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APPLICATION OF THE AMADO MODEL TO THE REACTION $^6Li(^3He, ^3He\,^3H)^3He$

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Abstract: The Faddeev three body formalism is applied to the reaction $^6Li(^3He, ^3He\,^3H)^3He$. ^3He and ^3H are treated as structureless particles interacting via an S wave separable interaction. The predictions of this model are compared with the plane wave impulse approximation. It is suggested that the PWIA is not valid for the reaction $^6Li(^3He, ^3He\,^3H)^3He$ and that from the study of this reaction information on short range trion-trion interaction could be obtained.

1. Introduction

Multiparticle reactions provide a rich source of information on reaction mechanisms, nuclear structure and nuclear interactions. Experimentally, multiparticle processes have been studied[1] for systems ranging from A = 3 to A = 200 focussing on quasifree processes and sequential decays, but also demonstrating the importance of other processes, e. g. double spectator, double final state interaction and knock out of unbound correlated pairs of particles. While the three particle system is rigorously described by Faddeev equations and there are by now theoretical predictions using fairly sophisticated interactions[2, 3], the theoretical development of the 4, 5 etc. particle problem is still inadequate. The present status of the theoretical work can be summarized in the following way:

— considerable progress has been achieved[4] in extending[5] the Faddeev formulation to four, . . . and N-body problem, and even some specific calculations (e. g. ^4He binding energy, the reactions D(d,p)t and ^3He(p,p)^3He) have been carried out[4,6], but there is still no model which can be applied to multiparticle reactions;

— Hackenbroich and collaborators[7] have developed a microscopic model which uses a realistic nucleon-nucleon interaction, but it is at present restricted to low centre of mass energies and to sequential processes;

— various intuitive models:

a) sequential decay models, e. g. Watson-Migdal enhancement[8],

b) plane wave impulse approximation (PWIA) has successfully predicted the shape of the energy spectra at quasifree conditions particularly higher energies, but it has been unable to predict the absolute cross section,

c) several modifications as radial cutoff, L-cutoff etc. have been introduced[9] and critically evaluated[10],

d) an obvious improvement represents the distorted wave impulse approximation (DWIA) which has been applied to many multiparticle reactions[11].

The accuracy of the DWIA for the reaction $^4He(p,2p)t$ has been tested in a three body model and it has been found[12] that the DWIA is adequate at 100 MeV, but that it overestimates the cross section at lower energies.

We present here another model for multiparticle reactions leading in a final state to three particles two of which can be considered to compose one of the initial state particles. For a specific case of the reaction $^6Li(^3He,{}^3He\,^3H)^3He$, since trions have spin 1/2, and ^6Li has spin 1, it is possible to use the existing Ebenhöh code[13] with appropriate modifications of the input parameters.

2. ^3He-^3He and ^3H-^3He interactions

In this model trions are treated as structureless particles interacting through a separable S wave interaction of the Yamaguchi type. The system ^3He-^3H (h-t) has a bound state: ^6Li, with the binding energy of 15.793 MeV and the triplet h-t parameters are chosen so as to reproduce this binding energy. Other parameters are adjusted to provide a reasonable fit to the 90° differential trion-trion cross section (Table 1.). For separable potentials the momentum space matrix elements are given by

$$v(k,k') = \lambda\, g(k)\, g(k') \qquad (1)$$

and the Yamaguchi form factors are

$$g(p) = N/(p^2 + \beta^2), \qquad (2)$$

with

$$N = \sqrt{\beta\,\varkappa\,(\beta + \varkappa)^3/\pi}, \qquad (3)$$

Table 1.

