YU ISSN 0015—3206
FZKAAA 14 (1) 63 (1982)

LETTER TO THE EDITOR

COLLISIONAL DAMPING OF TRANSVERSE OSCILLATIONS IN AN ELECTRON PLASMA

BHIMSEN K. SHIVAMOGGI

Department of Theoretical Physics, Research School of Physical Sciences, Australian National University, Canberra, Australia

Received 1 September

UDC 533.951

Original scientific paper

The collisional effects on the transverse oscillations in an electron plasma are studied using the Bhatnagar-Gross-Krook model for the collision term in the Boltzmann equation. For the sake of illustration, a Maxwellian distribution for the electron is considered. The results show a collisional damping of the transverse oscillations.

Kinetic theory of electromagnetic oscillations in a warm plasma was developed by Sitenko and Stepanov[1], Pradhan[2], and Bernstein[3] among others.Their method of analysis involved posing an initial-value problem for linearised disturbances in a uniform collisionless plasma, using the collisionless Boltzmann equation with a self-consistent electromagnetic field. The effect of particle-collisions on the transverse plasma oscillations was studied by Jain[4] using the Bhatnagar-Gross-Krook[5] model for the collision term in the Boltzmann equation. The method of analysis adopted by Jain[4] is cumbersome and involves using a Fourier transformation in velocity space, and the results are apparently not completely correct. The purpose of this paper is to give a simpler method of analysis that does not involve any Fourier transformation in velocity space.

One has for perturbations (denoted by subscript 1) in electric and magnetic fields correspoding to pure transverse oscillations

$$\vec{E}_1 = -\frac{1}{c}\frac{\partial \vec{A}_1}{\partial t}, \quad \vec{B}_1 = \nabla \times \vec{A}_1. \tag{1}$$

Using (1), the linearised Vlasov equation for the electrons (the ions being infinitely massive, stationary and merely forming the neutralising background) with the Bhatnagar-Gross-Krook collision term is given by

$$\frac{\partial F_1}{\partial t} + \vec{v} \cdot \nabla F_1 - \frac{e}{m}\left[\frac{1}{c}\frac{\partial \vec{A}_1}{\partial t} - \frac{1}{c}\vec{v} \times (\nabla \times \vec{A}_1)\right] \cdot \nabla_v F_1 = -\nu_c F_1 \tag{2}$$

where ν_c is a phenomenological collision frequency, and the subscript o denotes the equilibrium conditions. The Bhatnagar-Gross-Krook model used in Eq. (2) describes the relaxation-type collisional effects in a weakly-ionised plasma wherein one may assume that the electrons are deflected by single short-range collisions with neutral particles (which are assumed to be uniformly distributed in space) rather than by multiple scatterings off other charged particles.

From Maxwell's equations for the self-consistent electromagnetic field,

$$\nabla \cdot \vec{E}_1 = -4\pi e \int F_1 \, d\vec{v} \tag{3}$$

$$\nabla \times \vec{E}_1 = -\frac{1}{c}\frac{\partial \vec{B}_1}{\partial t} \tag{4}$$

$$\nabla \times \vec{B}_1 = \frac{1}{c}\frac{\partial \vec{E}_1}{\partial t} + \frac{4\pi e}{c}\int \vec{v} F_1 \, d\vec{v} \tag{5}$$

one obtains, on using (1)

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\vec{A}_1 = \frac{4\pi e}{c}\int \vec{v} F_1 \, d\vec{v}. \tag{6}$$

Let $F_0(\vec{v})$ be isotropic. Upon taking the Fourier-Laplace transforms of Eq. (2), and letting $g(\vec{k}, \vec{v})$ and $\vec{h}(\vec{k})$ be the Fourier transforms of F_1 and $\vec{A}_1$ at $t = 0$, one obtains

$$F_1 = \frac{\frac{e}{mc} s \vec{A}_1 \cdot \nabla_v F_0}{s + i\vec{k} \cdot \vec{v} + \nu_c} + \frac{g(\vec{k},\vec{v}) - \frac{e}{mc}\vec{h} \cdot \nabla_v F_0}{s + i\vec{k} \cdot \vec{v} + \nu_c}. \tag{7}$$

Let $\hat{\varepsilon}$ be a unit vector parallel to $\vec{A}_1$ and perpendicular to $\vec{k}$. Then (7) gives

$$\int (\hat{\varepsilon} \cdot \vec{v}) F_1 \, d\vec{v} = -\frac{e}{mc} s A_1 \int \frac{F_0(\vec{v})}{s + i\vec{k} \cdot \vec{v} + \nu_c} d\vec{v} + $$

$$+ \int \frac{[(\hat{\varepsilon} \cdot \vec{v}) g(\vec{k}, \vec{v}) + \frac{e}{mc} h(\vec{k}) F_0(\vec{v})]}{s + i\vec{k} \cdot \vec{v} + \nu_c} d\vec{v}. \tag{8}$$

