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DERIVATION OF SEA LEVEL MUON SPECTRA AT ZENITH ANGLES 0°, 65° AND 80° USING OWW SPECTRUM AND MACHINE INTERACTION PARAMETERS

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Sea level muon spectra at zenith angles 0°, 65° and 80° have bean calculated from the primary cosmic ray spectrum of Olejniczak, Wdowczyk and Wolfendale in the spectral range 10—250 GeV. The accelerator data on $pp \rightarrow \pi^{\pm} X$ and $pp \rightarrow K^{\pm} X$ inclusive reactions at high energies have been used in the calculation of fractional hadronic energy moments. The conventional pion and kaon atmospheric diffusion equations based on the modified formulation of Judge and Nash have been used in the derivation of muon spectra at different zenith angles. The estimated muon spectra are well in accord with the magnetic spectrograph data of different authors.

1. *Introduction*

The investigation on the spectral shapes of different particles is of interest for quantitative understanding of the cosmic ray phenomenology and also for the search of the validity of particle models at energies beyond the reach of machines.

Several authors have calculated the vertical sea level muon intensity from the predicted primary intensity[1–8]. The primary nucleon spectrum was used by them as a hadron source. However, the spectral index and its amplitudes are controversial till now. In a critical survey Olejniczak, Wdowczyk and Wolfendale[9)] (to be referred to as *OWW*) predicted the all particle primary cosmic ray spectrum by summing up the spectra of protons and different nuclei. Later Popova [10)] has given the power law fit of the all particle (total) primary cosmic ray spectrum. This spectrum can be used as a hadron source for the creation of secondary nuclear active particles in the atmosphere. Earlier Erlykin et al.[11)] have shown that the CERN ISR data for nucleon-nucleon collisions are applicable to the actual cosmic ray propagation in case of nucleon or meson-air nucleus collisions. This fact has also been successfully estimated by Wdowczyk and Wolfendale [12)] and they concluded that the available information suggests that cascading is very small for the comparatively large x values ($x \approx 0.2$) which is relevant to the muon problem. The Feynman[13)] scaling hypothesis principle appears to be borne by accelerator experiments in the range 10—1500 GeV at least for positive pion production. For negative pion production the approach to scaling is slower and full scaling appears to be reached only above 70 GeV energy. Thus the scaling hypothesis can predict the hadronic fractional energy moments which can be used to estimate the pion and kaon spectra. The primary cosmic ray particles being incident on the top of the atmosphere collide with the air nuclei and produce the first generation nucleons and mesons (pions and kaons) which are taken as the source of muons. These produced particles again generate secondary particles and the production processes continue. These informations are used to compute the fluxes of nucleons, pions and kaons produced in the atmosphere in hadronic cascades. The differential intensities of muons produced by pion and kaon decays can then be derived from the meson fluxes as a function of energy and zenith angles. We follow the conventional procedure after Judge and Nash[14)] to estimate the sea level muon intensity at different zenith angles after doing some modifications. The calculated spectra have been compared with the experimental data of different groups. In this calculation a flat isothermal atmosphere is assumed whose density decreases exponentially and this may be valid for the present muon spectra calculations up to 80 ° zenith angles.

In the present paper we have used the *OWW* [9)] primary spectrum as the source of hadrons incident in the atmosphere along with the Fermilab accelerator data of Johnson et al.[15)] for hadronic energy moments estimation. Using the conventional meson atmospheric diffusion equation after Judge and Nash[14)] the sea level muon spectra at zenith angles 0°, 65° and 80° have been estimated. The derived results have been compared with the magnetic spectrograph data of different authors[16-24)] up to 250 GeV muon energy.

2. *Kinematical aspects*

We have considered the differential primary cosmic ray spectrum up to TeV energy of the form

$$N(E)\,\mathrm{d}E = A\;E^{-\gamma}\,\mathrm{d}E \text{ per } (\mathrm{cm}^2\,\mathrm{sec\,sr\,GeV}) \qquad (1)$$

where A is the amplitude and E is the energy in GeV units with spectral index γ. This power law form of primary spectrum can be used as the source of hadrons in the present propagation analysis.