Effective range parameters and Yamaguchi parameters for the triton-^{3}He and ^{3}He-^{3}He interaction

	a (fm)	r_0 (fm)	$\varkappa^2$ (keV)	$\varkappa$ (fm^{-1})	β (fm^{-1})
triplet t-h	1.291	0.514	15793.33	1.067	4.484
singlet t-h	−21.076	0.651	30.27	−0.0467	4.670
singlet h-h	−12.168	0.651	88.91	−0.0801	5.1576

where parameters β and $\varkappa$ are given in terms of effective range parameters[13]

$$(\varkappa + \beta)^2 (r_0 \beta - 1) - 2 \beta^2 = 0$$

$$\left(\varkappa - \frac{1}{2} r_0 \varkappa^2\right) = \frac{1}{a}.$$

Since already at $E_{lab} \approx 10$ MeV[14] the trion-trion differential cross sections are not isotropic, it is obvious that this force can hardly simulate the phenomenology of the interaction. Consequently, in this crude model there is no reason why one should insist on reproducing exactly σ (90°) and indeed the interaction we are using in this model underestimates the data and it does not reproduce the increase at higher energies (Fig. 1). A more realistic interaction would certainly require higher partial wave interactions.

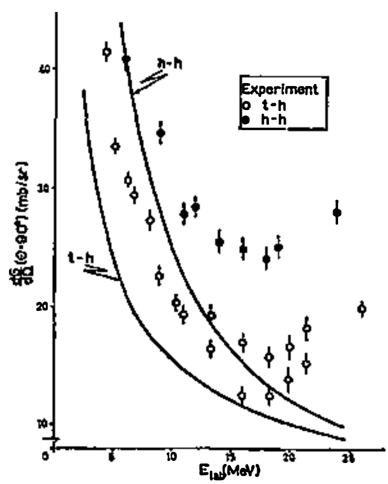

Fig. 1. The predictions of the differential cross sections $d\sigma^{h-h}/d\Omega$ cm (90°) and $d\sigma^{t-h}/d\Omega$ cm (90°) by the S wave separable potential with the Yamaguchi form factor compared with the experimental data: open circles (triton- He elastic scattering)[14] and dots (^{3}He- ^{3}He elastic scattering)[14].

3. *Results*

The predictions of this model are given in Figs. 2 and 3 for $E_{inc} = 35$ and 45 MeV for symmetric coplanar angles close to quasifree scattering kinematic conditions (solid curves). The reaction $^6Li(^3He, ^3He^3H)^3He$ has not been measured at any energy, so it is not possible to compare our results with the data. There are data for quasifree processes $^6Li(p,p^3He)^3H$ at 100 MeV[15]) and 156 MeV[16]) and for ^6Li $(\alpha, \alpha^3He)^3H$ at 50 MeV[17]). The peak cross sections are 20[15]) and 700 $\mu b/sr^2$ MeV[17]) for the reactions (p,p^3He) at 100 MeV and (α, α^3He), respectively.