Using (8), Eq. (6) gives

$$\vec{A}_1 = \frac{\frac{s}{c^2} h(\vec{k}) + \frac{1}{c^2} \dot{h}(\vec{k}) + \frac{4\pi e}{c} \int \frac{[(\hat{\varepsilon} \cdot \vec{v}) g(\vec{k}, \vec{v}) + \frac{e}{mc} h(\vec{k}) F_0(\vec{v})]}{s + i\vec{k} \cdot \vec{v} + \nu_c} d\vec{v}}{k^2 + \frac{s^2}{c^2} + \frac{\omega_p^2 s}{c^2} \int \frac{F_0(\vec{v})}{s + i\vec{k} \cdot \vec{v} + \nu_c} d\vec{v}} \tag{9}$$

where the dot overhead signifies the time derivate, and

$$\omega_p^2 = \frac{4\pi e^2 n}{m},$$

n being the electron density.

Upon introducing the Landau[6] prescription for the contour of integration[c] (9) gives the dispersion relation

$$1 = - \frac{\omega_p^2 s}{s^2 + k^2 c^2} \int \frac{F_0(\vec{v})\, dv}{s + i\vec{k} \cdot \vec{v} + \nu_c} d\vec{v} \tag{10}$$

or

$$1 = \frac{\omega_p^2 \omega}{k^2 c^2 - \omega^2} \int \frac{F_0(\mu)}{ku - (\omega + i\nu_c)} du \tag{11}$$

where

$$s = - i\omega, F_0(u) = \int F_0(\vec{v})\, \delta(u - \vec{k} \cdot \vec{v})\, d\vec{v}. \tag{12}$$

For a Maxwellian distribution

$$F_0(u) = \frac{e^{-u^2/2v_T^2}}{\sqrt{2\pi v_T^2}} \tag{13}$$

v_T being the electron thermal speed, Eq. (11) becomes

$$1 = \frac{\omega_p^2}{\omega^2 - k^2 c^2} \left(\frac{\omega}{\sqrt{2} k v_T}\right) Z\left(\frac{\omega + i\nu_c}{\sqrt{2} k v_T}\right) \tag{14}$$

where $Z(\beta)$ is the Fried-Conte[7] dispersion function

$$Z(\beta) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-\zeta^2}}{\beta - \zeta} d\zeta. \tag{15}$$

For the transverse oscillations given by (14), $\omega \gg k\, v_T$, (which rules out Landau damping of the transverse oscillations) so noting that

$$Z(\beta) = \frac{1}{\beta}\left(1 + \frac{1}{2\beta^2} + \dots\right) \quad |\beta| \gg 1 \tag{16}$$

Eq. (14) gives

$$1 = \frac{\omega_p^2}{\omega^2 - k^2 c^2} \cdot \frac{\omega}{\omega + i\nu_c}\left[1 + \left(\frac{k v_T}{\omega + i\nu_c}\right)^2 + \dots\right] \tag{17}$$

from which on putting

$$\omega = \omega_r + \omega_i \tag{18}$$

one obtains

$$\omega_r^2 \approx \omega_p^2 + k^2 c^2 + \frac{k^2 v_T^2 \omega_p^2}{\omega_p^2 + k^2 c^2} \tag{19}$$

$$\omega_i \approx -\frac{\nu_c}{2}\left(\frac{\omega_p^2}{\omega_p^2 + k^2 c^2}\right)\left[1 + \frac{3\, k^2 v_T^2}{\omega_p^2 + k^2 c^2}\right]. \tag{20}$$

(19) agrees with that given in Refs. 1—3. However, (20) shows differences with the result given by Jain[4], and latter's calculation does not seem to be completely correct. (20) describes the collisional damping of the transverse oscillations in an electron plasma.

References

1) A. G. Sitenko and K. N. Stepanov, Soviet Phys. JETP **31** (1956) 642;
2) T. Pradhan, Phys. Rev. **107** (1957) 1222;
3) I. B. Bernstein, Phys. Rev. **109** (1958) 10;
4) R. K. Jain, Nucl. Fusion **6** (1966) 15;
5) P. L. Bhatnagar, E. P. Gross, and M. Krook, Phys. Rev. **94** (1954) 511;
6) L. D. Landau, J. Phys. (U. S. S. R.) **10** (1946) 25;
7) B. D. Fried and S. D. Conte, *The Plasma Dispersion Function*, Academic Press, 1962.

KOLIZIONO PRIGUŠENJE TRANSVERZALNIH OSCILACIJA U ELEKTRONSKOJ PLAZMI

B. K. SHIVAMOGGI

Research School of Physical Sciences, Australian National University, Canberra, Australia

UDK 533.951

Originalni znanstveni rad

Razmotren je utjecaj sudara na transverzalne oscilacije elektronske plazme primjenom Bhatnagar-Gross-Krookova modela za koliozni član u Boltzmannovoj jednadžbi. Izvedeni rezultati primijenjeni su na Maxwellovu elektronsku raspodjelu.