$\mathrm{Im}\, A_{\pi\pi\to K\bar{K}} = \varrho_{\pi\pi}\,\theta(q^2-4m_\pi^2)\, A^*_{\pi\pi\to\pi\pi} A_{\pi\pi\to K\bar{K}}\,, \qquad (5)$$

$$\mathrm{Im}\, F_K^{\,V} = \varrho_{\pi\pi}\,\theta(q^2-4m_\pi^2)\, F_\pi^{*}\, A_{\pi\pi\to K\bar{K}}\,. \qquad (6)$$

When combined with equations (2) and (3), these equations lead immediately to the equality (1). As the model holds only at low momentum transfers, the arbitrary polynomial is again of no practical importance.

The obtained results are actually expected, because in a phase dispersion representation all involved amplitudes and form factors have the same Omnès function, which contains the essence of dynamics, the remaining multiplicative polynomials being more or less fixed at low energies by normalization.

There is, unfortunately, no similar model for the isoscalar part of the kaon form factor, $F_K^{\,S}$. One may, however, assume on physical grounds (at least at low momentum transfers) that

$$F_{K^0} = \frac{1}{2}\,(F_K^{\,S} - F_K^{\,V}) = 0, \qquad q^2 \text{ small}, \qquad (7)$$

so that, taking the account of (1), we find

$$F_{K^+} = \frac{1}{2}\,(F_K^{\,V} + F_K^{\,S}) = F_{\pi^+}\,. \qquad (8)$$

In this way the approximate equality of the pion and the kaon radii is also supported by the S-matrix approach.

R e f e r e n c e s

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