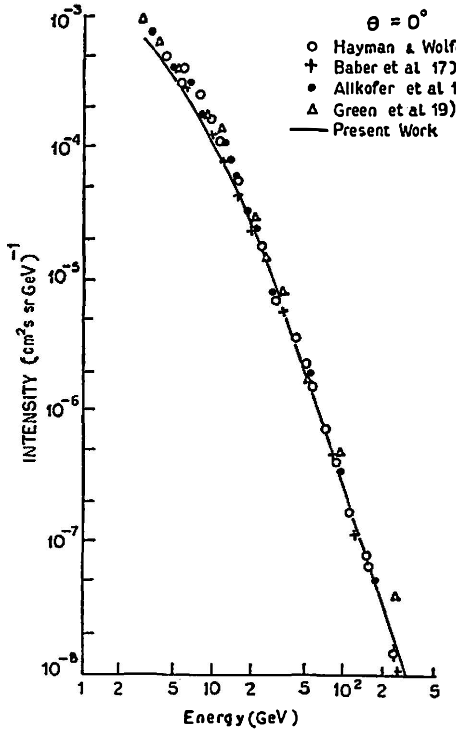

Fig. 1. Vertical sea level muon spectrum: Full curve is the derived spectrum from the *OWW*[9] primary spectrum using scaling hypothesis[13]. Magnetic spectrograph data: ○ - Hayman and Wolfendale [16], + - Baber et al.[17]. ● - Allkofer et al.[18] and Δ - Green et al.[19]. Data of Hayman and Wolfendale[16] were normalized to the Kiel muon intensity[18].

The fractional hadronic energy moments for $pp \to \pi^\pm X$ and $pp \to K^\pm X$ inclusive reactions can be estimated by the following expression

$$Z_{pi} = \frac{1}{\sigma_{in}} \int_0^1 x^{\gamma-2} \left(x \frac{d\sigma}{dx} \right) \bigg|_{pi} dx \tag{2}$$

where σ_{in} is the total inelastic cross section; the incident interaction particle is p and the produced secondary particle is i (pions or kaons). The slope of the incident differential energy spectrum of particle p, the spectral index γ is taken as constant and x is the usual Feynman variable $= 2\,p_L/s^{1/2}$ where p_L is the longitudinal centre of mass momentum of the secondary particle p, $s^{1/2}$ is the total centre of mass energy of the colliding nucleons and $x\ d\,\sigma/dx$ is taken from the invariant cross section $E(d^3\sigma/d^3p)$ in the following way:

$$x\frac{d\sigma}{dx} = \pi \int_0^\infty E(d^3\sigma/d^3p)\, dp_T^2. \tag{3}$$

The charged muon production spectrum $P(E_l)$ at energy E_l at the top of the atmosphere follows the form

$$P(E_l)\,\mathrm{d}E_l = N(E_l)\,\mathrm{d}E_l\,(Z_{pl}^{+} + Z_{pl}^{-}) \approx A_l\,E_l^{-\gamma_l}\,. \tag{4}$$

Using the conventional pion atmospheric diffusion equation after Judge and Nash[14] of the form

$$\frac{\mathrm{d}P_l}{\mathrm{d}y} = \frac{A_l\,E_l^{-\gamma_l}\exp\left(-y/\Lambda_p\cos\Theta\right)}{\Lambda_p\cos\Theta} - \frac{P_l}{\Lambda_l\cos\Theta} - \frac{P_l\,m_l\,c^2}{\varrho(y)\,\tau_l\,c\,E} \tag{5}$$