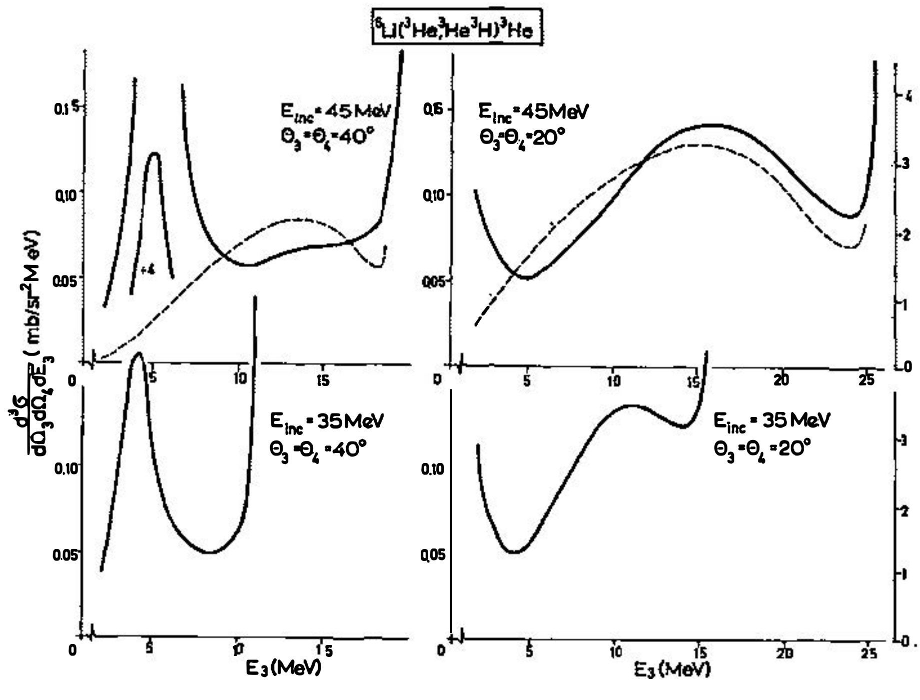

Fig. 2. The prediction of the model (solid curves — left hand ordinate) at $E_{inc} = 35$ MeV and 45 MeV at symmetic angles $\Theta_3 = \Theta_3 = 20°$ and 40° compared with the results of the PWIA (dashed curves — right hand orginate).

The results of our model (solid curves in Figs. 2 and 3, and dashed-dotted curve in Fig. 4a) are compared with the PWIA calculations (dashed curves in Fig. 2, Fig. 3a and Fig. 4a). The PWIA predicts about 10—30 times larger cross section. There is some difference in the shape of the energy spectra (Figs. 2 and 3) and in the shape of the angular variation of the cross section at q_5^{min} (smallest value of the spectator momentum) for $\Theta_3 = \Theta_4$ (Fig. 4: dashed curve — PWIA vs. dashed-dotted curve-our model).

4. *Discussion and conclusion*

It is known[1,18] that the predicted cross section in the quasifree scattering region sensitively depends on the on-energy shell interaction of the two particles that are involved in the quasifree process. The experimental t-h cross section at conditions defined by the quasifree process are compared with the input cross

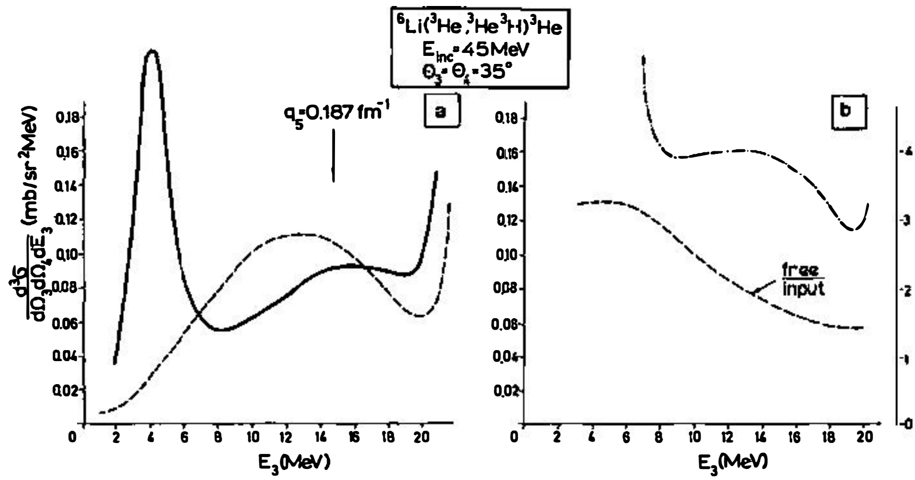

Fig. 3. a) The prediction of the model (solid curve — left hand ordinate) compared with the results of the PWIA (dashed curve — right hand ordinate in Fig. 3. b)

b) The ratio (dashed curve) of the free ^3He-^3H cross section with the cross section predicted by the S wave separable ^3He-^3H interaction given by parameters in Table 1. and used as an input in the present model (right hand ordinate).
Dashed-dotted curve represents the prediction of the model modified by multiplying the cross section with the ratio of free/inpunt ^3He-^3H cross section (left hand ordinate).

