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PSEUDOSCALAR MESON DECAY CONSTANTS IN A QUARK CONFINEMENT MODEL*

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We calculate numerical values of the weak leptonic decay constants f_P of the pseudoscalar mesons, using a model of quark confinement in which, in a first approximation, the constituent quark and antiquark each obey a confining relativistic potential.

1. Introduction

The major test ground of *QCD* is its plausible realizability of the quark confinement[1]. There are numerous attempts, however, to bypass the confinement--existence proof and instead to start with a confining mechanism explicitly built into a theory that suitably approximates *QCD*. The best known example is the *MIT* bag model[2], where quarks are treated as the Dirac quanta, free within the bag, and kept inside by the appropriate vacuum pressure. The successful history of the model, with a variety of improving versions, resulted in numerous applications to the meson and baryon spectroscopy and led to the reliable determination of free parameters like the quark masses and the particle radii. In contradistinction to the bag model the confining-potential models of the quark confinement[3] leave much to further elaboration, in particular in the domain of meson spectroscopy.

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An attractive feature of such models is their fixed, and relatively small, number of free parameters. Namely, when using the linear type of potential, $V = V_0 + \lambda r$, or harmonic, $V = V_0 + \lambda r^2$, there are two undetermined parameters, and if one includes also the Coulomb term $-\gamma/r$ one has a third free parameter. This situation, as compared to the determination of the bag radius, separately for each particle under consideration, offers some definite advantage. The absence of the necessity for an empty bag analysis, reflecting the *QCD* phase transition problem, also makes things calculationally simpler.

The decay constants f_P, of the pseudoscalar mesons P, enter the consideration via the matrix element

$$\langle 0 \,|\, A^P_\mu(0) \,|\, P\rangle = \mathrm{i}\,\frac{1}{\sqrt{V}}\,\frac{1}{\sqrt{2E_P}}\, f_P\, p^P_\mu, \tag{1}$$

and their theoretic determination has been approached by many authors[3,4]. In most cases, thereby, the chiral extension of the quark classification group is used to provide information on the values of the ratio between the decay constant of the meson under consideration and the pion. Also, within the *MIT* bag model, there have been a few attempts to explicitely determine f_P values[5].

In the present paper we view the problem from the standpoint of the relativistic confining potential, introduced in the quark confinement models in Ref. 3. The basic aim is to confront and test the model in a comparison with the *MIT* bag approach.

2. *The decay width in the confining-potential model*

The basic ingredient of the model is the assumption that the quark-antiquark pair of which a meson is composed may be viewed as motion of two independent particles, each subjected, in the *CM* frame of the meson, to a relativistic central potential

$$V(\vec{x}) = \frac{1}{2}(1+\beta)\, V(r), \tag{2}$$

where the specified form of $V(r)$ has to be chosen in such a way that an appropriate quark confinement results. Consequently, quark wave functions have to satisfy the Dirac equation extended to an external potential

$$\left[\vec{\alpha}\cdot\vec{p} + \beta m_i + \frac{1}{2}(1+\beta)\, V(r)\right] \psi_i(\vec{x}) = E_i\, \psi_i(\vec{x}), \tag{3}$$

with index i denoting the corresponding quark flavours. Solutions to Eq. (3) can be put into the form:

$$\psi_{im}(x) = N_i \begin{bmatrix} \Phi_i(\vec{x})\, U_m \\ \dfrac{\vec{\sigma}\cdot\vec{p}}{E_i + m_i}\, \Phi_i(x)\, U_m \end{bmatrix}, \tag{4}$$

where $\vec{p}$ is $(-i\,\vec{\nabla})$, m the spin projection and U_m the corresponding Pauli spinor. The functions $\Phi_l(\vec{x})$ have to satisfy the equation

$$\Delta\Phi_l(\vec{x}) + (E_l + m_l)(E_l - m_l - V(r))\,\Phi_l(\vec{x}) = 0. \tag{5}$$

We select the linear potential[3])

$$V(r) = V_0 + \lambda r, \tag{6}$$

and are interested in the ground state solutions, $J^P = 1/2^+$, normalized to unity.

In order to determine f_P we have to find out the width, $\Gamma(P \to \bar{\nu}\, l)$, of the leptonic decay of the pseudoscalar meson P.