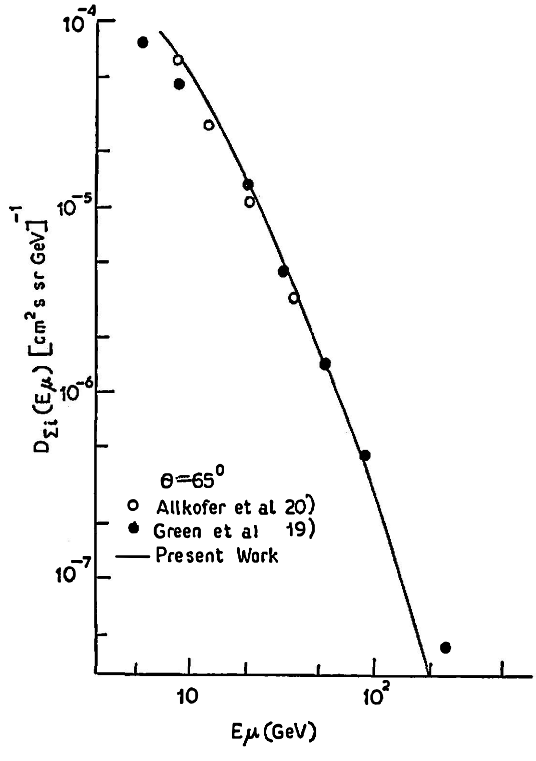

Fig. 2. Sea level muon spectrum at 65° zenith angle: Full curve is the derived spectrum from *OWW*[9] spectrum using scaling hypothesis[13]. Magnetic spectrograph data: ● - Green et al.[19] and ○ - Allkofer and Clausen[20] (at $\Theta \cong 60°$).

where $P_l\,\mathrm{d}E_l\,\mathrm{d}\Omega$ is the number of mesons per (cm^2 sec) of energy E_l in $\mathrm{d}E_l$ travelling within solid angle $\mathrm{d}\Omega$, m_l and τ_l are the mass and mean life time of a meson at rest, Λ_l is the absorption free path of mesons, viz. pions and kaons, $\varrho(y)$ is the atmospheric density at a vertical depth $y\,\mathrm{g\cdot cm^{-2}}$, γ_l is the spectral index of the meson production spectrum. The first term of the relation (5) accounts the production of meson by primary interaction and the second is for the removal of mesons by absorption and the third for the decay of mesons. For an isothermal atmosphere

$$\varrho(y) = y\,\varrho(0)/y_0 \tag{6}$$

the solution of the diffusion equation after Judge and Nash[14] follows

$$P_l = \frac{A_l E_l^{-\gamma_l}}{\Lambda_p \cos\Theta} \cdot y \exp(-y/\Lambda_l \cos\Theta) \sum_{n=0}^{\infty} \frac{(y/\Lambda \cos\Theta)^n}{n!\,(1+n+H_l/E_l\cos\Theta)} \tag{7}$$

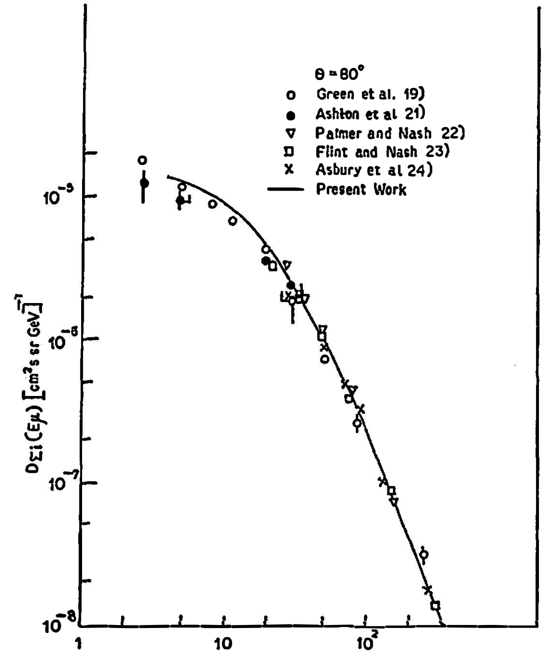

Fig. 3. Sea level muon spectrum at large zenith angle 80°: Full curve is the derived spectrum from the OWW[9] primary spectrum using scaling hypothesis[13]. Magnetic spectrograph data: ○ - Green et al.[19]; ● - Ashton et al.[21]; ▽ - Palmer and Nash[22], □ - Flint and Nash[23] and × - Asbury et al.[24].

where Λ_p and Λ_l are the absorption mean free path of protons and mesons in air and $1/\Lambda = 1/\Lambda_l - 1/\Lambda_p$; the critical energy of the meson decay follows

$$H_l = m_l c^2 y_0 / c\,\tau_l\,\varrho(0) \tag{8}$$

where $m_l c^2$ is the rest energy of meson whose life time is τ_l at rest. From the Feynman scaling[13] we find $\gamma_\pi = \gamma_K = \gamma$ i. e. the spectral indices secondary mesons and primary particles are identical.

