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ELECTRON DIELECTRIC FUNCTION IN THE SECOND-ORDER RPA

Ž. Crljen

Institute "Rudjer Bošković", Zagreb*

and

V. Šips

Faculty of Science and Institute "Rudjer Bošković", Zagreb

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The random phase approximation (RPA) is a very useful technique for finding an approximate solution of the many-body problem. It was first developed by Bohm and Pines[1-4] in a sequence of papers devoted to the collective behaviour of an electron plasma, and later applied to various many-body systems[5]. The essential feature of RPA is that it leads to linearized equations which determine the ground-state energy and the elementary-excitation spectrum. Suhl and Werthamer[6-7] showed how the most general linear solution of the problem under consideration could be achieved. This approach is known as the generalized RPA. An equivalent procedure was proposed by Watabe[8] using the Green function method.

Confining themselves to second-order processes, Suhl and Werthamer derived an explicit expression for the electron dielectric function. Even at that stage of approximation, the mathematical formalism was so complicated that one class of terms, which are

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*Present address: Chalmers Tekniska Hogskola, Göteborg, Sweden.

equally important as those included into the calculation, were left out of consideration. In the present paper we attempt to find a correct form of the dielectric function valid in the second-order RPA.

Let us consider an electron gas in a uniform distributed sea of positive charge. Further, let electrons interact with a weak external test charge oscillating with a frequency ω. Then the Hamiltonian may be expressed with the help of electron creation and annihilation operators as

$$H = \sum_{sk} E_k N_{sk} + \frac{1}{2} \sum_{q} V_q \rho_q^{\dagger} \rho_q + \sum_{q} V_q \rho_q^{\dagger} r_q \, , \qquad (1)$$

where $E_k = \hbar^2 k^2/2m$, $N_{sk} = c_{sk}^{\dagger} c_{sk}$, $V_q = 4\pi e^2/q^2$, $\rho_q = \sum_{sk} c_{sk}^{\dagger} c_{sk+q}$ and r_q is the density fluctuation of the test charge.

The frequency- and wave-vector-dependent dielectric function of the electron gas is defined by[9]

$$\frac{1}{\varepsilon(q,\omega)} = 1 + \frac{\rho_q}{r_q} \, . \qquad (2)$$

Now introduce the function

$$\xi_q = \sum_{sk} F_{qk} c_{sk}^{\dagger} c_{sk+q} \, . \qquad (3)$$

From the equation of motion for ξ_q it follows

$$\sum_{sk} F_{qk} (\hbar\omega + E_k - E_{k+q}) c_{sk}^{\dagger} c_{sk+q} + V_q (\rho_q + r_q) \sum_{sk} F_{qk} (N_{sk+q} - N_{sk}) +$$
$$+ \sum_{skk'} V_{k'-k} (N_{sk'+q} - N_{sk'}) (F_{qk} - F_{qk'}) c_{sk}^{\dagger} c_{sk+q} +$$
$$+ \sum_{ss'kk'q'} V_{q'} (F_{qk-q'} - F_{qk}) c_{s'k'}^{\dagger} c_{s'k'+q'} c_{sk}^{\dagger} c_{sk+q-q'} = 0. \qquad (4)$$

In the usual RPA only the first and the second terms in (4) are retained. Then taking F_{qk} to be

$$F^{(o)}_{qk} = \frac{1}{\hbar\omega + E_k - E_{k+q}} , \qquad (5)$$

one obtains the expression for the RPA dielectric function

$$\varepsilon_{RPA}(q,\omega) = 1 + V_q \sum_{sk} \frac{N_{sk+q} - N_{sk}}{\hbar\omega + E_k - E_{k+q}} . \qquad (6)$$

Instead of neglecting the last two terms in (4), in the higher-order RPA we consider the equation of motion for the two-pair operator $c^\dagger_{s'k'} c_{s'k'+q'} c^\dagger_{sk} c_{sk+q-q'}$. It is a matter of somewhat lengthy and tedious calculations to show that the linearized form of this equation is

$$c^\dagger_{s'k'} c_{s'k'+q'} c^\dagger_{sk} c_{sk+q-q'} (\hbar\omega + E_k + E_{k'} - E_{k+q-q'} - E_{k'+q'}) =$$
$$= V_{q'-q}\{c^\dagger_{s'k'} c_{s'k'+q}[N_{sk}(1-N_{s'k'+q'}-N_{sk+q-q'}) + N_{s'k'+q'}N_{sk+q-q'}] -$$
$$- c^\dagger_{s'k'+q'-q} c_{s'k'+q'}[N_{sk+q-q'}(1-N_{sk}-N_{s'k'}) + N_{sk}N_{s'k'}]\} +$$
$$+ V_{q'}\{c^\dagger_{sk} c_{sk+q}[N_{s'k'}(1-N_{s'k'+q'}-N_{sk+q-q'}) + N_{s'k'+q'}N_{sk+q-q'}] -$$
$$- c^\dagger_{sk-q'} c_{sk-q'+q}[N_{s'k'+q'}(1-N_{sk}-N_{s'k'}) + N_{sk}N_{s'k'}]\} . \qquad (7)$$

The approximation used in writing Eq. (7) plays the central role in the second-order RPA.

Inserting (7) into (4) and choosing F_{qk} to be given by

$$F_{qk}(\hbar\omega + E_k - E_{k+q}) + \sum_{k'} V_{k'-k}(N_{sk'+q} - N_{sk'})(F_{qk} - F_{qk'}) +$$
$$+ \sum_{s'k'q'} V_{q'} \{\frac{N_{s'k'}(1-N_{sk+q-q'}-N_{s'k'+q'}) + N_{sk+q-q'}N_{s'k'+q'}}{\hbar\omega + E_k + E_{k'} - E_{k+q-q'} - E_{k'+q'}} \cdot$$
$$\cdot [V_{q'-q}(F_{qk'+q'-q} - F_{qk'}) + V_{q'}(F_{qk-q'} - F_{qk})] -$$
$$- \frac{N_{s'k'+q'}(1-N_{sk+q'}-N_{s'k'}) + N_{sk+q'}N_{s'k'}}{\hbar\omega + E_{k+q'} + E_{k'} - E_{k+q} - E_{k'+q'-q}} \cdot$$

$$\cdot \left[V_{q'-q}(F_{qk'+q'-q} - F_{qk'}) + V_{q'}(F_{qk} - F_{qk+q'})\right]\Big\} = 1, \tag{8}$$

we arrive at

$$\rho_q + V_q(\rho_q + r_q) \sum_{sk} F_{qk}(N_{sk+q} - N_{sk}) = 0 . \tag{9}$$

Hence in the second-order RPA the dielectric function becomes

$$\varepsilon(q,\omega) = 1 + V_q \sum_{sk} F_{qk}^{(2)} (N_{sk+q} - N_{sk}) , \tag{10}$$

$F_{qk}^{(2)}$ being the solution of the integral equation (8) obtained by the second iteration.

It is important to note that our expression for the dielectric function reduces to that derived by Suhl and Werthamer provided that all second-order contributions that are in (8) multiplied by V_q^2, are left out.

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