We use the phenomenological weak interaction Lagrangian

$$\mathscr{L}(x) = \frac{G}{\sqrt{2}} j_\mu^{lep^+}(x)\, j^{\mu\ had}(x), \tag{7}$$

with the currents

$$j_\mu^{lep}(x) = \bar{\nu}(x)\,\gamma_\mu(1-\gamma_5)\,l(x), \tag{8}$$

and

$$j_\mu^{had}(x) = \bar{q}(x)\,A_{KM}\,\gamma_\mu(1-\gamma_5)\,q(x). \tag{9}$$

In the hadronic current $q(x)$ denotes the six-flavour-quark column, (u, d, s, c, t, b) — transposed, and A_{KM} is the Kobayashi-Maskawa matrix. (Alternatively, in the four-quark approach, one uses the Cabibbo angle introducing matrix).

The state of the pseudoscalar meson P is defined as

$$|P\rangle = \frac{1}{\sqrt{2}}(a_{1/2}^+ b_{-1/2}^+ - a_{-1/2}^+ b_{1/2}^+)\,|0\rangle, \tag{10}$$

where a and b, respectively, denote the creation operators of the corresponding quark (antiquark) flavours, with the spin projections $\pm 1/2$.

The amplitude for this process is given as

$$\begin{aligned} S_{fi} = \frac{G}{\sqrt{2}}(c.\,a.)\int \mathrm{d}^4x\,\langle l\bar{\nu}\,|\,\bar{l}(x)\,\gamma_\mu(1-\gamma_5)\,\nu(x)|\,0\rangle\,\cdot \\ \cdot\,\langle 0\,|\,\bar{q}_i(x)\,\gamma^\mu(1-\gamma_5)\,q_j(x)\,|\,P\rangle, \end{aligned} \tag{11}$$

where $(c.\ a.)$ stands for the appropriate combination of the Cabibbo angles and the indices i, j denote the quark-antiquark flavour composition of the specified pseudoscalar meson.

The decay width is given as

$$\Gamma(P \to \bar{\nu}\, l) = \frac{G^2}{(2\pi)^2}\ (c.\ \ a.)^2 \sum_{s_l, s_\nu} \int \mathrm{d}^3 p_l\, \mathrm{d}^3 p_\nu\, \delta(M - E_l - E_\nu) \cdot$$
$$\frac{m_l m_\nu}{E_l E_\nu} \left| u_{s_l}(p_l)\, \gamma_\mu (1 - \gamma_5)\, v_{s_\nu}(p_\nu) \int \mathrm{d}^3 x\, \mathrm{e}^{-\mathrm{i}(\vec{p}_l^{\,2} + \vec{p}_\nu^{\,2}) \cdot \vec{x}}\, \overline{\psi}_{\bar{j}-1/2}(\vec{x})\, \gamma_\mu (1 - \right.$$
$$\left. - \gamma_5)\, \psi_{l\,1/2}(x) \right|^2. \tag{12}$$

The symbol $\psi_{\bar{j}m}(\vec{x})$ stands for the antiquark wave function and is obtained from the corresponding quark wave function by charge conjugation

$$\psi_{\bar{j}m}(x) = \mathrm{i}\, \gamma_2\, \psi^*_{jm}(x). \tag{13}$$

Integration over the lepton momenta $\vec{p}_l$, $\vec{p}_\nu$, is carried out by transferring to $\vec{p}_+$, $\vec{p}_-$ i. e. the sum and the difference of the momenta, respectively, and then setting $\vec{p}_+ = 0$, in the leptonic part of the amplitude and in $\delta(M - E_l - E_\nu)$. In the hadronic part, however, since the total momentum in this approach is not conserved, this sum can only be considered as randomly distributed around the zero value, corresponding to the decaying meson being phenomenologically at rest.