The mean differential muon spectrum at sea level at a zenith angle Θ can be estimated by the relation

$$D_l(E_\mu, \Theta) = \int_0^{y_0 \sec\Theta} P\, W_\mu \frac{\mathrm{d}t}{\mathrm{d}y} \frac{\mathrm{d}y}{\tau}$$

$$\approx \frac{A_l W_\mu}{\Lambda_p \cos\Theta} E_l^{-(\gamma+1)} \Lambda_l H_l \sum_{n=0}^{\infty} \left(\frac{\Lambda_p - \Lambda_l}{\Lambda_p}\right)^n \Big/ \left(n + 1 + \frac{H_l}{E_l \cos\Theta}\right) \tag{9}$$

where E_μ is the sea level muon energy, τ is the dialated life time of muon; W_μ is the survival probability of muons produced at a depth y reaching ground level y_0 as

$$W_\mu = [y E_\mu / (y_0 r_\pi E_\pi)]^{B'} \tag{10}$$

where

$$B' = H_\mu / (r_\pi E_\pi \cos\Theta + q\, y)\;; \tag{10a}$$

m_μ and τ_μ are the muon rest mass and life time of a muon, $r_\pi E_\pi$ is the muon energy at production depth y and q is the mean energy loss of the muon in the atmosphere which usually obeys the relation

$$q = a_{ion.} + b\, E_\mu \tag{11}$$

where $a_{ion.}$ is the energy loss of muon by collision in the atmosphere and b is the energy loss of muon by radiation, electron pair production and nuclear interaction. The critical energy for muon decay follows $H_\mu = y_0 m_\mu c^2 / [\varrho(0)\, \tau_\mu c]$ and the meson energy at production E_l follows

$$E_l = [E_\mu + q(y_0 - y) \sec\Theta] / r_l. \tag{12}$$

In an earlier investigation Thompson and Whalley$^{4)}$ have assumed that all muons are generated at an atmospheric mean depth $y = 100\ \mathrm{g\cdot cm^{-2}}$ and this simplifies the solution of the equation (10) considerably and leads to an error of $\lesssim 1\ \%$ in the muon flux in the spectral range $10 - 10^4$ GeV compared with the exact evaluation of muon survival probability for sea level depth $y_0 = 1033\ \mathrm{g\cdot cm^{-2}}$. The ratio of energies of parent and daughter nucleon have been estimated after Rathgeber$^{25)}$ which follows

$$r_l = (m_l^2 + m_\mu^2) / (2\, m_l^2) \tag{13}$$

and from the kinematics of the muon decay processes, the pion and kaon energies E_π and E_k related to muon energies at production $E_{\mu p}$ by the following expressions:

$$E_\pi = E_{\mu p} / r_\pi \tag{14}$$

and

$$E_k = E_{\mu p} / r_k. \tag{15}$$

The ground level muon intensity $D_\mu(E_\mu)$ can be estimated by taking into account the branching ratio $b_{k\mu}$ for $K \to \mu_2$ decay by the following expressions:

$$D_l(E_\mu) = D_{\pi\to\mu}(E_\mu) + b_{k\mu} D_{K\to\mu}(E_\mu). \tag{16}$$

3. Results and discussions

The primary cosmic ray spectrum has been fairly surveyed by *OWW*[9)] and gave a composite spectrum of primary cosmic ray particles with atomic numbers 1, 4, 10, 26 and 56. Later Popova[10)] has given a power law fit to the sum of all particle intensity (composite spectrum) which follows the form:

$$N(E)\,\mathrm{d}E = A\,E^{-\gamma}\,\mathrm{d}E\,(\mathrm{cm}^2\,\mathrm{sec\,sr\,GeV})^{-1} \tag{17}$$

where the spectral amplitude is A = 1.9 and the spectral index is $\gamma = 2.6$, respectively.

The fractional hadronic energy moments have been estimated from the Fermilab data after Johnson et al.[15)] on the Lorentz invariant cross sections for $pp \to \pi^{\pm} X$ and $pp \to K^{\pm} X$ inclusive reactions. They are shown in Table 1.