section (dashed curve in Fig. 3b gives the »free« input ratio, while solid curves in Fig. 4b show the input and »free« cross sections as defined above). Though the particle-particle t-matrix enters in the model in a much more complex way (off--energy shell and integration over all energies), one can obtain some insight into the effect of the more realistic trion-trion force by multiplying the prediction of the model with the ratio of the »free« and input t-h cross sections. The cross sections modified in this way are larger and the shape of the energy spectra and angular distributions are more similar to the PWIA predictions (dashed-dotted curve in Fig. 3b and solid curve in Fig. 4a).

The breakup amplitudes $T(q_f, q_i)$ at 35 and 45 MeV in various spin and angular momentum states are shown in Figs. 5 and 6. The total spin is S and L is the trion — ^6Li (or two trions subsystem) relative angular momentum. The total c. m. energy E is

$$E = p_f^2 + q_f^2,$$

where p_f is the relative momentum of a two-body substate, and q_f is the relative momentum between a two-body subsystem and a third particle. In these figures p_f is related to a subsystem described as ^{6}Li and ^{6}Li* (or d and d* in case of the ^{2}H (n, 2n) p reaction). Comparing the breakup amplitudes at 35 and 45 MeV one can see that the relative contribution of higher angular momentum states ($L = 4$) increases with the energy. Comparing the breakup amplitudes at 35 MeV ($E_{c.m.} = 7.54$ MeV) with those from the reaction ^{2}H (n,2n)p at 14.1 MeV ($E_{c.m.} = 7.17$

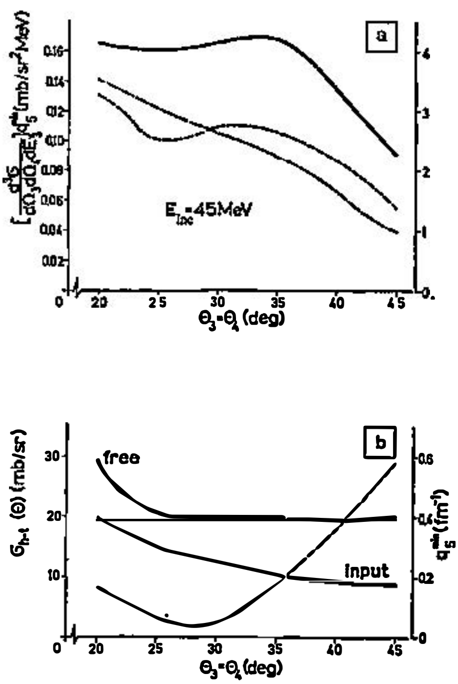

Fig. 4. a) The cross section $d^3\sigma/d\Omega_3\, d\Omega\, dE_3$ at (q_5) min for symmetric coplanar angles ($\Theta_3 = \Theta_4$) for PWIA (dashed curve-right hand ordinate), for the model (dasheddotted-left hand ordinate) and for the model corrected by multiplying with the ratio of the free vs. input ^{3}He-^{3}H cross section (solid curve-left hand ordinate).

b) The free and input ^{3}He-^{3}H cross sections (left hand ordinate) and (q_5) min (right hand ordinate).

MeV) one can see that higher angular momentum states contribute less in the reaction ^{6}Li(^{3}He,^{3}He^{3}H)^{3}He. Also, the p_f — dependence of various terms is different for the two reactions. It is the $L = 0$ part which is primarily responsible for the divergence of the multiple scattering series. It is also the $L = 0$, $S = 1/2$ part where one expects the cross section to be sensitive to the short range nuclear interaction. Therefore, one can conclude that:

— the reaction ^{6}Li(^{3}He,^{3}He^{3}H)^{3}He can be even less adequately described by the PWIA than the reaction ^{2}H(n,2n)p, and