To compute the numerical value of f_P we make use of the approximate form for the wave function $\Phi_l(\vec{x})$, given as

$$\Phi_l(x) = \left(\frac{\xi_l^2}{\pi}\right)^{3/4} \mathrm{e}^{-\frac{1}{2}\xi_l^2 r^2}, \tag{14}$$

with

$$\xi_l = \left\{\left(\frac{2}{3\sqrt{\pi}}\right) \lambda\, (E_l + m_l)\right\}^{1/3}, \tag{15}$$

where the ground state energy is determined by

$$E_l = m_l + V_0 + 2.3448 \left(\frac{\lambda^2}{E_l + m_l}\right)^{1/3}. \tag{16}$$

As a justification for the adapted approximation we note that in the analysis of the baryon properties[3], this form for the wave function yielded only about 6% discrepancy when compared to the results obtained with the exact solutions for $\Phi_l(\vec{x})$.

The decay width becomes

$$\Gamma (P \to \bar{\nu}_l\, l) = \frac{G^2}{2\pi} (c.\ a.)^2 \left(1 - \frac{m_l^2}{M_P^2}\right)^2 N_i^2 N_j^2 \cdot$$
$$\cdot \int \left(\frac{\xi_i^2 \xi_j^2 r^2}{(E_i + m_i)(E_j + m_j)} - 1\right)^2 \cdot \Phi_i^2 (\vec{x}) \Phi_j^2 (\vec{x})\, \mathrm{d}^3 x \tag{17}$$

where the normalization to unity implies

$$N_i^{-2} = \int \left(1 + \frac{\xi_i^4 r^2}{(E_i + m_i)^2}\right) \Phi_i^2 (\vec{x})\, \mathrm{d}^3 x. \tag{18}$$

Comparison between Eq. (17) and the standard expression for the decay width

$$\Gamma (P \to \bar{\nu}_l\, l) = \frac{G^2}{8\pi} (c.\ a.)^2 M_P m_l^2 \left(1 - \frac{m_l^2}{m_P^2}\right)^2 f_P^2, \tag{19}$$

where f_P enters via its definition through the axial current matrix element (1), yields

$$f_P^2 = \frac{4}{m_P} N_i^2 N_j^2 \int \left(\frac{\xi_i^2 \xi_j^2}{(E_i + m_i)(E_j + m_j)} - 1\right)^2 \Phi_i^2 (\vec{x}) \Phi_j^2 (\vec{x})\, \mathrm{d}^3 x \tag{20}$$

3. *Numerical results and conclusion*

When the quark masses and the confining-potential parameters are taken over from the baryon spectroscopy analysis[3], $m_u = m_d = 0.272$ GeV, $m_s = 0.703$ GeV, $V_0 = -0.281$ GeV, $\lambda = 0.201$ GeV^2, we obtain: $f_\pi = 0.268$ GeV, $f_K = 0.177$ GeV. Confronted with the experimental data, $f_\pi^{exp} = 0.129$ GeV, $f_K^{exp} = 0.165$ GeV, the pion decay constant f_π poorly stands the test. Changing the value of λ to $\lambda \approx \approx 0.050$ GeV^2, however, and leaving other parameters unchanged sets $f_\pi \approx f_\pi^{exp}$, but we discard that option on the ground that it would lead to an unsatisfactory baryon spectroscopy. This brings to the mind the *MIT* bag model calculation[4] which produces, same as in the present approach, a fairly good f_K value and a poor f_π. This situation further confirms the conclusion, made within the spectroscopic analysis of the *MIT* bag model limitations, that the pion can not be adequately treated solely within the frame of a quark confinement model.

As regards the decay constants f_P for the pseudoscalar mesons which contain heavy quarks, the presented method, when the heavy quark masses are assigned the usual values[6]: $m_c = 1.5$ GeV, $m_b = 4.5$ GeV, yields: $f_D = 0.108$ GeV, $f_F = 0.132$ GeV, $f_B = 0.078$ GeV. We note that the results for f_D and f_F are somewhat different from our earlier estimate[5], obtained within the scope of a modified bag model, while f_B agrees with the corresponding value there.

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KONSTANTE RASPADA PSEUDOSKALARNIH MEZONA U MODELU KVARKNOG KONFAJNMENTA

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Numeričke vrijednosti konstanata slabih leptonskih raspada pseudoskalarnih mezona f_P određene su u okviru modela kvarknog konfajnmenta u kome se konstitucioni kvark i antikvark, pojedinačno, podvrgavaju jednom usrednjenom relativističkom potencijalu.