TABLE 1

γ	$Z_{p\pi^+}$	$Z_{p\pi^-}$	Z_{pK^+}	Z_{pK^-}
2.6	0.053	0.027	0.010	0.0028

Table shows the spectral index of the differential *OWW*[9)] primary spectrum along with the estimated hadronic fractional energy moments for pion and kaon production inclusive reactions after Johnson et al.[15)].

The calculated pion and kaon production spectra have been estimated and follow the relation

$$P(E_\pi)\,\mathrm{d}E_\pi = (Z_{p\pi^+} + Z_{p\pi^-})\,N(E_\pi)\,\mathrm{d}E_\pi \tag{18}$$

and

$$P(E_k)\,\mathrm{d}E_k = (Z_{pK^+} + Z_{pK^-})\,N(E_k)\,\mathrm{d}E_k. \tag{19}$$

The kinematical expressions given by Rathgeber[25)] yields the energy degradation factors viz., $r_\pi = 0.78$, $r_k = 0.52$. The mean energy loss of muons due to ionization, bremmstrahlung and nuclear interactions has been estimated by the following relation

$$E_{\mu p} = E_\mu + (y_0 - y)\,(2.5 \cdot 10^{-3} + 2.78 \cdot 10^{-6}\,E_\mu). \tag{20}$$

The muon energy at production $E_{\mu p}$ relates the pion and kaon energies E_π and E_k by the expressions (14) and (15).

The absorption lengths of protons Λ_p and pions Λ_π have been estimated by the relations

$$\Lambda_p = \lambda_n / (1 - Z_{pp} - Z_{pn}) \tag{21}$$

$$\Lambda_\pi = \lambda_n / (1 - Z_{\pi^+\pi^+} - Z_{\pi^+\pi^-}) \tag{22}$$

where λ_n and λ_π are nucleon and pion interaction mean free paths in air, taken to be 80 $\text{g}\cdot\text{cm}^{-2}$ and 120 $\text{g}\cdot\text{cm}^{-2}$, respectively. The respective hadronic energy moments and for meson production by mesons in different inclusive reactions have been estimated from the data of Breitenlohner[26)] and Abdrakhamanov et al.[27)] and the calculated values are $Z_{pp} = 0.163$, $Z_{pn} = 0.11$ and the energy moments for $\pi\pi \to \pi^\pm X$ inclusive reactions after Erlykin et al.[11)] yield $Z_{\pi^+\pi^+} = 0.192$, $Z_{\pi^+\pi^-} = 0.0585$. The calculated absorption lengths of protons and pions are $\Lambda_p = 110$ $\text{g}\cdot\text{cm}^{-2}$ and $\Lambda_\pi = 160$ $\text{g}\cdot\text{cm}^{-2}$. Due to the insufficient information available for the evaluation of Z_{kk} we assumed that $Z_{k^+k^+} \approx Z_{k^-k^-} \approx Z$ and $Z_{k^+k^-} \approx Z_{k^-k^+} \approx Z_{\pi^+\pi^-}$. We have calculated Λ_k by using above identities and the following relation

$$\Lambda_k = \lambda_k / (1 - Z_{k^+k^+} - Z_{k^+k^-}) \tag{23}$$

with $\lambda_k = 150$ $\text{g}\cdot\text{cm}^{-2}$ and accepting the fractional energy moments for $\pi\pi$ interactions Λ_K comes out to be 200 $\text{g}\cdot\text{cm}^{-2}$. The critical energies for pion, kaon and muon decays have been estimated by the relation (8) where the scale height of the isothermal atmosphere considered here is 6.7 Km and the result follows $H_\pi = 119.3$ GeV, $H_K = 891$ GeV and $H_\mu = 1.07$ GeV.