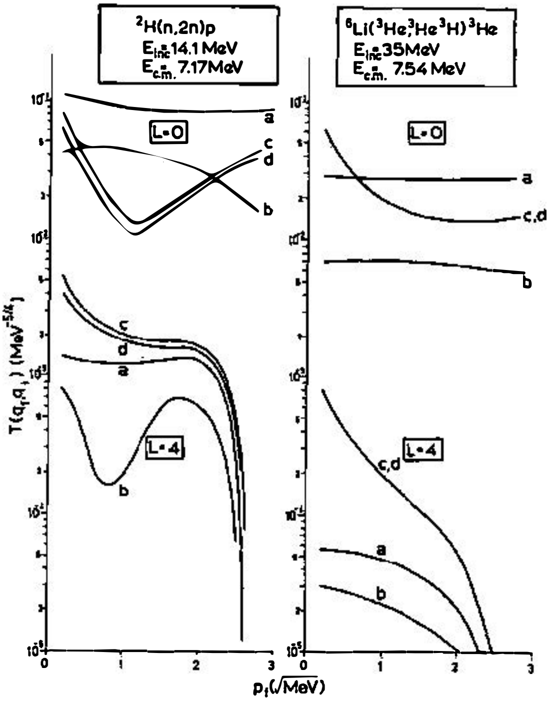

Fig. 5. ^{6}Li(^{3}He,^{3}HeH)^{3}He, at E_{inc} = 45 MeV Breakup amplitudes in various spin and angular momentum states: upper portion L (^{3}He-^{6}Li or ^{3}He-^{6}Li* orbital angular momentum in the final state) = 0, lower portion for L = 4. a: S = 3/2, ^{3}He-^{6}Li, b: S = 1/2, ^{3}He-^{6}Li, c: S = 1/2, ^{3}He-^{6}Li*, d: S = 1/2, ^{3}H-^{6}Be*.

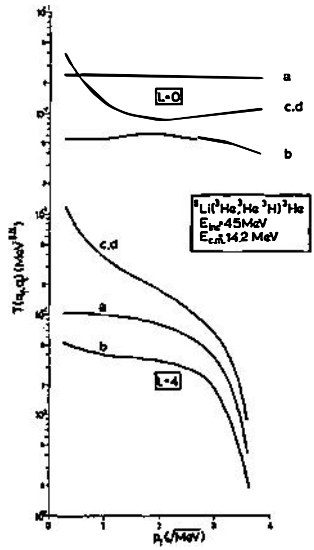

Fig. 6 a) ^{2}H(n,2n)p at E_{inc} = = 14.1 MeV
b) ^{6}Li(^{3}He,HeH)^{3}He at E_{inc} = 35 MeV. Symbols are the same as in Fig. 5 except that for Fig. 6 a) a: S = 3/2, n-d, b: S = 1/2, n-d, c: S = 1/2, n-d* and d: S = 1/2, p-n^2.

— the study of the reaction ^{6}Li(^{3}He,^{3}He^{3}H)^{3}He might provide useful information on trion-trion short range interaction.

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PRIMJENA AMADOVOG MODELA NA REAKCIJU ^{6}Li (^{3}He, ^{3}He ^{3}H) ^{3}He

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Sadržaj

Faddeevljev formalizam za tri tijela primijenjen je na reakciju ^{6}Li(^{3}He,^{3}He^{3}H)^{3}He. ^{3}He i ^{3}H su opisane kao čestice bez unutrašnje strukture, dok se za njihovo međudjelovanje koristi S-valna separabilna interakcija. Rezultati tog modela su uspoređeni s predviđanjima impulsne aproksimacije s ravnim valovima (PWIA).

Može se zaključiti, da se reakcija ^{6}Li(^{3}He,^{3}He^{3}H)^{3}He ne može opisati pomoću PWIA. Iz studija spomenute reakcije mogle bi se dobiti informacije o sili kratkog dosega između dva triona.