Using the relation (10), whose simplified form represents the muon survival, probability follows:

$$W_\mu = \left[\frac{y}{y_0} \frac{E_\mu}{E_\mu + q(y_0 - y)\sec\Theta}\right]^{\frac{H_\mu \sec\Theta}{E_\mu + q y_0 \sec\Theta}}. \tag{24}$$

The survival probability of muons produced at mean depth $y = 100$ $\text{g}\cdot\text{cm}^{-2}$ reaching sea level vertical incidence atmospheric depth $y_0 = 1033$ $\text{g}\cdot\text{cm}^{-2}$ and also at different zenith angles viz. . 65° and 80° have been estimated using above expressions. We have used the branching ratio for $K\mu_2$ decay as 0.63 and the derived sea level muon spectra at different zenith angles viz. at $\Theta = 0^\circ$, 65° and 80° and have been displayed in Figs. 1, 2 and 3. We have used the Feynman scaling hypothesis to estimate the hadronic energy moments for different inclusive reactions like $pp \to \pi^\pm X$, $pp \to K^\pm X$, $\pi\pi \to \pi^\pm X$. We have assumed the validity of scaling hypothesis at ISR energies. The superposition model in the primary nucleon-air nucleus collision is neglected in the present analysis. The charge transfer distribution after Abdrakhmanov et al.[27)] and Baldin et al.[28)] for hadron-hadron interactions infers that irrespective of the nature of projectile particle, the quantum numbers were conserved in the target and projectile hemisphere in the centre of mass system.

One can assume without any loss of generality that the hadron can be replaced by the hadronic single matter and the projectile matter breaks up independent of the energy and the mass number of the target material i. e. the projectile matter behaves as stated above according to the hypothesis of limiting fragmentation[29, 30] for x > 0 which is a similar of Feynman scaling hypothesis for forward region. Following above informations we have used hadronic energy moments for *pp* collisions which is equivalent for hadron-hadron or hadron-nucleus interactions. The nucleons in a nucleus behave like a single hadronic matter and *p*-nucleus collision may be treated as the *p-p* collision with the same projectile hadron energy E_L in the laboratory system. We have neglected the regeneration of nucleons in the present study. But the good agreement of the predicted spectra with the experimental data suggest that primary spectrum with a single spectral index (in the TeV region) via nuclear interaction model viz. Feynman scaling can predict a secondary spectrum adequately. Our hadron source spectrum is assumed to be proton incident and the primary flux of *OWW*[9] is taken to be the proton equivalent flux[10]. At low energies below 10 GeV the discrepances between the experimental muon spectra and our calculated values occur. The disagreement at lower muon energies may be attributed due to the local atmospheric effects and the time-dependent variation of the primary intensity below 50 GeV energy on account of the variation in the sunspot numbers. We can infer from the present analysis that the scaling hypothesis is valid for primary interactions for incident median primary energies up to 2 TeV. The controversial concept on the break in the primary spectrum is not essential for the present investigation i. e. for intensity calculations at moderate muon energies.

4. *Conclusion*

The hadronic scaling in the fragmentation region holds at ISR energies. This concept can be used to derive the atmospheric muon spectra at different zenith angles viz. 0°, 65° and 80°. The derived sea level muon spectra agrees satisfactorily the measured data of different authors[16-24]. The hardening of muon spectrum increases with zenith angles which supports the measured findings of Judge and Nash[14]. Finally we can conclude that the primary *OWW*[9] spectrum can predict well the sea level muon spectra at different zenith angles in the spectral range 10—250 GeV.

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IZVOD SPEKTRA MUONA NA MORSKOJ RAZINI POD ZENITNIM KUTEM 0°, 65° i 80° POMOĆU OWW SPEKTRA I PARAMETARA INTERAKCIJE DOBIVENIH NA MAŠINAMA

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Spektar muona na morskoj razini pod zenitnim kutem od 0°, 65° i 80° izračunat je iz primarnog spektra kozmičkih zraka Olejniczaka, Wdowczyka i Wolfendalea u spektralnom rasponu od 10—250 GeV. Za račun frakcionalnih hadronskih energetskih momenata upotrebljeni su akceleratorski podaci za inkluzivne reakcije $pp \rightarrow \pi^{\pm} X$ i $pp \rightarrow K^{\pm} X$ na visokim energijama. Za izvod muonskog spektra pod različitim zenitnim kutem upotrebljena je konvencionalna pionska i kaonska atmosferska jednadžba difuzije osnovana na modificiranoj formulaciji Judgea i Nasha. Ocijenjen muonski spektar je u dobrom slaganju s podacima magnetnih spektrografa drugih autora.